Sudakov Shoulder Resummation in Thrust and Heavy Jet Mass

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Theory Seminar
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Based on
arXiv:2205.05702 (PRD106.074011)
with Arindam Bhattacharya (Harvard) and Xiaoyuan Zhang (Harvard)
arXiv:2306.08033 (JHEP11.2023.080)
with Arindam Bhattacharya (Harvard), Xiaoyuan Zhang (Harvard), Iain Stewart (MIT) and Johannes Michel (MIT)

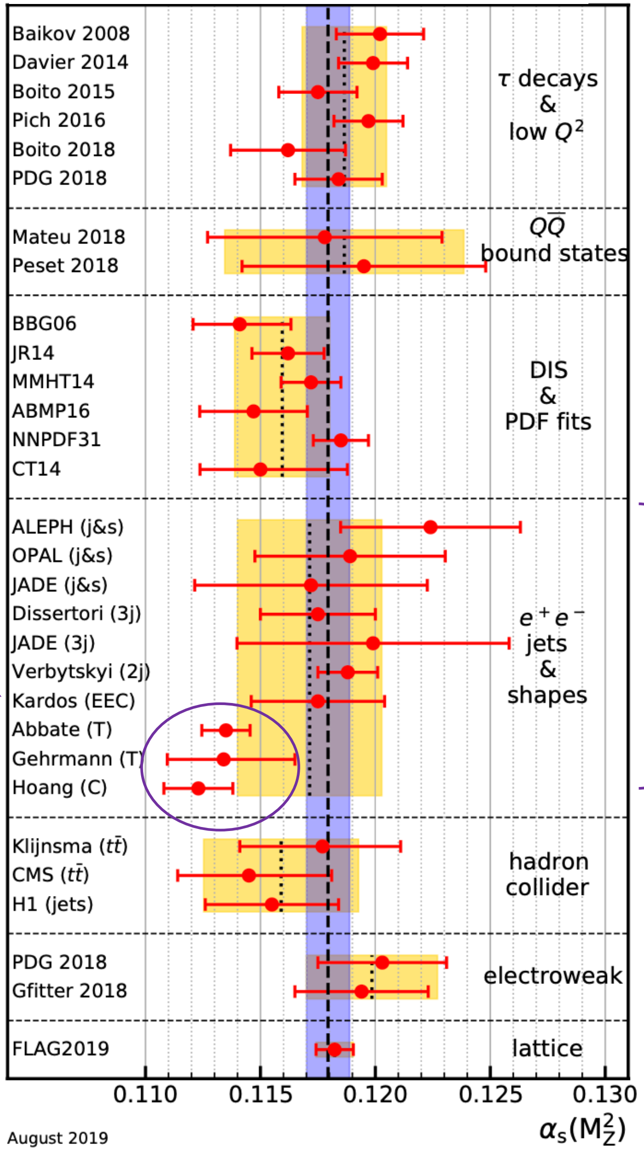
Outline

- Overview of α_s measurements
- Heavy Jet Mass at e+e- colliders
  - Unusual behavior compared to thrust
- Sudakov shoulder resummation
- Position-space resummation

PDG 2019

Overall global fit

$$\alpha_s(M_Z^2) = 0.1179 \pm 0.0010\,.$$



EEC
(NNLL + NNLO
+ MC power corrections)

Thrust
(N^3LL + NNLO
+ theory power corrections)

$$\alpha_s(M_Z^2) = 0.1135 \pm 0.0011$$

C-parameter

Refs [...] quote surprisingly small overall experimental, hadronization, and theoretical uncertainties of only 2, 5, and 9 per-mille, respectively, which calls for an independent confirmation.

“unbiased” e^+e^- average

$$\alpha_s(M_Z^2) = 0.1171 \pm 0.0031.$$

Ignores theorists results?

LHC:
only CMS only tt total cross section

$$\alpha_s(M_Z^2) = 0.1145^{+0.0036}_{-0.0031}.$$

Flavor Lattice Averaging Group

$$\alpha_s(M_Z^2) = 0.1182 \pm 0.0008\,,$$

PDG 2019

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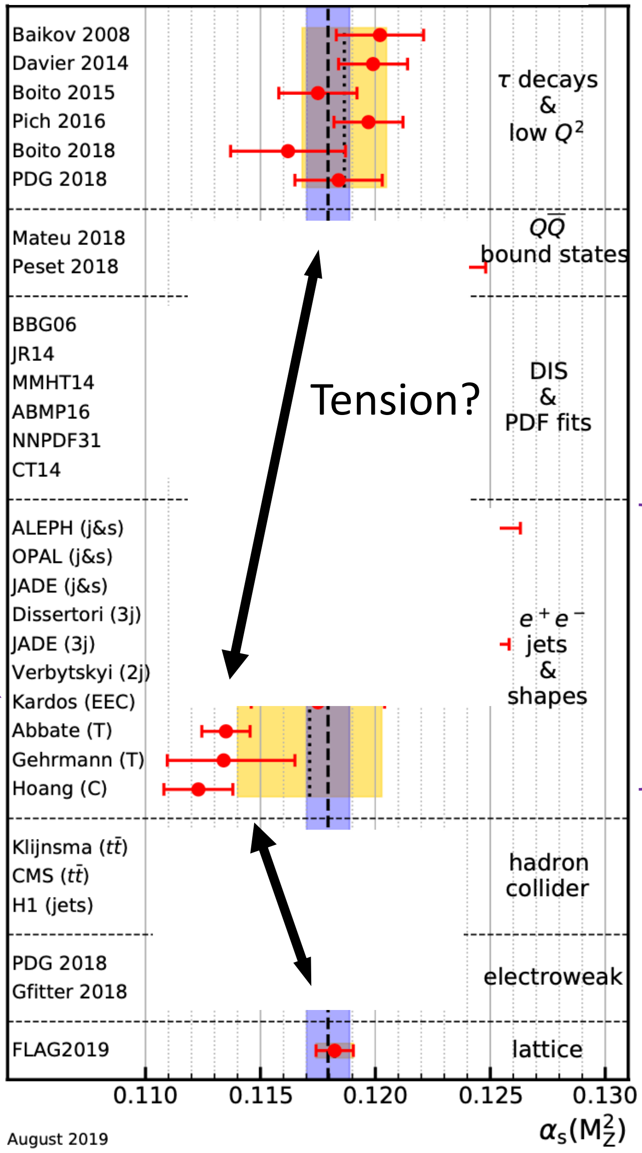

