Machine Learning the S-Matrix

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Outline

1. S-matrix bootstrap

Based on “Reconstructing S-matrix Phases with Machine Learning”

- arXiv:2308.09451
- with Aurelian Dersy (Harvard) and Sasha Zhiboedov (CERN)

2. Simplifying spinor-helicity amplitudes

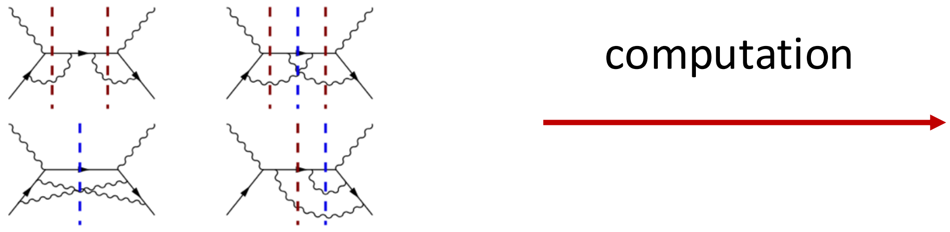
Based on “Learning the simplicity of scattering amplitudes”

- arXiv:2408.04720
- with Aurelian Dersy (Harvard) and Cliff Cheung (Caltech)

The S-Matrix

The S-matrix is the fundamental object of Quantum Field Theory

- A lot is known about it from pertubation theory (Feynman diagrams)



- spinor-helicity amplitudes
- twistor space
- mulitple-polylogarithms
- amplituhedron

- Some things are known/conjectured about it non-perturbatively
  - e.g. it should be unitary, analytic and satisfy crossing relations
  - it cannot grow with energy faster than $\log^2 E$ (Froissart bound)

Can ML help understand the S-matrix?

1. Can ML help with the non-perturbative S-matrix?

[Dersy, MDS, Zhiboedov, 2308.09451]

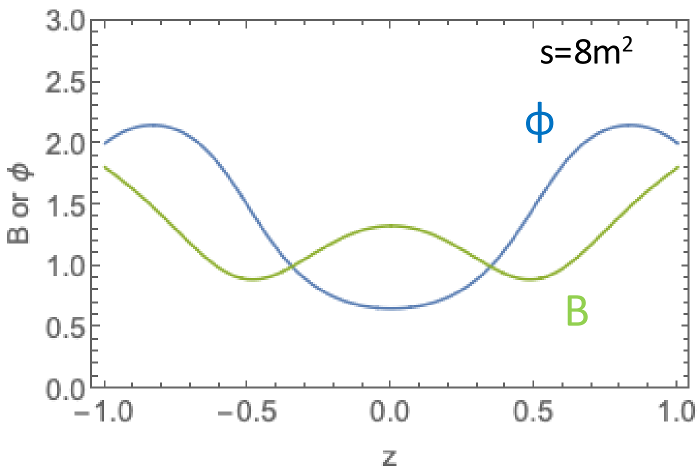
What does this mean?

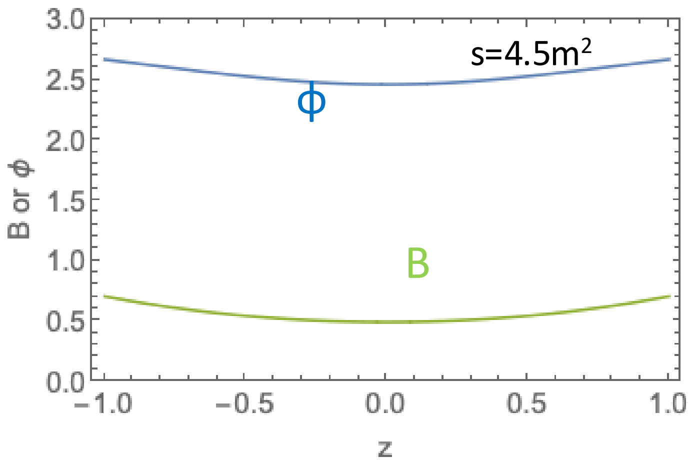
- Use analyticity, unitarity, crossing symmetry to determine S

Example questions:

For a given cross section $\sigma = |A|^2$

- Does there always exist a phase ϕ so that $A = B\, e^{i\phi}$?
- e.g. Penedones et al 1708.06756, Fitzpatrick et al 2207.12448 found (B, ϕ) pairs





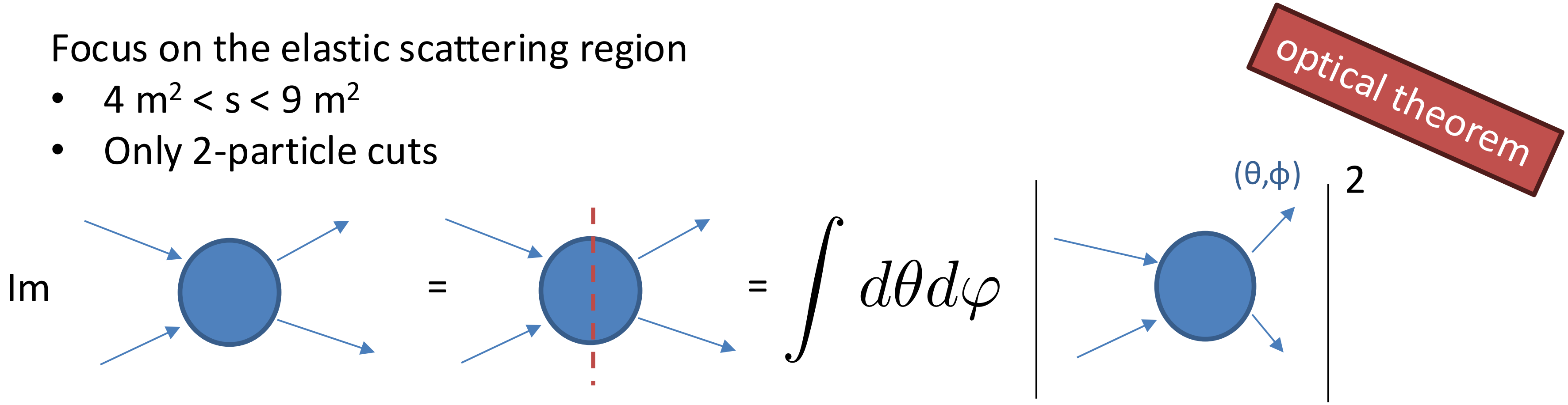
Two questions ML can help with

1. Can we determine ϕ from B?
2. Can there be many phases ϕ_1, ϕ_2, ... for the same B?

Elastic scattering

Focus on the elastic scattering region

- $4\,m^2 < s < 9\,m^2$
- Only 2-particle cuts



- Only states with fixed total energy are relevant
- States characterized by scattering angle z=cosθ

We write $\mathcal{M}(z) = B(z) e^{i\phi(z)}$

Unitarity constraint $\mathrm{Im}\mathcal{M}(z) = \int d\Pi \, |B|^2$

$$\Rightarrow \; B(z_{12}) \sin\phi(z_{12}) = \frac{1}{4\pi}\int d\Omega_3 B(z_{13}) B(z_{23}) \cos(\phi(z_{13}) - \phi(z_{23}))$$

$$z_2 = z z_1 + \sqrt{1-z^2}\sqrt{1-z_1^2}\cos\phi_1$$

$$\Rightarrow \; \sin\phi(z) = \frac{\int_{-1}^{1} dz_1 \int_0^{2\pi} d\phi_1 B(z_1) B(z_2) e^{i\phi(z_1) - i\phi(z_2)}}{4\pi B(z)}$$

Does ϕ(z) exist satisfying this equation for a given B(z)?

Elastic scattering

set phase to zero

$$\sin\phi(z) = \frac{\int_{-1}^{1} dz_1 \int_0^{2\pi} d\phi_1 B(z_1) B(z_2) e^{i\phi(z_1) - i\phi(z_2)}}{4\pi B(z)} \leqslant \max_z \underbrace{\frac{\int_{-1}^{1} dz_1 \int_0^{2\pi} d\phi_1 B(z_1) B(z_2)}{4\pi B(z)}}_{\equiv \sin\mu}$$

Martin (1969) proved that

- If sinμ < 1 for a given B(z) then there always exists a phase ϕ(z)
- easy to find examples of B(z) with sin μ > 1 for which no phase exists

e.g. if B(z) = B and ϕ(z)= ϕ are constants then

$$\sin\varphi = \frac{\int_{-1}^{1} dz_1 \int_0^{2\pi} d\phi_1 B^2 e^{i(\varphi - \varphi)}}{4\pi B} = B \;= \sin\mu$$

⇨ If B >1 then no phase exists

- If sinμ < 0.79 then the phase is unique
  - Later, Gangal and Kupsch (1984) extended uniquess proof to sin μ < 0.89
  - Crichton (1966) found an example with multiple phases for same the B with sin μ =3.2
  - Atkinson (1977) found an example with multiple phases with sinμ = 2.15

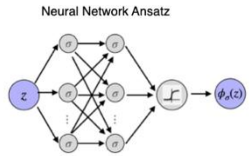
Open questions

- Can **0.89** be raised?
- Can **2.15** be lowered?
- How can we construct ϕ when it exists?