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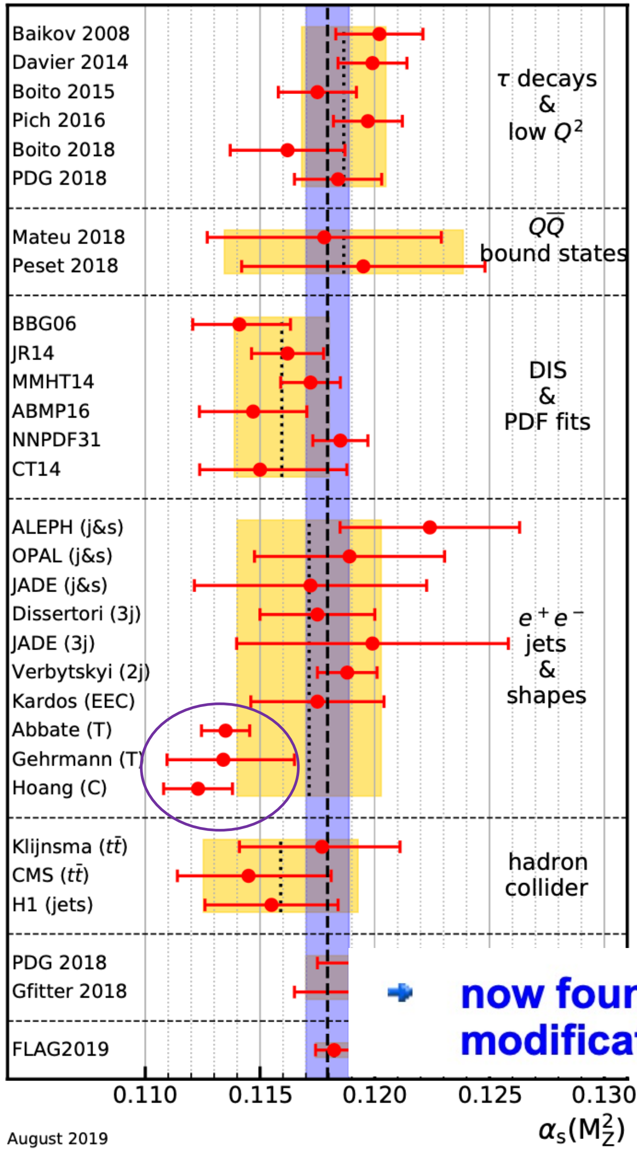
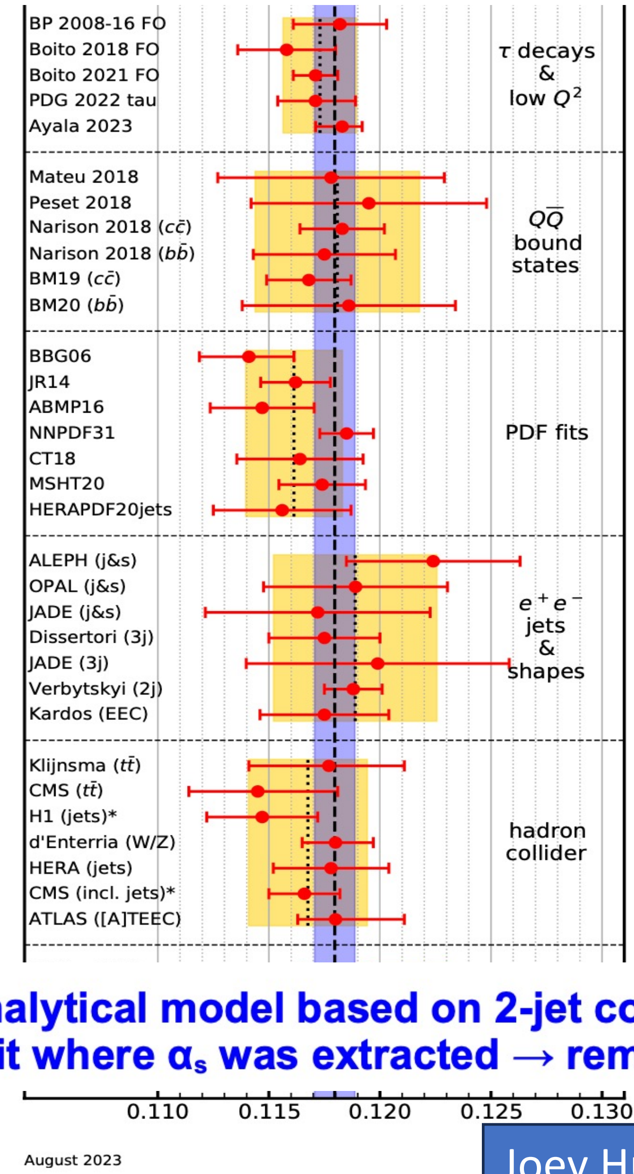


removed 2023

PDG 2023

$$\alpha_s(m_Z^2) = 0.1180 \pm 0.0009$$



now found that use of analytical model based on 2-jet configuration needs modification for 3-jet limit where α_s was extracted → removed for now

Why were they removed?

- New insights into power corrections in 3-jet region (Nason et al.)

"These findings are **inconsistent** with **the very small experimental, hadronization,** and theoretical uncertainties of only 2, 5, and 9 per-mille, respectively, as reported in Refs. [683,685]. For these reasons, **we exclude the results** of Refs. [683–685] from the average. "

--PDG QCD p. 35

Two issues here

1. All event shapes seem to give low values of α
   - Are we modelling the **power corrections** correctly?

2. Heavy jet mass gives a very low value
   - Are we modeling the **tail** correctly?

Event shapes

Thrust

Thrust [Farhi, 1977]

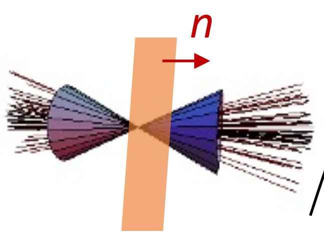
$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$

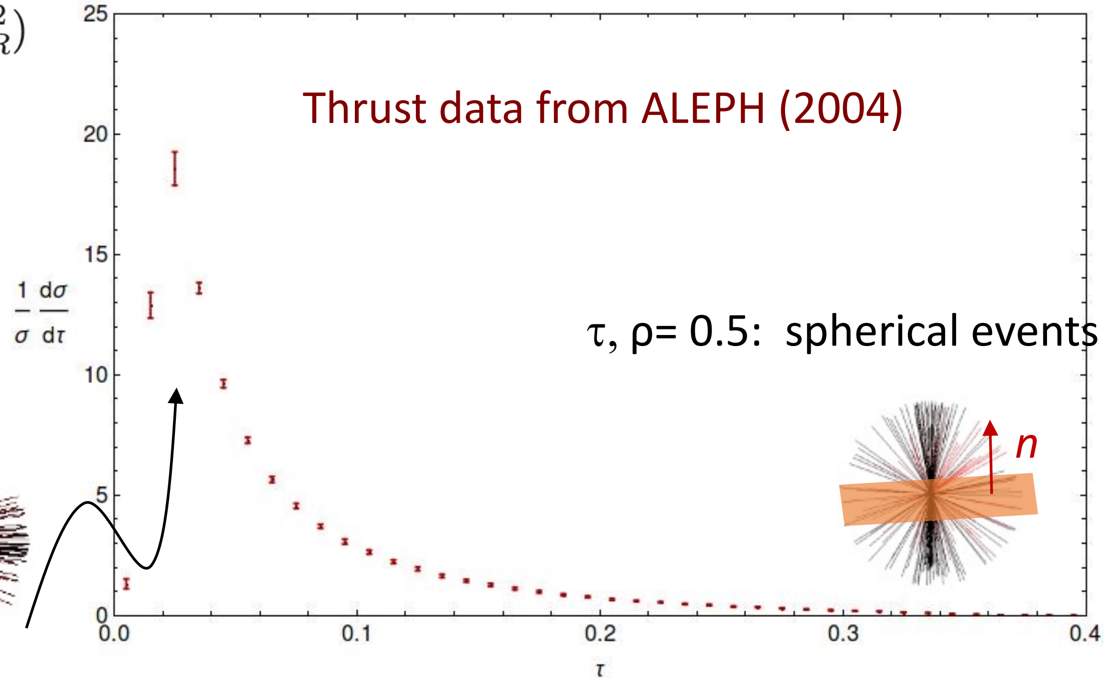
$$\tau = 1 - T$$

- Split event into two hemispheres
  - thrust axis *n* maximizes Thrust
- Thrust axis effectively minimizes sum of hemisphere masses