Can we find ϕ(z) given B(z) with ML? ... Yes!

[Dersy, MDS, Zhiboedov, 2308.09451]

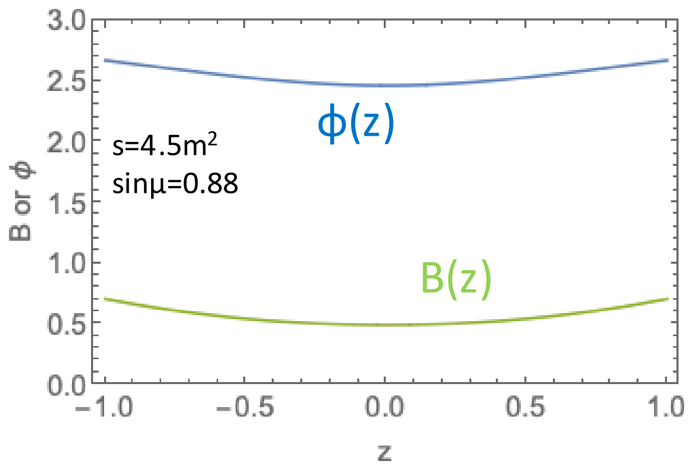
- Parametrize ϕ(z) as a neural network

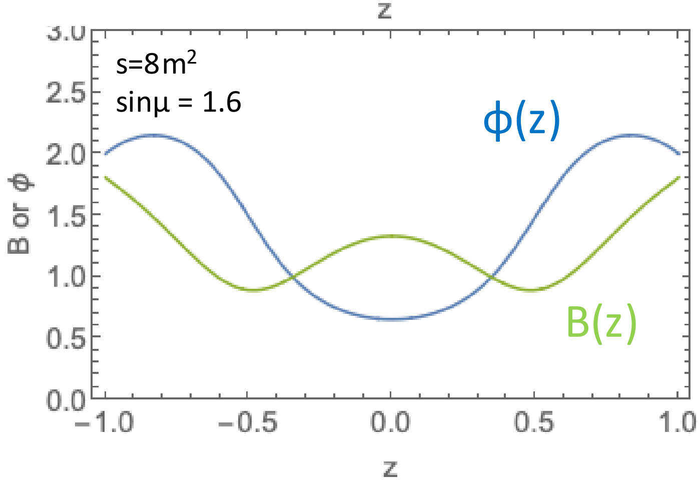


- Loss function is unitarity condition

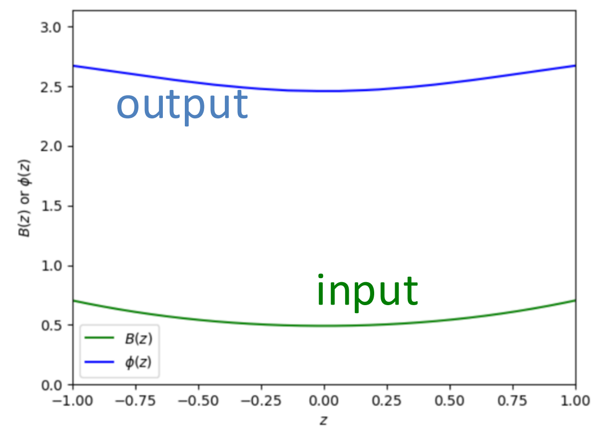
$$\mathscr{L} = \mathbb{E}\left|\left|B(z)\sin\phi(z) - \frac{1}{4\pi}\int_{-1}^{1} dz_1 \int_0^{2\pi} d\phi_1 B(z_1)B(z_2)\cos\left(\phi(z_1)-\phi(z_2)\right)\right|\right|^2$$

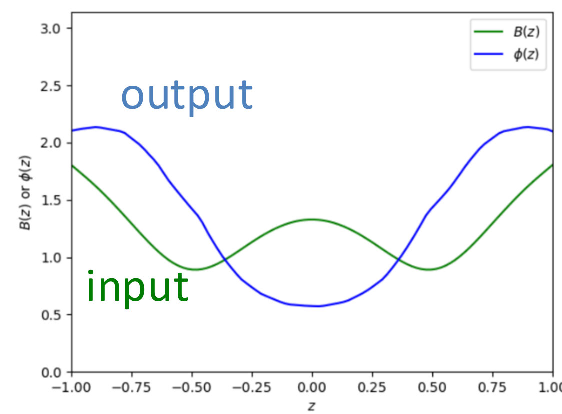
some known examples





ML solutions





excellent agreement with known results

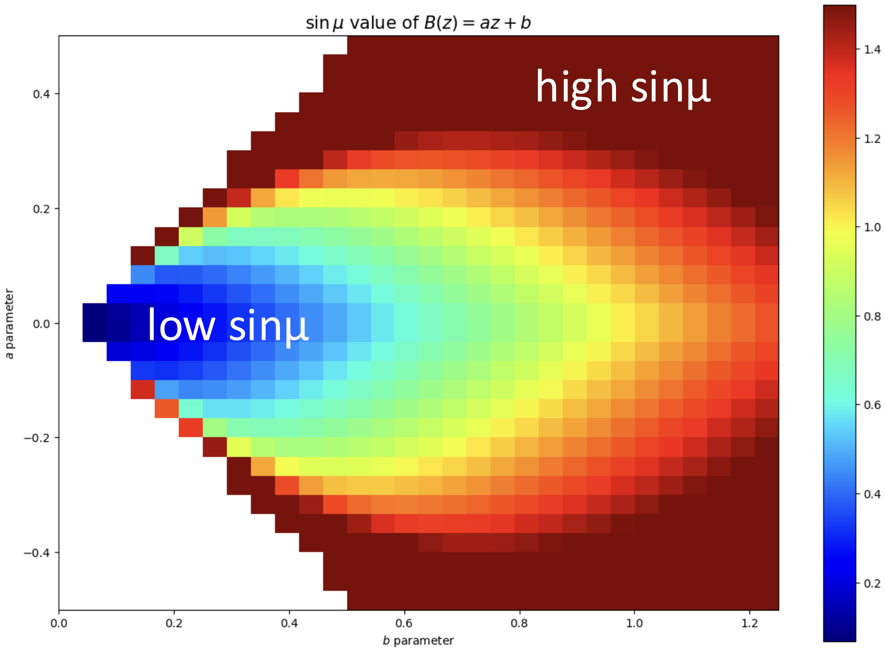
Loss as a proxy for sinμ

Consider functions B(z) = a z + b

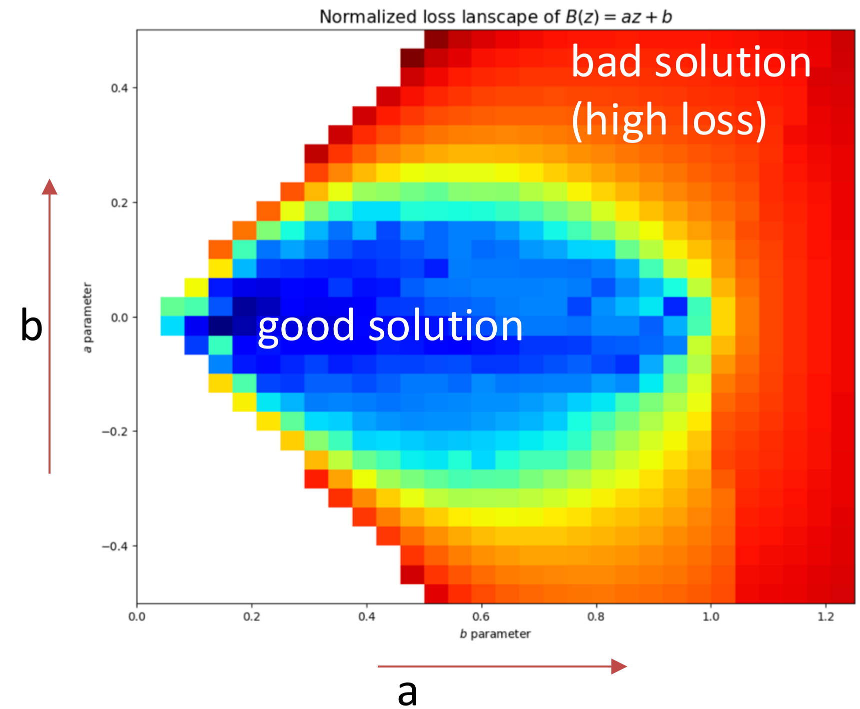
- Use ML to find ϕ(z)

$$\sin\mu = \max_z \frac{\int_{-1}^{1} dz_1 \int_0^{2\pi} d\phi_1 B(z_1) B(z_2)}{4\pi B(z)}$$