Heavy jet mass

$$\rho \equiv \frac{1}{Q^2} \max(M_L^2, M_R^2)$$



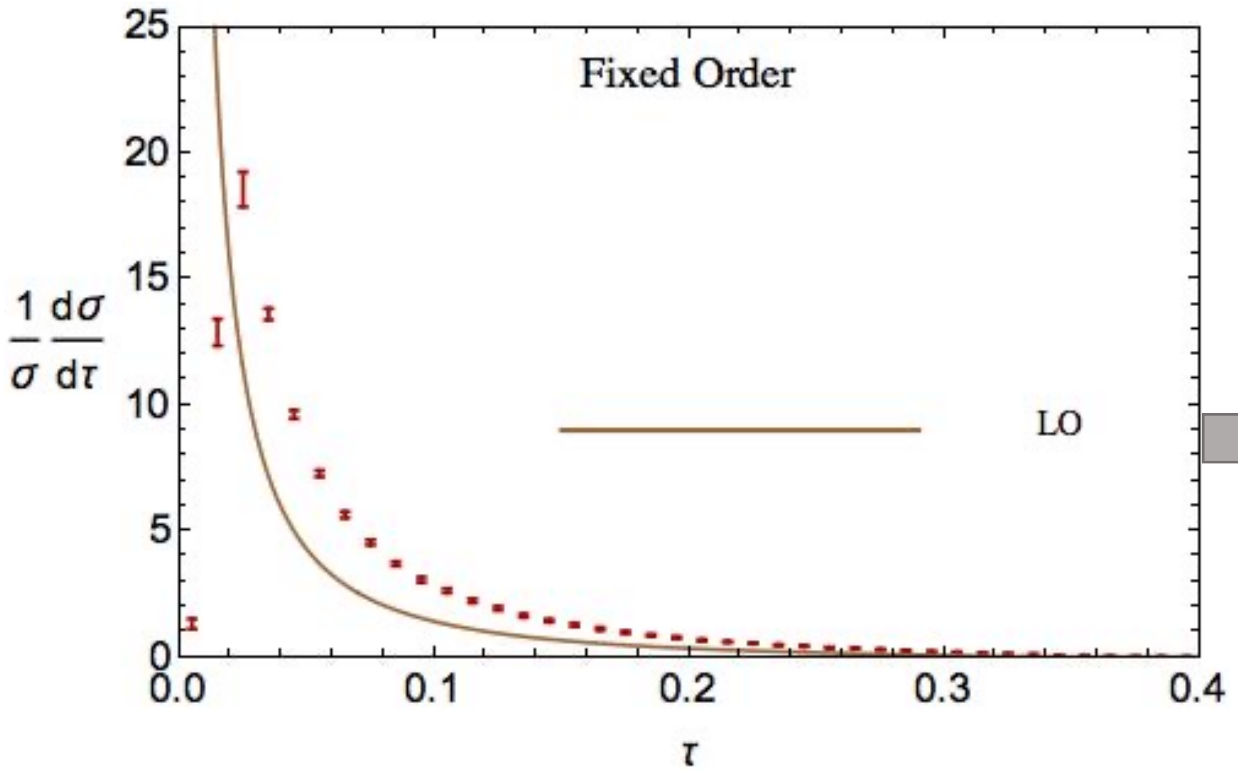


τ, ρ= 0.5: spherical events

τ,ρ << 1: small jets

Thrust from QCD

$$\left[\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\right]_{\mathrm{LO}} = \delta(\tau) + \frac{C_F}{2\pi}\alpha_s\left[\frac{-4\log\tau - 3}{\tau} - 8 + 2\log\tau + 23\tau - \frac{44}{3}\tau^2 + \cdots\right]$$

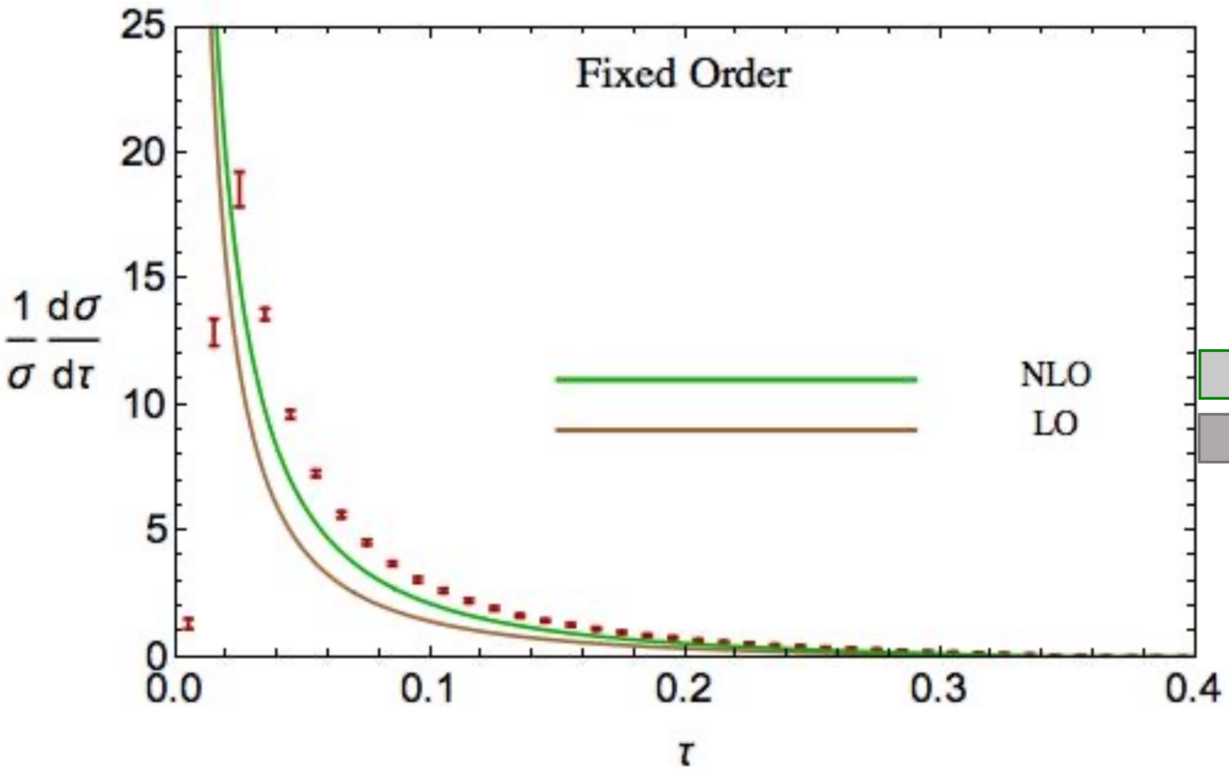


Farhi (1977)

Theory doesn’t look anything like the data!

Thrust from QCD

$$\left[\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\right]_{\mathrm{LO}} = \delta(\tau) + \frac{C_F}{2\pi}\alpha_s\left[\frac{-4\log\tau - 3}{\tau} - 8 + 2\log\tau + 23\tau - \frac{44}{3}\tau^2 + \cdots\right]$$



Ellis, Sexton (1981)

Farhi (1977)

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Thrust from QCD

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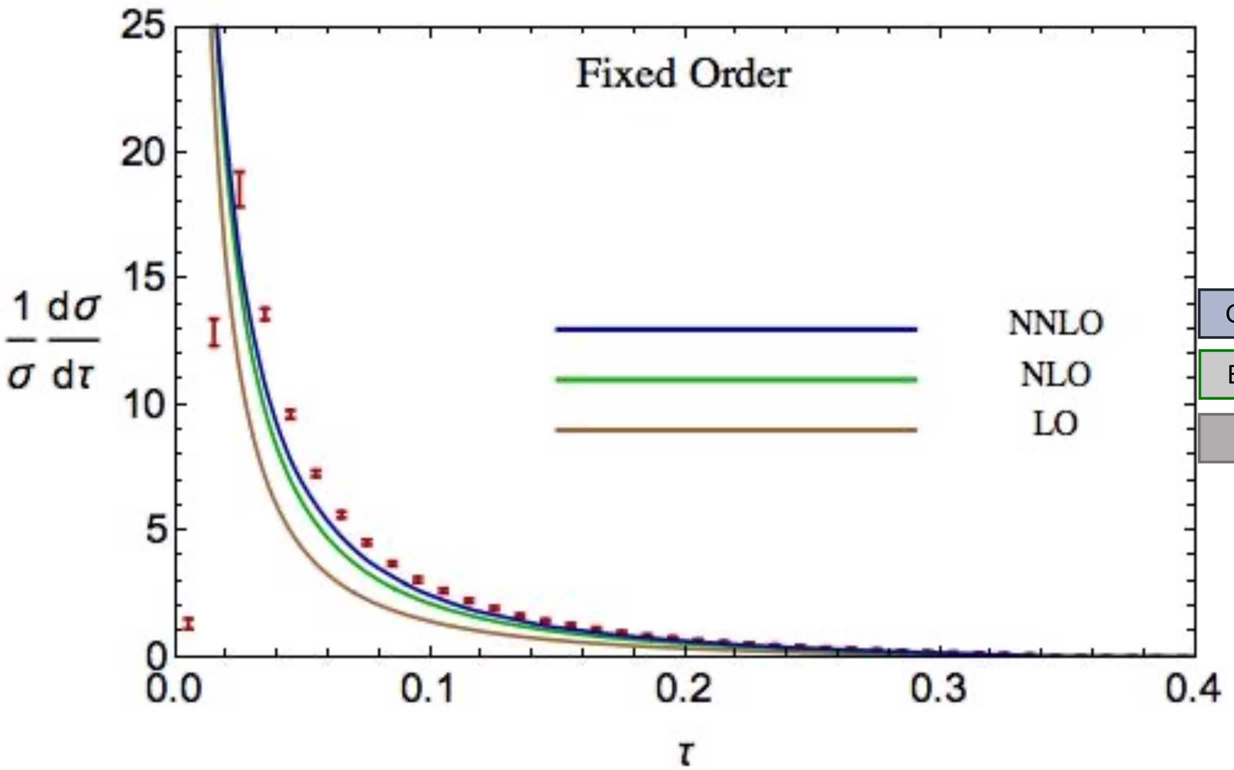


Gehrmann et al (2007)

Ellis, Sexton (1981)

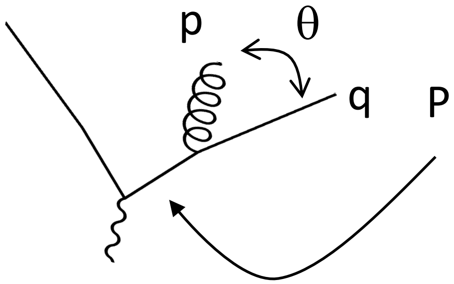
Farhi (1977)

Theory doesn't look anything like the data!

- If α_s is small, why does perturbation theory not work?

Large Logarithms!!

Why are there large logarithms?



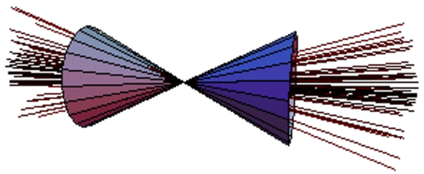
Propagator factor: $$\frac{1}{(p+q)^2} = \frac{1}{E_p E_q \sin^2\theta}$$

Blows up when E=0 (soft divergence)

or θ = 0 (collinear divergence)

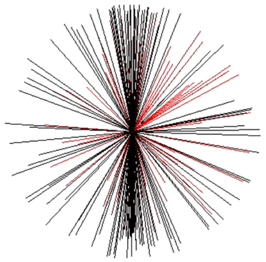
Cross section is: $$\frac{d\sigma}{d\theta} \approx \alpha_s \frac{\log\theta}{\theta}$$
$$\sim \alpha_s \frac{\log\tau}{\tau}$$