Contours of sin μ



Loss landscape from ML search for ϕ



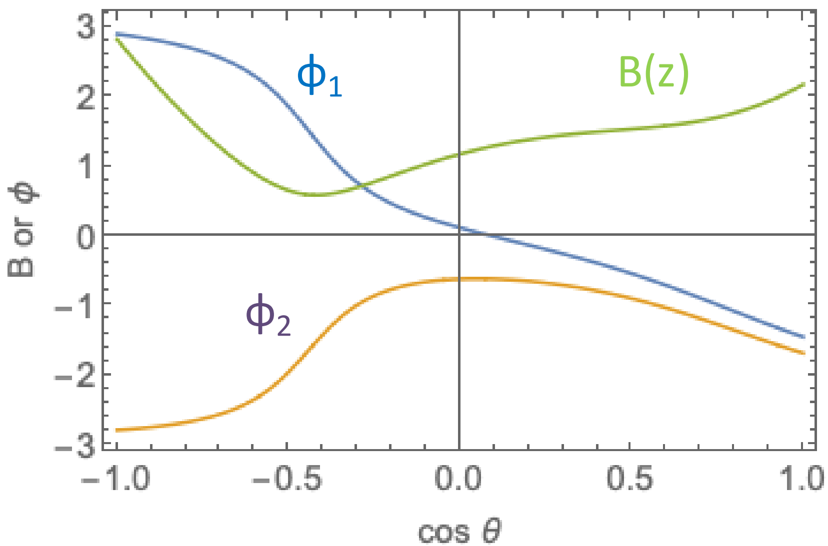
- Totally different things have similar contours
- Don't need an exact solution to learn something:
  - suggests high sin μ solutions will be hard to find

Is the phase always unique? No!

Crichton (1966) found a phase-ambigous amplitude with only 2 partial waves

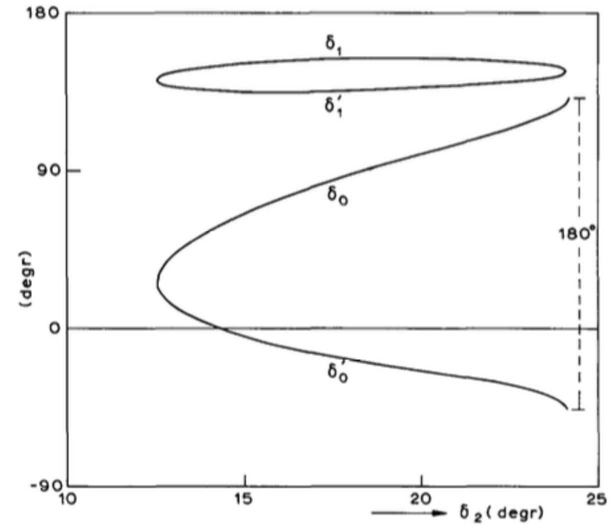
$$\delta_2 = \delta_2' = 20^\circ = 0.349066$$

$$\delta_0 = -0.407703, \quad \delta_1 = -0.758247, \quad \delta_0' = 1.72571 \quad \delta_1' = -0.463483$$

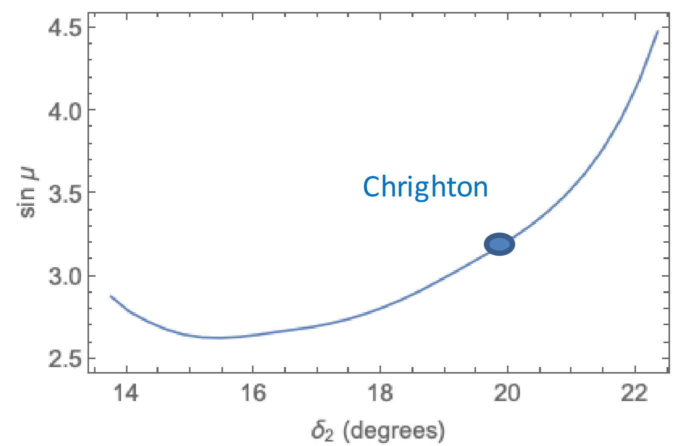


[Atkinson 1973]

found a family of solutions



Changing δ_2 changes the solution along a curve [Atkinson 1973]



Phase ambiguities with ML

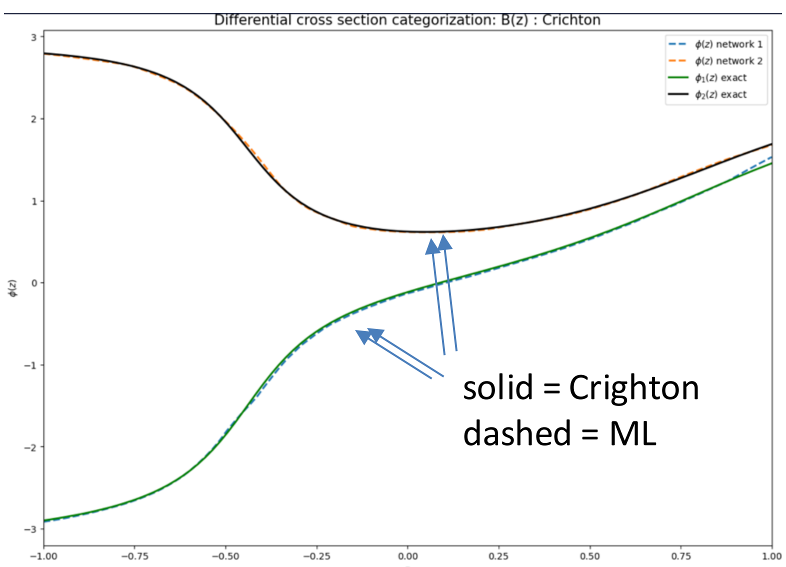
Given B(z), find two phases $\phi_1(z)$ and $\phi_2(z)$

- Unitarity condition loss for each ϕ:

$$\mathscr{L} = \mathbb{E}\left|\left|B(z)\sin\phi(z) - \frac{1}{4\pi}\int_{-1}^{1} dz_1 \int_{0}^{2\pi} d\phi_1 B(z_1)B(z_2)\cos\left(\phi(z_1) - \phi(z_2)\right)\right|\right|^2$$

- Add repulsive loss to keep solutions apart

$$\mathcal{L}_R = \mathbb{E}_z\,||d\left(\phi_1(z), \phi_2(z)\right)||^{-p} + \mathbb{E}_z\,||d\left(\pi - \phi_1(z), \phi_2(z)\right)||^{-p}$$



- Reduce repulsion to zero to find exact solutions

- Two solutions found with ML agree exactly with Crighton's
- We input B(z) and find phases

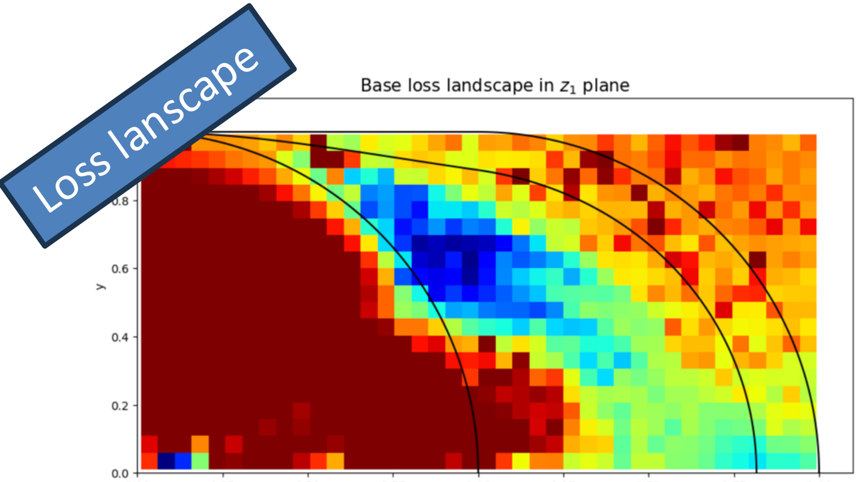
Machine Learning approach

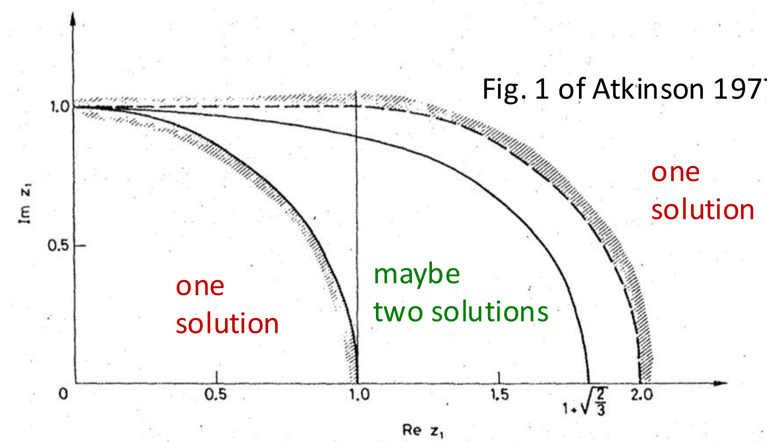
Atkinson (1977): consider a class of functions based on a single complex number z_1

$$A_1(z) = \frac{z - z_1}{1 - z_1} F(z) \qquad A_2(z) = \frac{z - z_1^\star}{1 - z_1^\star} F(z)$$

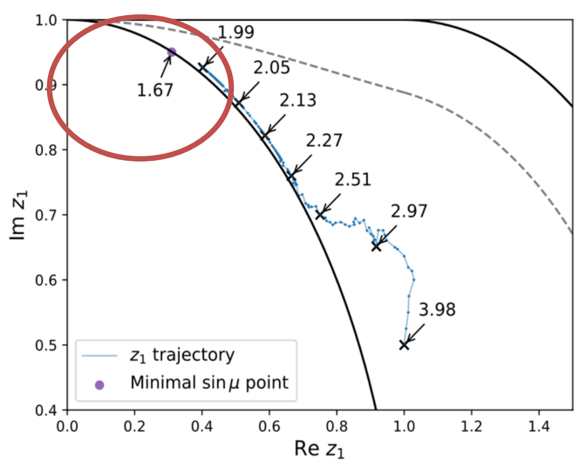
$$B(z) = |A_1(z)| = |A_2(z)|$$

Loss = unitarity for ϕ1 + unitarity for ϕ2 + ϕ1-ϕ2 repulsive term





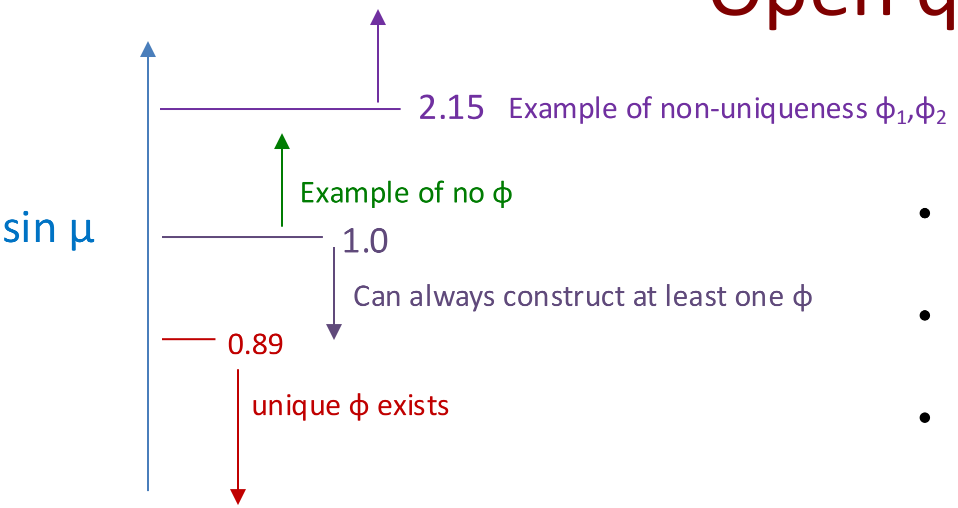
Gradient descent in sin μ



First improvement on phase-ambiguity bound in 50 years!

- Crichton (1966), sin μ =3.2
- Atkinson (1977), sin μ = 2.3
- MDS et al. (2023), sinμ = 1.67

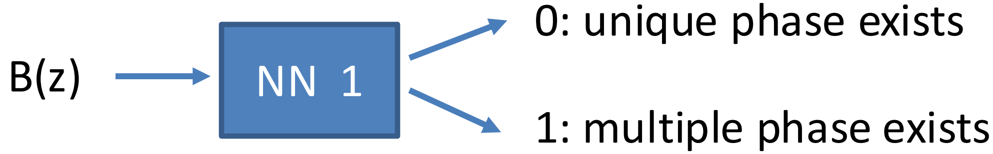
Open questions



- Can 2.15 be lowered?
  - Yes: to 1.64 at least
- Can 0.89 be raised?
  - Uniqueness proof hard to answer with ML
- How can we construct ϕ(z) given B(z)
  - Using ML ✓

New questions

- What properties to the phase ambiguous solutions have?
- Is there a better way to characterize solutions than sinμ?
  - Train NN 1



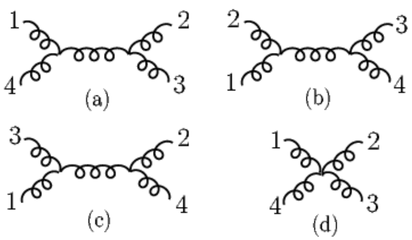
  - Train NN 2 to figure out what NN 1 is doing
    - Symbolic regression?
    - Can we learn a better indicator for phase determination than sinμ?

2. Spinor-helicity amplitudes

The S matrix is mostly understood perturbatively

calculate some Feynman diagrams for
4 gluon scattering

get a messy answer



$$= -\frac{\langle 12\rangle\langle 13\rangle\langle 24\rangle[13][24]}{\langle 23\rangle\langle 34\rangle^2[12][23]} + \frac{\langle 12\rangle\langle 13\rangle\langle 24\rangle[14]}{\langle 23\rangle\langle 34\rangle^2[12]} - \frac{\langle 12\rangle\langle 24\rangle[13][24]^2}{\langle 34\rangle^2[12][14][23]} + \frac{\langle 12\rangle\langle 24\rangle[24]}{\langle 34\rangle^2[12]} + \frac{\langle 12\rangle[13][24][34]}{\langle 34\rangle[12][14][23]}$$
$$- \frac{\langle 12\rangle[34]}{\langle 34\rangle[12]} - \frac{\langle 13\rangle\langle 14\rangle[13][34]}{\langle 34\rangle^2[12][23]} - \frac{\langle 13\rangle\langle 24\rangle[13][24][34]}{\langle 34\rangle^2[12][14][23]} + \frac{\langle 13\rangle\langle 24\rangle[34]}{\langle 34\rangle^2[12]} - \frac{\langle 13\rangle\langle 24\rangle[13][24]}{\langle 34\rangle^2[12]^2} + \frac{\langle 13\rangle\langle 24\rangle[14][23]}{\langle 34\rangle^2[12]^2}$$
$$+ \frac{\langle 13\rangle[13][34]^2}{\langle 34\rangle[12][14][23]} - \frac{\langle 14\rangle\langle 23\rangle[34]}{\langle 34\rangle^2[12]} - \frac{\langle 23\rangle\langle 24\rangle[13][24]^2}{\langle 34\rangle^2[12]^2[14]} + \frac{\langle 23\rangle\langle 24\rangle[23][24]}{\langle 34\rangle^2[12]^2} + \frac{\langle 23\rangle[13][24][34]}{\langle 34\rangle[12]^2[14]} - \frac{\langle 23\rangle[23][34]}{\langle 34\rangle[12]^2}$$

simplify!

$$= -\frac{\langle 12\rangle[34]^2}{\langle 34\rangle[14][23]}$$

Parke-Taylor Formula

6-point amplitude

really big mess
(9 pages Parke, Taylor 1985)

Our result has succesfully passed both these numerical checks. Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

Parke,Taylor (1985)

Six gluon scattering

Hundreds of Feynman diagrams

Simple !

$$|\mathcal{M}_6(1^-2^-3^+4^+5^+6^+)|^2 = \frac{(p_1 \cdot p_2)^3}{(p_2 \cdot p_3)(p_3 \cdot p_4)(p_4 \cdot p_5)(p_5 \cdot p_6)(p_6 \cdot p_1)}$$

We do not expect such a simple expression for the other helicity amplitudes. Also, we challenge the string theorists to prove more rigorously that eqn(3) is correct.

Parke and Taylor (1986)

- simpler form suggests deeper structure
- is there a better way to do the calculation?
    - In this case, yes! (BCFW recursion)

largely inspired the modern amplitudes program
(amplituhedron, tropical geometry, etc.)

Parke and Taylor saw a pattern

- Then they guessed a simplified form
- That’s just what transformers do!