Rate for

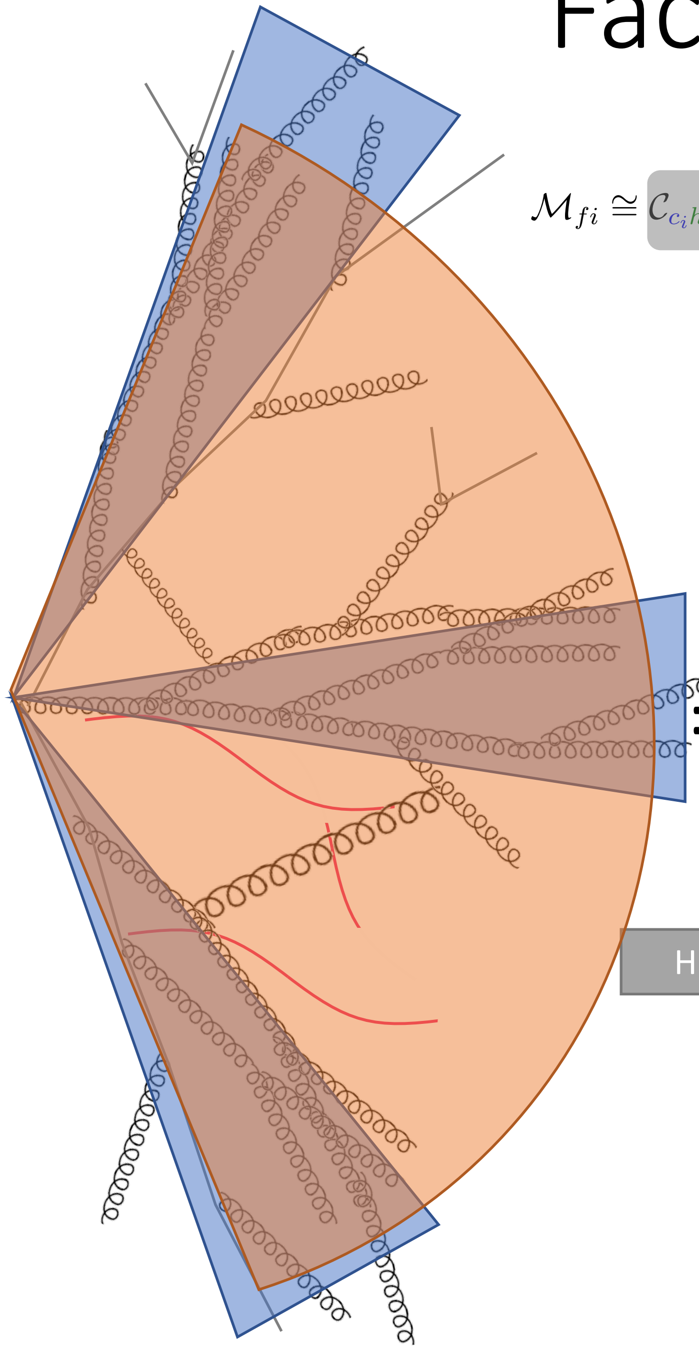


much greater than

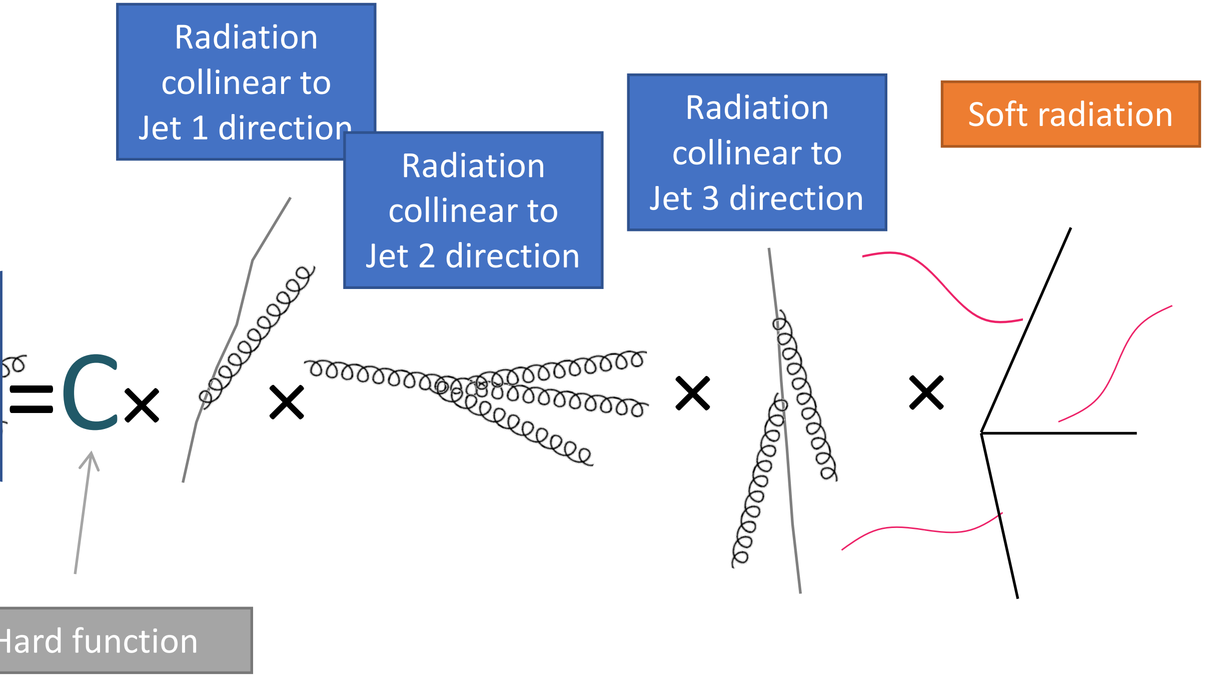
Rate for



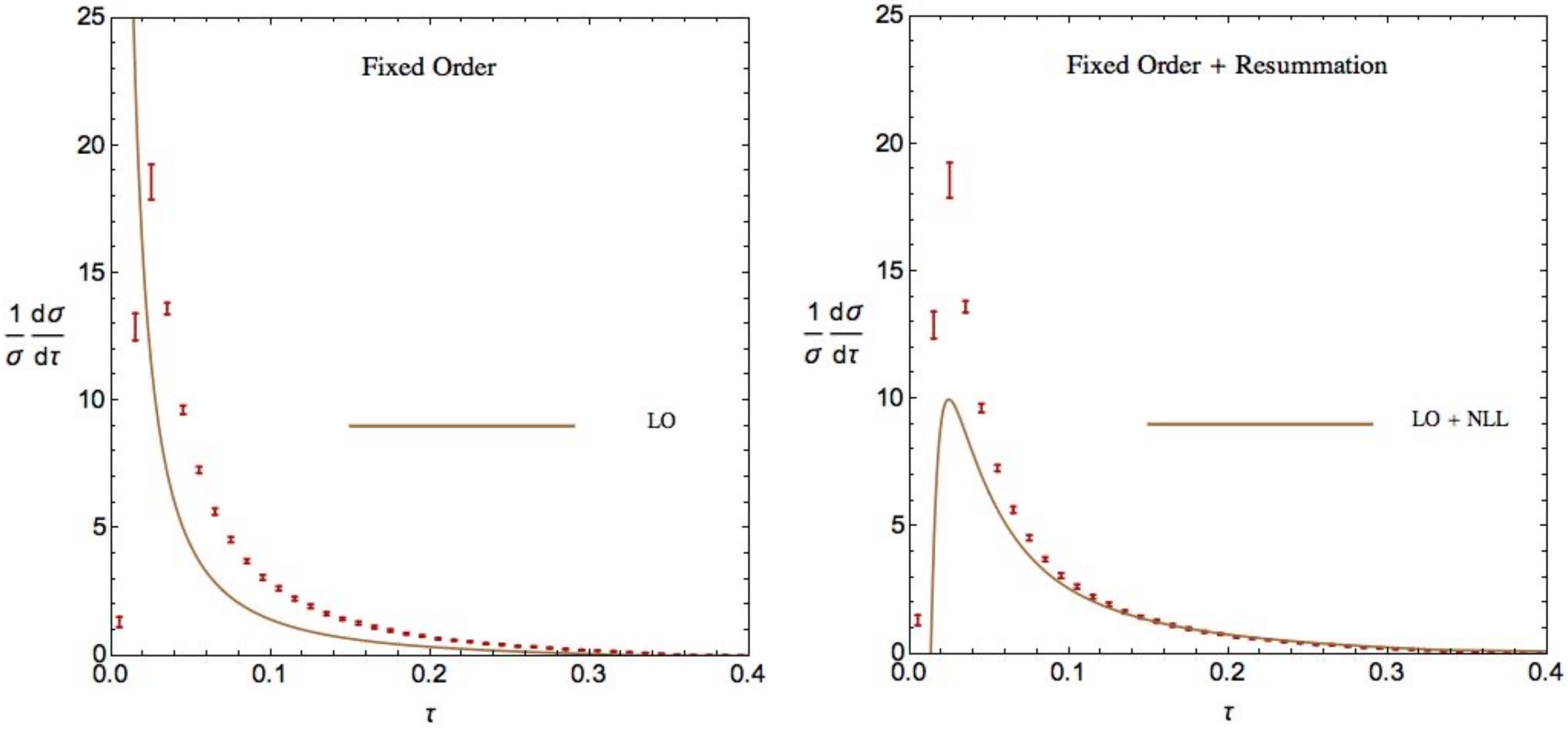
Factorization



$$\mathcal{M}_{fi} \cong \mathcal{C}_{c_i h_j}(S_{ij}) \frac{\langle X_1|\, \psi^* W_1\, |0\rangle^{h_1 c_1}}{\langle 0|\, Y_1^\dagger W_1\, |0\rangle} \cdots \frac{\langle 0|\, \overline{W}_N^\dagger \psi\, |X_N\rangle^{h_N c_N}}{\langle 0|\, \overline{W}_N^\dagger \overline{Y}_N\, |0\rangle} \langle X_s^f|\, Y_1^\dagger \cdots \overline{Y}_N\, |X_s^i\rangle^{c_1 \cdots c_N}$$