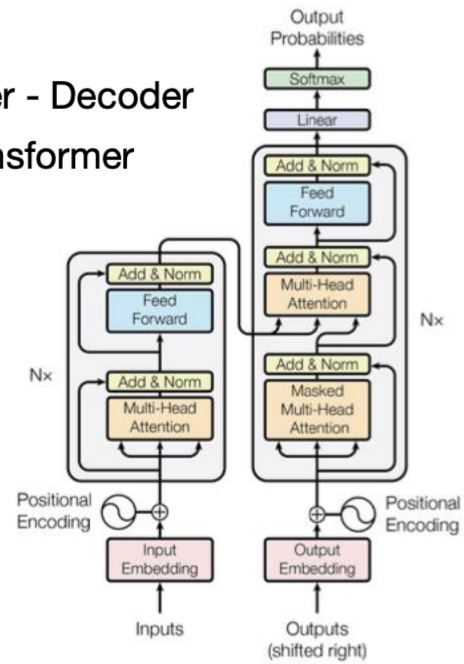
Simplification with transformers

Encoder - Decoder Transformer

Idea introduced for integration + ODE's

[1912.01412 by Lample, Charton]

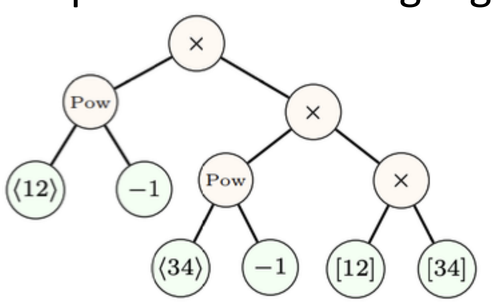
$$\mathcal{M} = \frac{\langle 12\rangle^3 [13]}{\langle 23\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [23]} + \frac{\langle 12\rangle^3 [14][25]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][45]} - \frac{\langle 12\rangle^2 [13][24][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][23][45]} + \frac{\langle 12\rangle^2 [13][25][34]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][23][45]} - \frac{\langle 12\rangle^2 \langle 23\rangle [13][45]}{\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [14][15]} - \frac{\langle 12\rangle^2 [45]}{\langle 15\rangle\langle 34\rangle\langle 35\rangle [15]}$$



$$\overline{\mathcal{M}} = -\frac{\langle 12\rangle^3}{\langle 15\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle}$$

Mathematical expressions are language:

$$\frac{[12][34]}{\langle 12\rangle\langle 34\rangle} =$$



$$= [\text{'mul'}, \text{'pow'}, \text{'ab12'}, \text{'-1'}, \text{'mul'}, \cdots]$$

Generate training data by scrambling

scramble

Input:
$$\frac{-\langle 34\rangle^2[12]^3[34][35] + \langle 34\rangle^2[12]^2[13][23][45] - \langle 34\rangle^2[12]^2[14][23][35]}{\langle 23\rangle\langle 34\rangle[13]^2[34][35] + \langle 23\rangle\langle 45\rangle[13]^2[35][45] - \langle 23\rangle\langle 45\rangle[13][14][35]^2}$$

$$[15] \to \frac{[14][35]-[13][45]}{[34]}$$

Schouten

$$\frac{-\langle 34\rangle^2[12]^3[35] - \langle 34\rangle^2[12]^2[15][23]}{\langle 23\rangle\langle 34\rangle[13]^2[35] - \langle 23\rangle\langle 45\rangle[13][15][35]}$$

$$[25] \to \frac{[12][35]+[15][23]}{[13]}$$

Schouten

$$\langle 24\rangle \to \frac{\langle 45\rangle[15]-\langle 34\rangle[13]}{[12]}$$

Momentum conservation

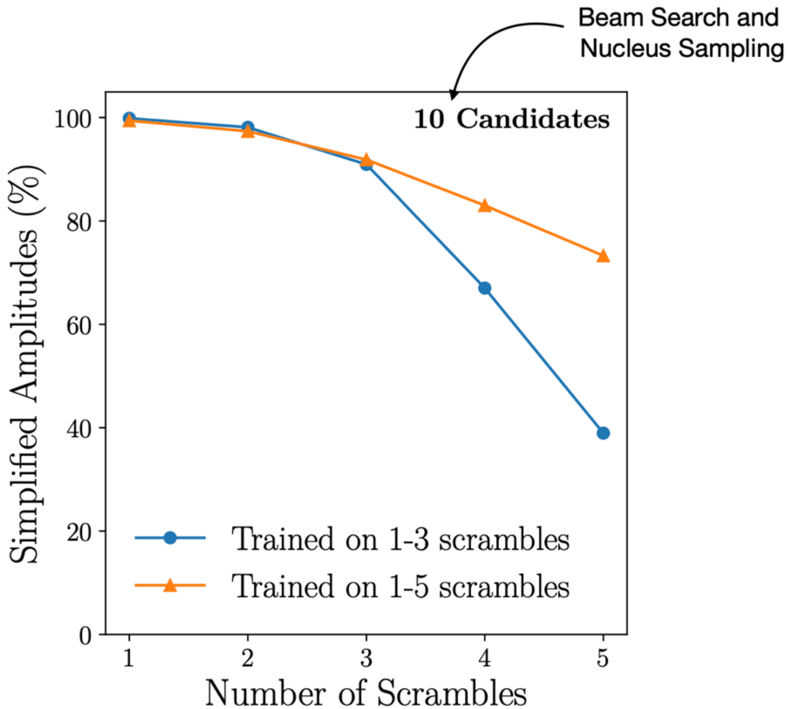
$$-\frac{\langle 34\rangle^2[12]^2[25]}{\langle 23\rangle\langle 34\rangle[13][35] - \langle 23\rangle\langle 45\rangle[15][35]}$$

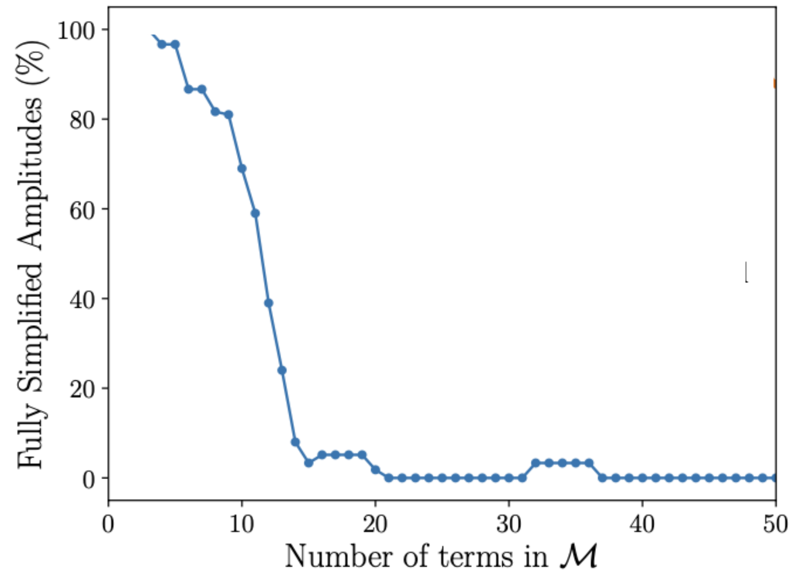
$$\frac{\langle 34\rangle^2[12][25]}{\langle 23\rangle\langle 24\rangle[35]}$$

Desired output

transformer will learn to unscramble (translate)

Transformer performance





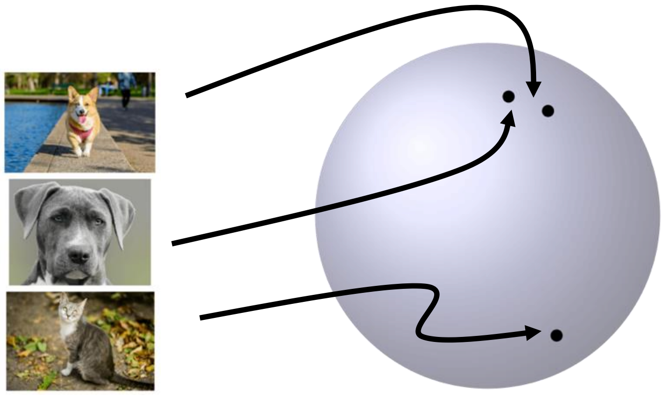
- Does not generalize well
- If trained on 3 scrambles can only unscamble 3 times
- Can't train for > 5 scrambles (exponential amount of training data required)

- Can't simplify expressions more than 15 terms
- Expressions of interest can be very long
  - hundres of terms
- Need new techniques for organizing transformer
  - We use **contrastive learning**