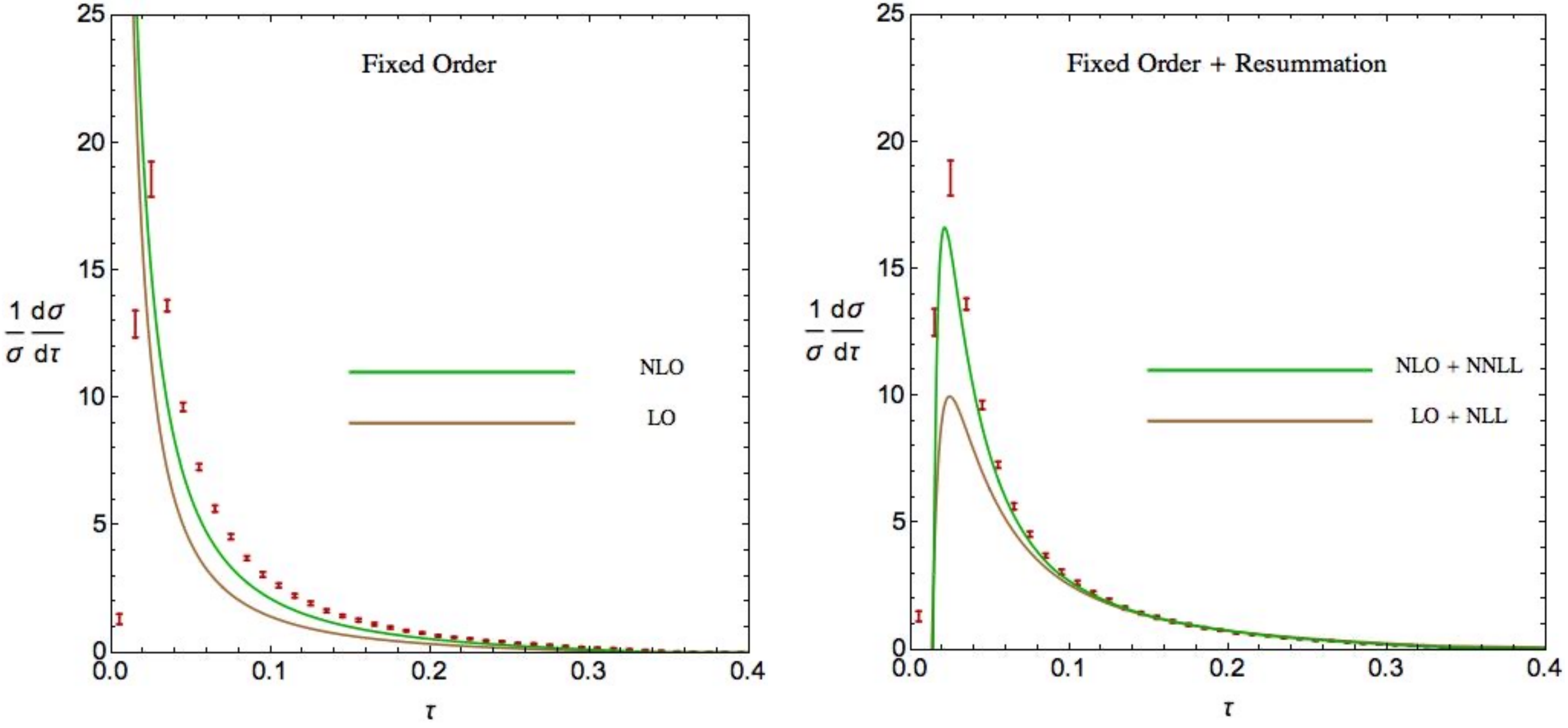


Thrust

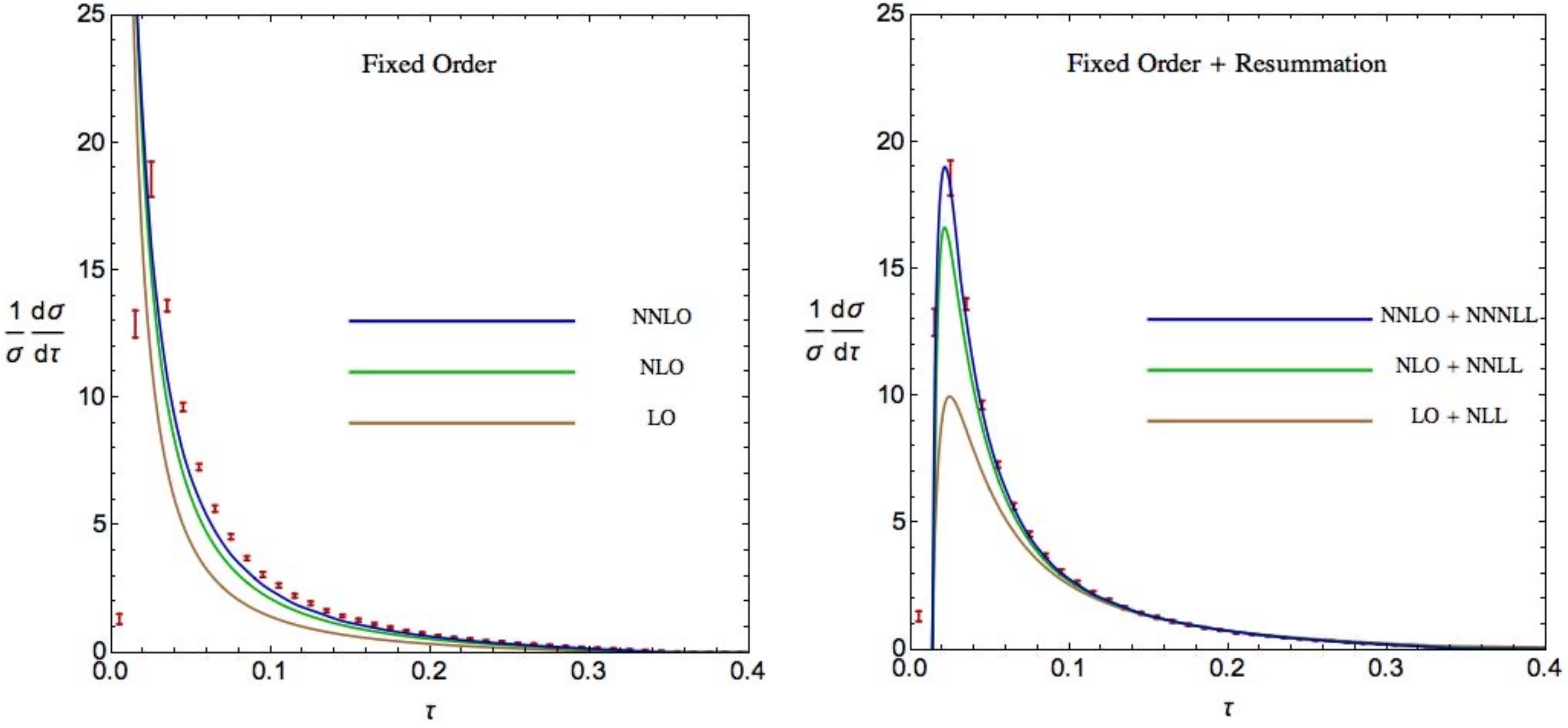


Fixed Order
LO
Fixed Order + Resummation
LO + NLL
$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
τ

Thrust

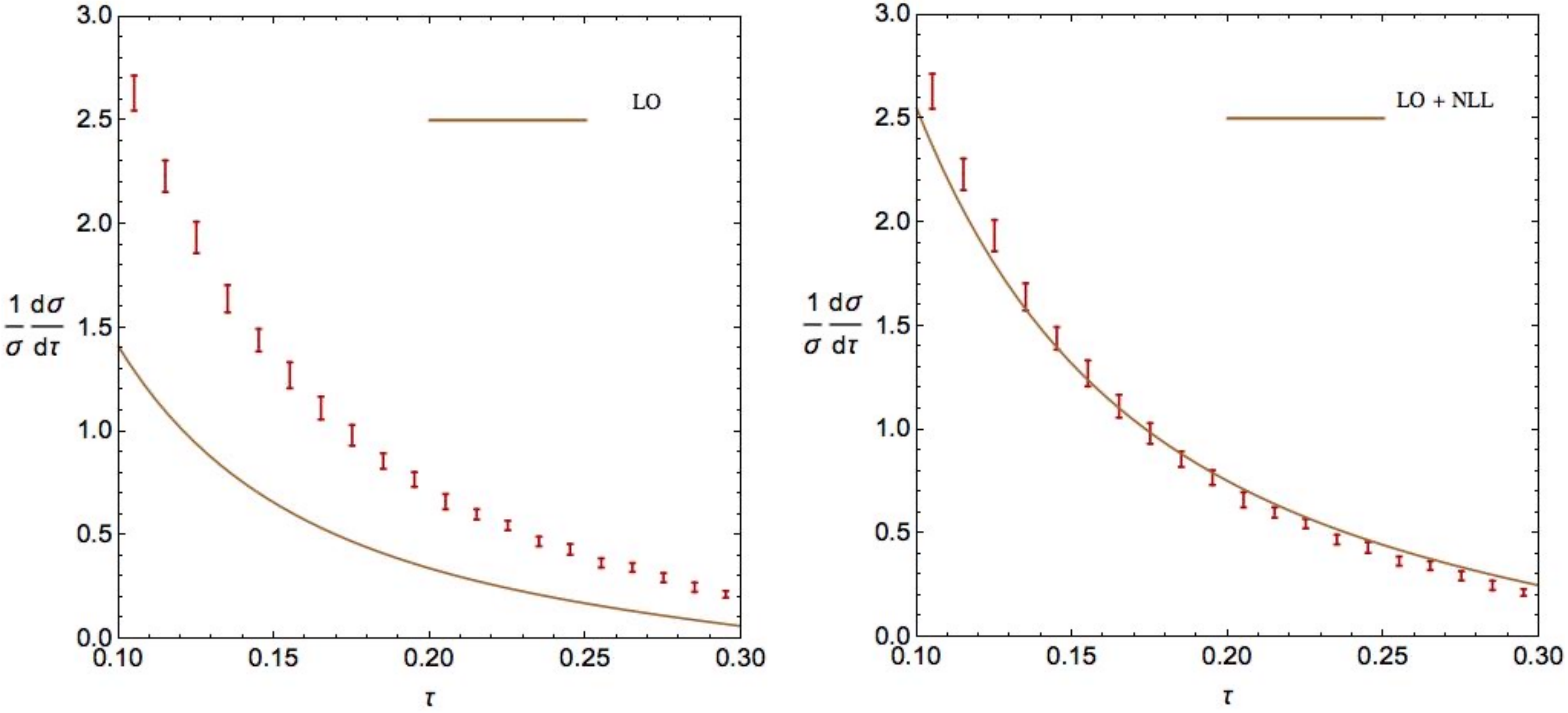

Fixed Order
NLO
LO
$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
τ
0.0
0.1
0.2
0.3
0.4
0
5
10
15
20
25
Fixed Order + Resummation
NLO + NNLL
LO + NLL

Thrust



Fixed Order
NNLO
NLO
LO
Fixed Order + Resummation
NNLO + NNNLL
NLO + NNLL
LO + NLL
$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
τ

Improved convergence



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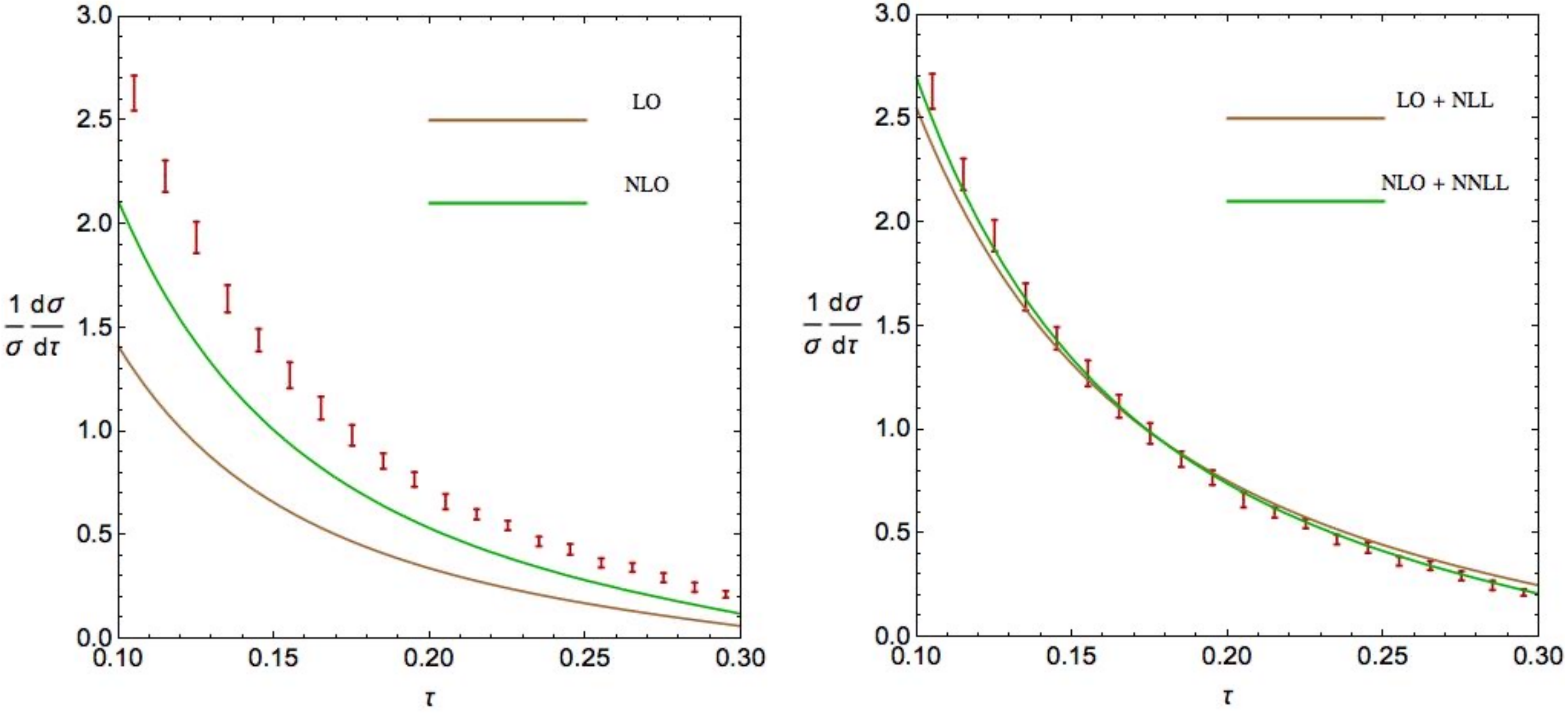

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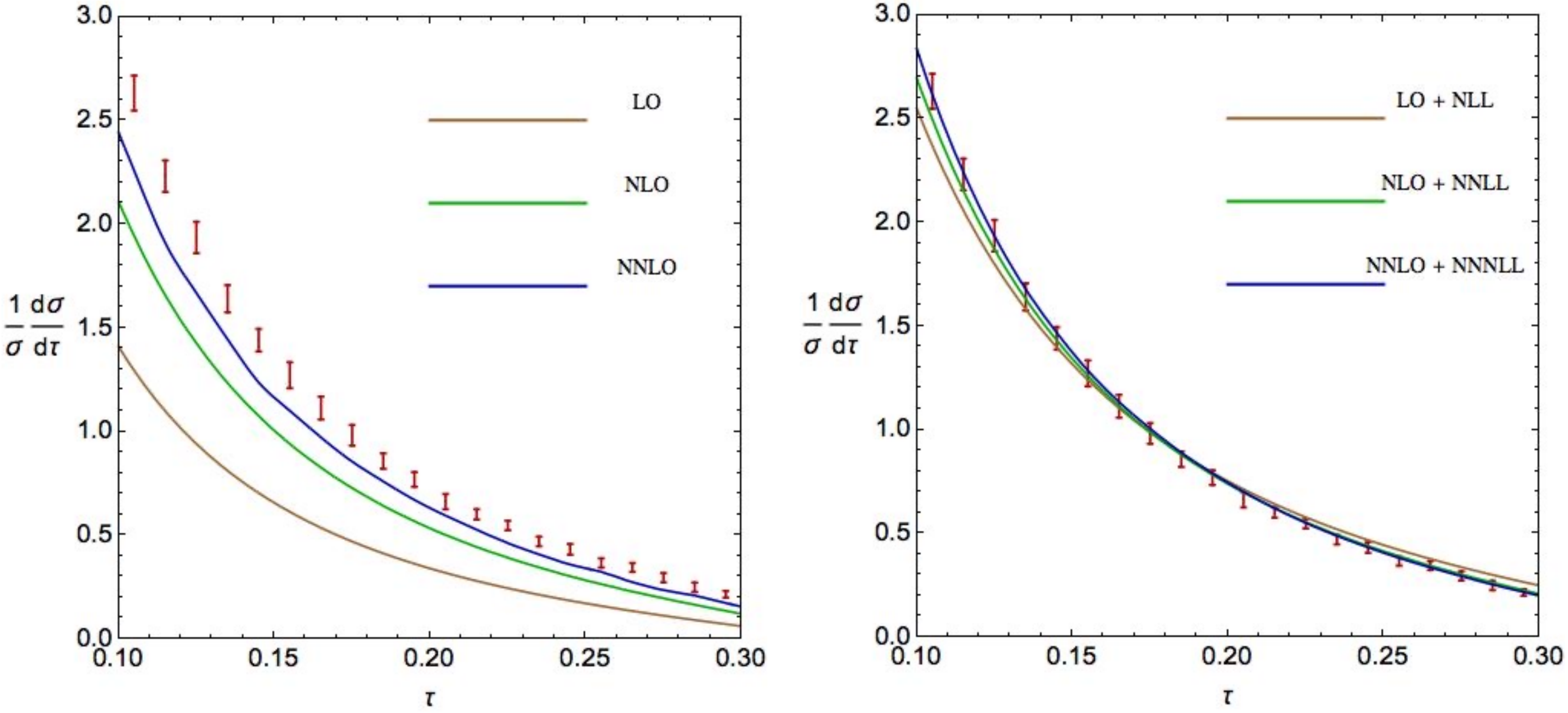
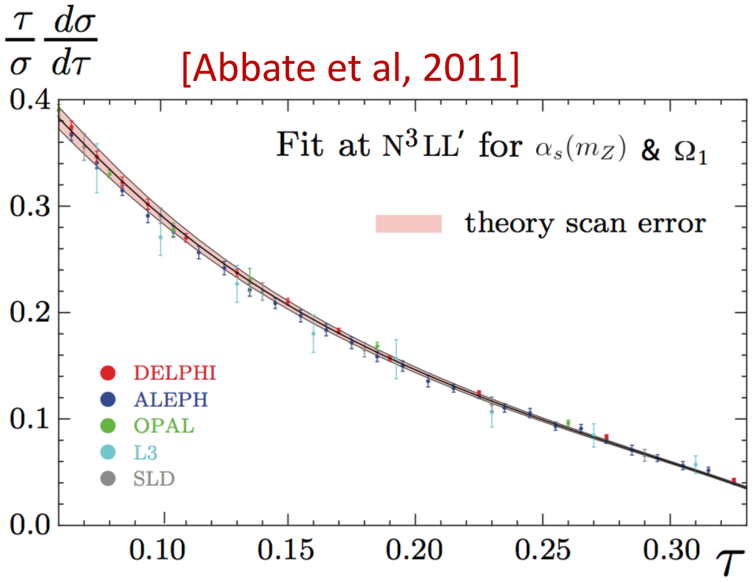
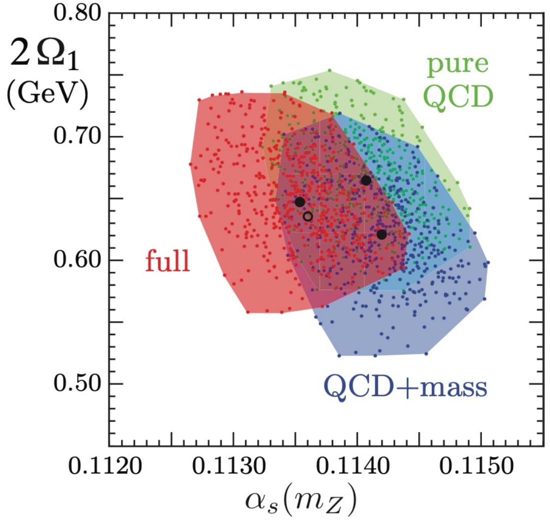