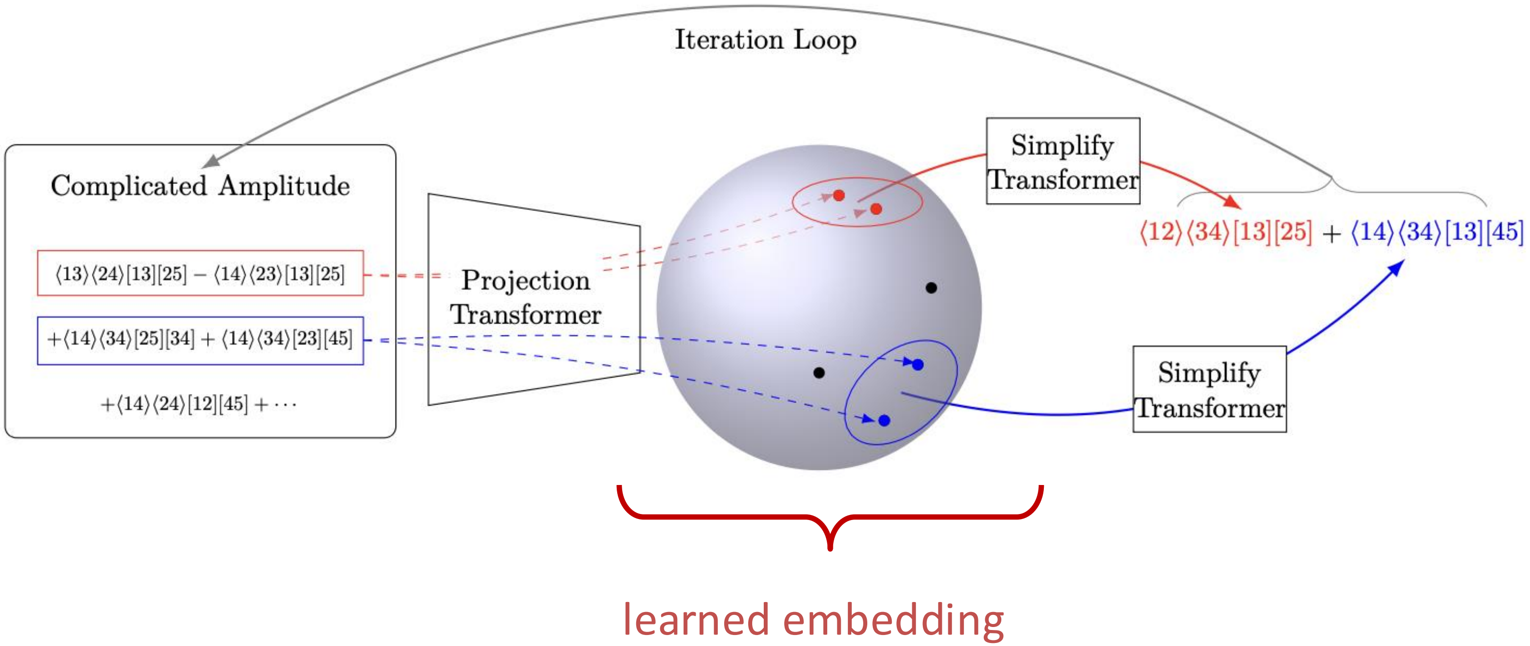
Contrastive learning

Learn an embedding so that terms that are similar are close

- similar = appear in some identity
- close = metric on embedding space



1. Pick subset of terms expected to simplify
2. Apply transformer
3. Repeat



Contrastive learning

Supervised contrastive loss

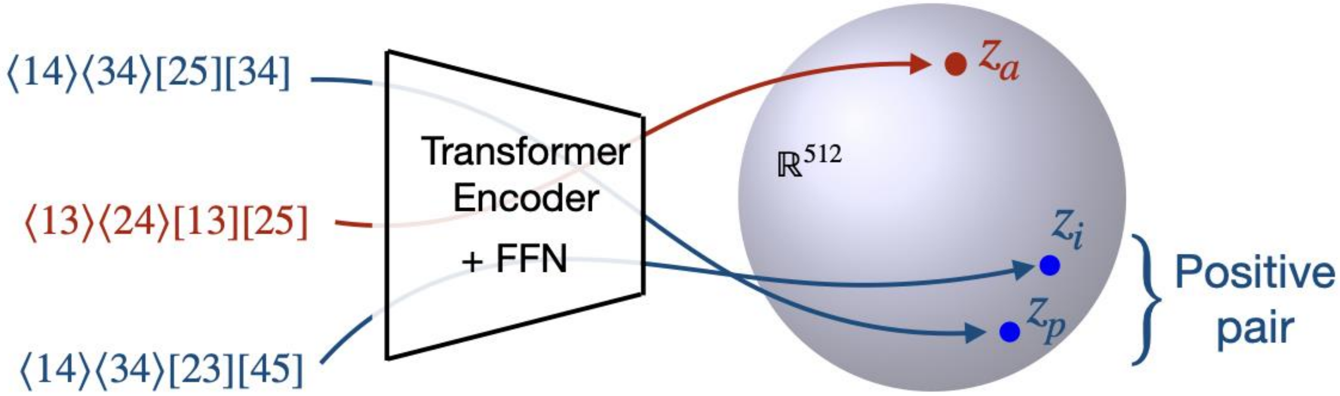
p and i in same identity

$$\mathcal{L}_{\text{con}} = -\frac{1}{|I|}\sum_{i\in I}\frac{1}{|P(i)|}\sum_{p\in P(i)}\log\frac{\exp(s(\boldsymbol{z}_i,\boldsymbol{z}_p)/\tau)}{\sum_{a\in A(i)}\exp(s(\boldsymbol{z}_i,\boldsymbol{z}_a)/\tau)}$$

a and i not in any identites

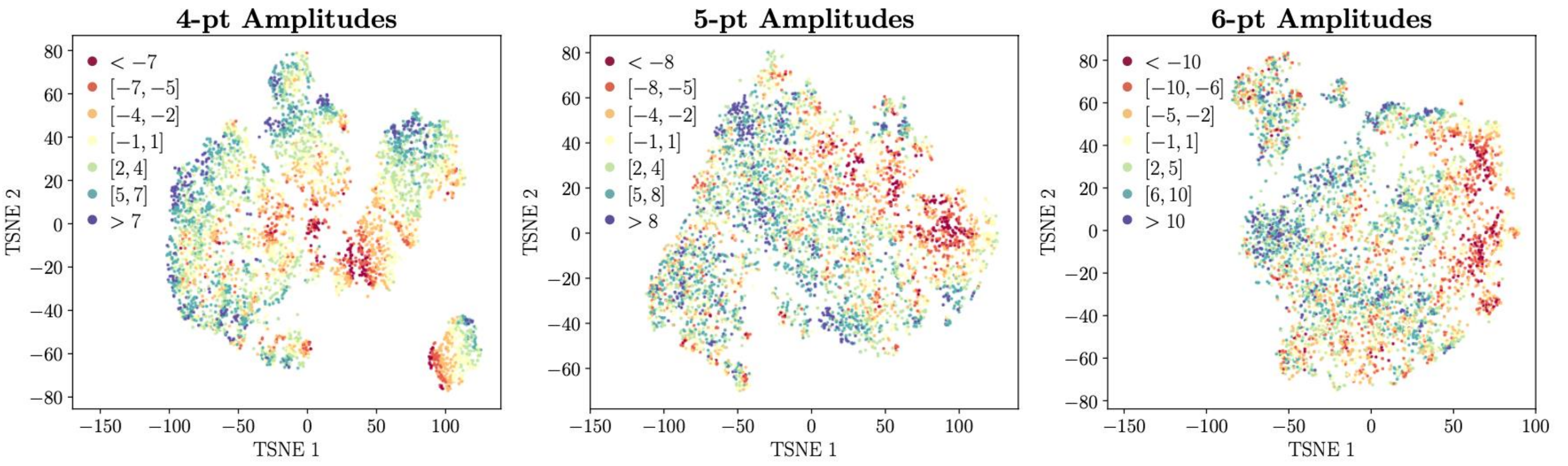
$$s(\boldsymbol{z}_1,\boldsymbol{z}_2) = \frac{\boldsymbol{z}_1\cdot\boldsymbol{z}_2}{||\boldsymbol{z}_1||\,||\boldsymbol{z}_2||}$$

“cosine” similarity



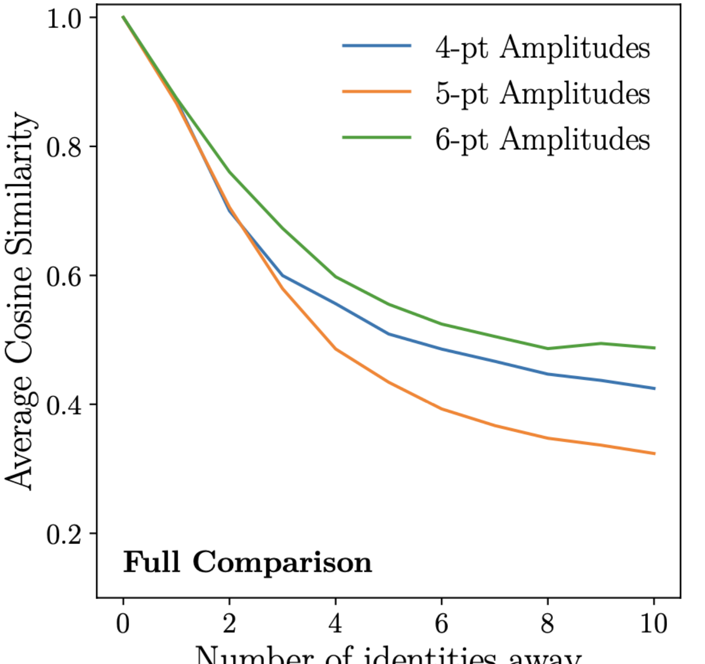
Cross-checks

- t-SNE on latent space



- color = mass dimension
  - Learns dimensional analysis
  - Learns other features as well

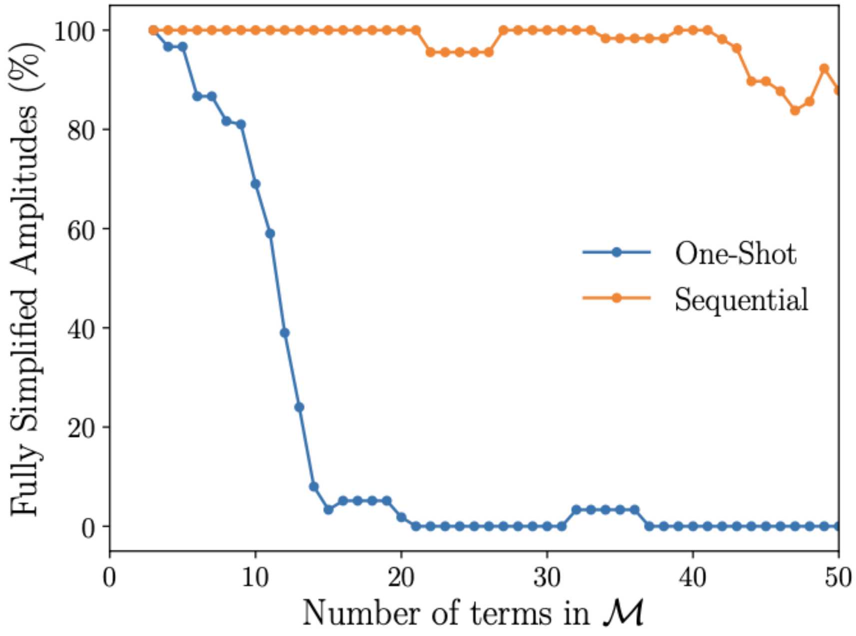
distance inversely correlated with complexity



Performance

Does it work?

Yes!
It can simplify long expressions now



Example application: 5 gluon amplitude

$$\mathcal{M} = \frac{\langle 12\rangle^3 [13]}{\langle 23\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [23]} + \frac{\langle 12\rangle^3 [14][25]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][45]} - \frac{\langle 12\rangle^3 [15][24]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][45]} - \frac{\langle 12\rangle^2 [13][34]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [14][23]} + \frac{\langle 12\rangle^2 [13]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]}$$

$$- \frac{\langle 12\rangle^2 [13][24][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][23][45]} + \frac{\langle 12\rangle^2 [13][25][34]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][23][45]} + \frac{\langle 12\rangle^2 [14][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][45]} - \frac{\langle 12\rangle^2 [15][34]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][45]} - \frac{\langle 12\rangle^2 [13][45]}{\langle 23\rangle\langle 24\rangle\langle 35\rangle [12][23]}$$

$$- \frac{\langle 12\rangle^2\langle 23\rangle [13][45]}{\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [14][15]} - \frac{\langle 12\rangle^2 [45]}{\langle 15\rangle\langle 34\rangle\langle 35\rangle [15]} - \frac{\langle 12\rangle^2 [13][34][45]}{\langle 15\rangle\langle 24\rangle\langle 35\rangle [14][15][23]} + \frac{\langle 12\rangle^2 [13][45]}{\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][15]} + \frac{\langle 12\rangle^2 [14][45]}{\langle 15\rangle\langle 23\rangle\langle 35\rangle [12][15]}$$

$$+ \frac{\langle 12\rangle^2\langle 34\rangle [13][34][45]}{\langle 15\rangle\langle 23\rangle\langle 24\rangle\langle 35\rangle [12][15][23]} - \frac{\langle 12\rangle^2\langle 15\rangle [15]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]} + \frac{\langle 12\rangle^2\langle 23\rangle [23]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]} - \frac{\langle 12\rangle^2\langle 23\rangle [25][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][45]}$$

$$+ \frac{\langle 12\rangle^2\langle 23\rangle [15][24][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][45]} + \frac{\langle 12\rangle^2\langle 23\rangle [14][23][25]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[45]} - \frac{\langle 12\rangle^2\langle 23\rangle [15][23][24]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[45]} - \frac{\langle 12\rangle^2\langle 23\rangle [25]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]}$$

$$+ \frac{\langle 12\rangle^2\langle 23\rangle [15][24]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][14]} + \frac{\langle 12\rangle^2 [24]}{\langle 13\rangle\langle 35\rangle\langle 45\rangle [12]} + \frac{\langle 12\rangle^2\langle 34\rangle [13][24][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][23]} - \frac{\langle 12\rangle^2 [45]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle [12]} + \frac{\langle 12\rangle^2\langle 23\rangle [13][24][45]}{\langle 13\rangle\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][14][15]}$$

$$+ \frac{\langle 12\rangle^2 [24][45]}{\langle 13\rangle\langle 15\rangle\langle 35\rangle [12][15]} + \frac{\langle 12\rangle^2\langle 34\rangle [13][24][34][45]}{\langle 13\rangle\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][14][15][23]} - \frac{\langle 12\rangle\langle 13\rangle [13][34][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][23][45]} + \frac{\langle 12\rangle\langle 13\rangle [13][15][34]^2}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][23][45]}$$

$$+ \frac{\langle 12\rangle\langle 13\rangle\langle 23\rangle [13][34]}{\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][14]} + \frac{\langle 12\rangle\langle 13\rangle [34]}{\langle 15\rangle\langle 34\rangle\langle 35\rangle [12]} + \frac{\langle 12\rangle\langle 13\rangle [13][34]^2}{\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][14][23]} + \frac{\langle 12\rangle\langle 14\rangle\langle 23\rangle [13][45]}{\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][15]}$$

$$+ \frac{\langle 12\rangle\langle 14\rangle [14][45]}{\langle 15\rangle\langle 34\rangle\langle 35\rangle [12][15]} + \frac{\langle 12\rangle\langle 14\rangle [13][34][45]}{\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][15][23]} - \frac{\langle 12\rangle\langle 15\rangle [13][15][34]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][23]} + \frac{\langle 12\rangle\langle 23\rangle [13][24][34][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][23][45]}$$

$$- \frac{\langle 12\rangle\langle 23\rangle [34][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][45]} + \frac{\langle 12\rangle\langle 23\rangle [15][34]^2}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][45]} - \frac{\langle 12\rangle\langle 23\rangle [13][24][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[45]} + \frac{\langle 12\rangle\langle 23\rangle [14][23][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[45]}$$

$$- \frac{\langle 12\rangle\langle 23\rangle [35]}{\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]} + \frac{\langle 12\rangle\langle 23\rangle [15][34]}{\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][14]} + \frac{\langle 12\rangle [34]}{\langle 35\rangle\langle 45\rangle [12]} + \frac{\langle 12\rangle\langle 34\rangle [13][34]^2}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][23]} - \frac{\langle 12\rangle\langle 23\rangle^2 [13][23][45]}{\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][14][15]}$$

$$+ \frac{\langle 12\rangle\langle 23\rangle^2 [13][24][35]}{\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][14][15]} - \frac{\langle 12\rangle\langle 23\rangle [23][45]}{\langle 15\rangle\langle 34\rangle\langle 35\rangle [12][15]} + \frac{\langle 12\rangle\langle 23\rangle [24][35]}{\langle 15\rangle\langle 34\rangle\langle 35\rangle [12][15]} + \frac{\langle 12\rangle\langle 23\rangle [13][24][34][35]}{\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][14][15][23]}$$

$$+ \frac{\langle 12\rangle\langle 23\rangle\langle 45\rangle [13][45]^2}{\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][14][15]} + \frac{\langle 12\rangle [34][45]}{\langle 15\rangle\langle 35\rangle [12][15]} + \frac{\langle 12\rangle\langle 45\rangle [45]^2}{\langle 15\rangle\langle 34\rangle\langle 35\rangle [12][15]} + \frac{\langle 12\rangle\langle 34\rangle [13][34]^2[45]}{\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][14][15][23]}$$

$$+ \frac{\langle 12\rangle\langle 45\rangle [13][34][45]^2}{\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][14][15][23]} + \frac{\langle 12\rangle\langle 14\rangle\langle 23\rangle [13][24][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][23]} - \frac{\langle 12\rangle\langle 14\rangle\langle 23\rangle [34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]} - \frac{\langle 12\rangle\langle 14\rangle\langle 23\rangle [45]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]}$$

$$+ \frac{\langle 12\rangle\langle 14\rangle\langle 23\rangle [14][25]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2} - \frac{\langle 12\rangle\langle 14\rangle\langle 23\rangle [15][24]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2} + \frac{\langle 12\rangle\langle 14\rangle\langle 23\rangle^2 [13][24][45]}{\langle 13\rangle\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][14][15]} + \frac{\langle 12\rangle\langle 14\rangle\langle 23\rangle [24][45]}{\langle 13\rangle\langle 15\rangle\langle 34\rangle\langle 35\rangle [12][15]}$$

$$+ \frac{\langle 12\rangle\langle 14\rangle\langle 23\rangle [13][24][34][45]}{\langle 13\rangle\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][14][15][23]} - \frac{\langle 12\rangle\langle 15\rangle\langle 23\rangle [15][23]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2} + \frac{\langle 12\rangle\langle 23\rangle^2 [23]^2}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2} - \frac{\langle 12\rangle\langle 23\rangle^2 [23][25][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[45]}$$

$$+ \frac{\langle 12\rangle\langle 23\rangle^2 [15][23][24][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[14][45]} - \frac{\langle 12\rangle\langle 23\rangle^2 [23][25]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2} + \frac{\langle 12\rangle\langle 23\rangle^2 [15][23][24]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2[14]} + \frac{\langle 12\rangle\langle 23\rangle [23][24]}{\langle 13\rangle\langle 35\rangle\langle 45\rangle [12]^2}$$

$$+ \frac{\langle 12\rangle\langle 23\rangle\langle 34\rangle [13][24][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[14]} + \frac{\langle 12\rangle\langle 23\rangle [13][24][45]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle [12]^2[14]} - \frac{\langle 12\rangle\langle 23\rangle [23][45]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle [12]^2} + \frac{\langle 12\rangle\langle 23\rangle [25][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle [12]^2} - \frac{\langle 12\rangle\langle 23\rangle [15][24][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle [12]^2[14]}$$

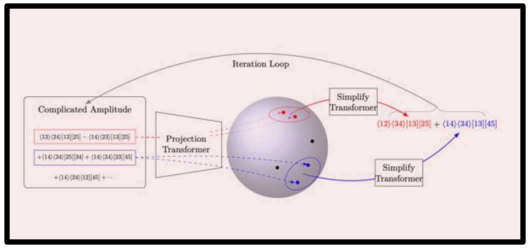
$$+ \frac{\langle 12\rangle\langle 23\rangle\langle 45\rangle [25][45]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2} - \frac{\langle 12\rangle\langle 23\rangle\langle 45\rangle [15][24][45]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2[14]} + \frac{\langle 13\rangle\langle 23\rangle [13][34]^2[35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][23][45]} + \frac{\langle 13\rangle\langle 23\rangle^2 [13][34][35]}{\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][14][15]}$$

$$+ \frac{\langle 13\rangle\langle 23\rangle [34][35]}{\langle 15\rangle\langle 34\rangle\langle 35\rangle [12][15]} + \frac{\langle 13\rangle\langle 23\rangle [13][34]^2[35]}{\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][14][15][23]} + \frac{\langle 14\rangle\langle 23\rangle [13][34]^2}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12][14][23]} + \frac{\langle 14\rangle\langle 23\rangle^2 [13][34][45]}{\langle 15\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12][14][15]}$$

$$+ \frac{\langle 14\rangle\langle 23\rangle [34][45]}{\langle 15\rangle\langle 34\rangle\langle 35\rangle [12][15]} + \frac{\langle 14\rangle\langle 23\rangle [13][34]^2[45]}{\langle 15\rangle\langle 24\rangle\langle 35\rangle [12][14][15][23]} - \frac{\langle 15\rangle\langle 23\rangle [15][35]}{\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2} + \frac{\langle 23\rangle^2 [13][24][34][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[14][45]}$$

$$- \frac{\langle 23\rangle^2 [23][34][35]}{\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[45]} + \frac{\langle 23\rangle^2 [13][24][35]}{\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2[14]} + \frac{\langle 23\rangle [24][35]}{\langle 34\rangle\langle 35\rangle [12]^2} + \frac{\langle 23\rangle [34][35]}{\langle 24\rangle\langle 35\rangle [12]^2} - \frac{\langle 14\rangle\langle 15\rangle\langle 23\rangle [15][45]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2}$$

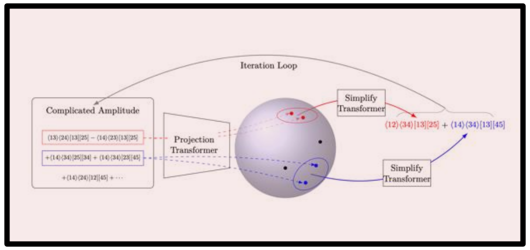
$$+ \frac{\langle 14\rangle\langle 23\rangle^2 [13][24][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2[14]} - \frac{\langle 14\rangle\langle 23\rangle^2 [23][34]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle\langle 45\rangle [12]^2} + \frac{\langle 14\rangle\langle 23\rangle^2 [13][24][45]}{\langle 13\rangle\langle 24\rangle\langle 34\rangle\langle 35\rangle [12]^2[14]} + \frac{\langle 14\rangle\langle 23\rangle [24][45]}{\langle 13\rangle\langle 34\rangle\langle 35\rangle [12]^2} + \frac{\langle 14\rangle\langle 23\rangle [34][45]}{\langle 13\rangle\langle 24\rangle\langle 35\rangle [12]^2}$$



$$\overline{\mathcal{M}} = -\frac{\langle 12\rangle^3}{\langle 15\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle}$$

Example application: graviton-scalar scattering

298 terms



simplifies to 2 terms

$$\overline{\mathcal{M}} = \frac{\langle 12\rangle\langle 13\rangle\langle 23\rangle}{\langle 24\rangle\langle 25\rangle\langle 45\rangle}\left(\frac{[14][35]}{\langle 14\rangle\langle 35\rangle} - \frac{[15][34]}{\langle 15\rangle\langle 34\rangle}\right)$$

Try it yourself!

https://spinorhelicity.streamlit.app/

Sampling Method

- [x] Nucleus Sampling
- [ ] Beam Search
- [x] Greedy Decoding

Beam Size (Beam Search and Nucleus Sampling)

5

1 20

Nucleus Cutoff (Nucleus Sampling)

0.95

0.80 1.00

Temperature (Nucleus Sampling)

1.50

0.50 4.00

Spinor-Helicity Simplification

This app simplifies spinor-helicity expressions. Enter the input in the syntax of Fortran (ab(1,2)**2*sb(1,2)), Mathematica (ab[1,2]^2*sb[1,2]), or S@M (Spaa[1,2]^2*Spbb[1,2]). All terms should have uniform scaling in mass dimension and little group, with purely monomial denominators. Specify the number of external particles and the mode of simplification, with additional tunable parameters on the sidebar. One-shot and iterative mode are effective for shorter (<10 terms) and longer (<40 terms) expressions, respectively. Apply even longer expressions at your own risk. See our paper for details. For faster performance we encourage a local download from this GitHub repository.

For any inquiries or further information, please contact Aurélien Dersy

Amplitude Type

5-pt

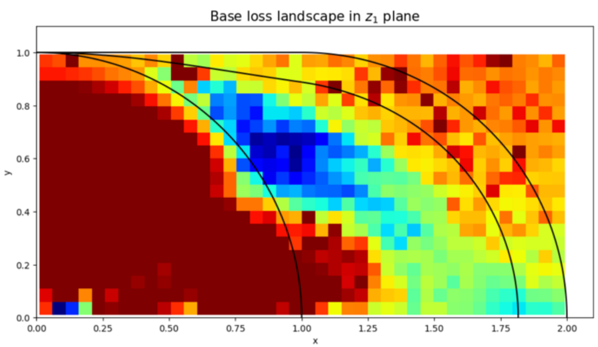
Input Equation

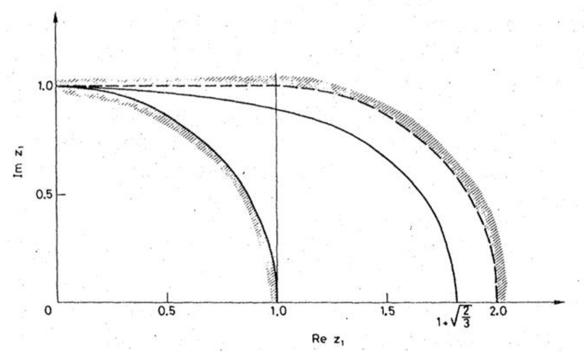
(-ab(1,2)**2*sb(1,2)*sb(1,5)-ab(1,3)*ab(2,4)*sb(1,3)*sb(4,5)+ab(1,3)*ab(2,4)*sb(1,4)*sb(3,5)-ab(1,3)*ab(2

$$\frac{-\langle 12\rangle^3\,[12]\,[15]-\langle 12\rangle\langle 13\rangle\langle 24\rangle\,[13]\,[45]+\langle 12\rangle\langle 13\rangle\langle 24\rangle\,[14]\,[35]-\langle 12\rangle\langle 13\rangle\langle 24\rangle\,[15]\,[34]}{\langle 15\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\,[12]\,[15]}$$

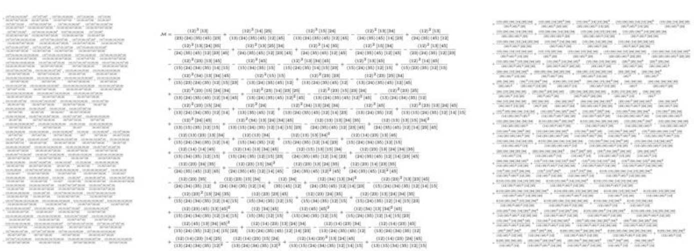
Conclusions

- Machine learning is a powerful tool to understand the S-matarix
  - New results on old non-perturbative S-matrix questions





  - Simplification of perturbative expessions



$$-\frac{\langle 12\rangle^3}{\langle 15\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle}$$

- Future of ML in high energy theory is symbolic
  - Lots to do!