Power corrections etc.

[Abbate et al, 2011]





$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat\sigma_s}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat\sigma_{\rm ns}}{\mathrm{d}\tau} + \frac{\Delta \mathrm{d}\hat\sigma_b}{\mathrm{d}\tau}\right)\left(\tau - \frac{k}{Q}\right) \times \boxed{S_\tau^{\rm mod}\big(k - 2\bar\Delta(R,\mu_S)\big)} + \mathcal{O}\left(\sigma_0\, \alpha_s \frac{\Lambda_{\rm QCD}}{Q}\right).$$

Non-pertubative model function

- Can be fit from data
- Dominated by first moment

$$2\bar\Omega_1 = \int \mathrm{d}k'\, k'\, S_\tau^{\rm mod}(k'),$$

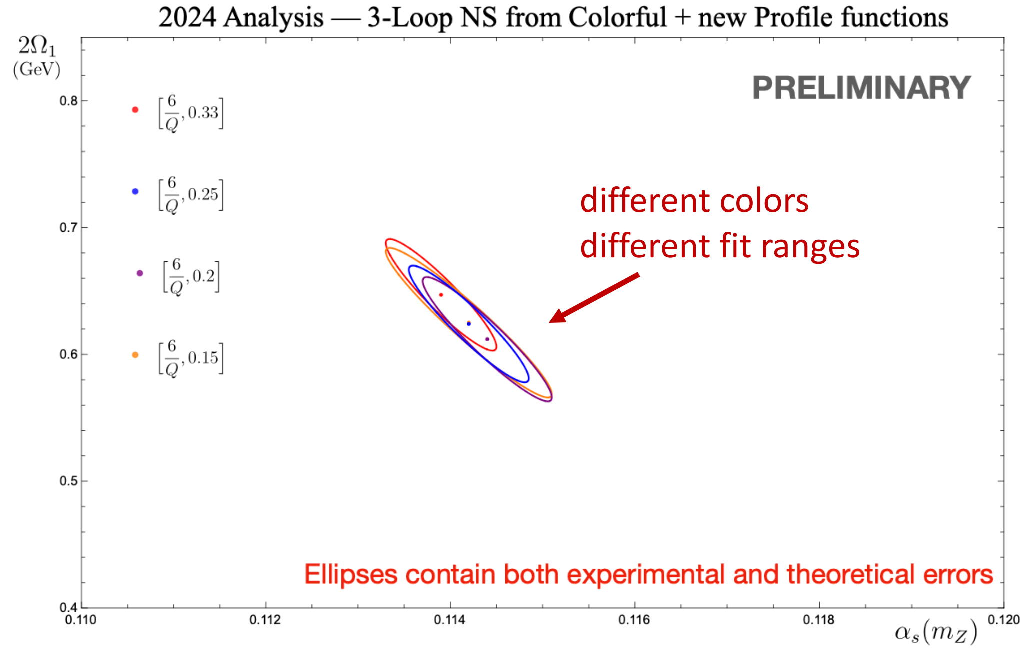
- Include bottom-quark corrections
- Include QED effects
- Use profile functions instead of canonical scales
- + 3-loop hard function
- R-gap subtraction to remove leading renormalon

$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\rm exp} \pm (0.0005)_{\rm hadr} \pm (0.0009)_{\rm pert},$$

(Abatte et al 2010 fit for thrust)

Thrust update (2024)

Miguel Benitez-Rathgeb et al, work in progress



- Added QED effects
- Added bottom quark effects
- Improved profile functions

$$\alpha_s = 0.1142 \pm 0.0006_{\text{pert}} \pm 0.0009_{\text{exp}} \pm 0.0004_{\text{had}} = 0.1142 \pm 0.0012_{\text{tot}}$$

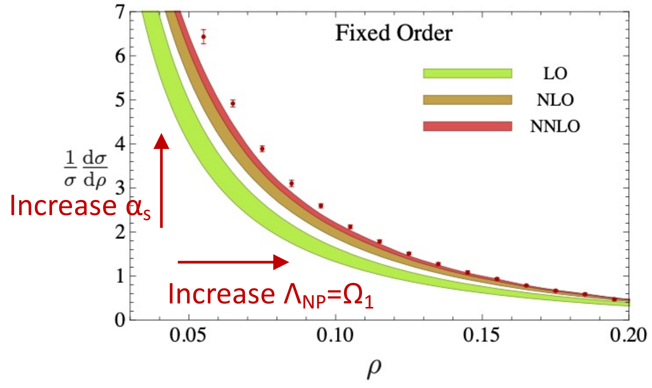
- Still lower than world average
- Good stability to fit range

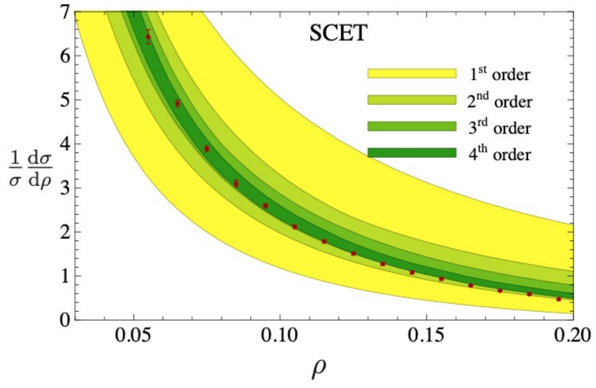
Heavy jet mass

Try the same procedure for heavy jet mass $\rho \equiv \frac{1}{Q^2}\max(M_L^2, M_R^2)$

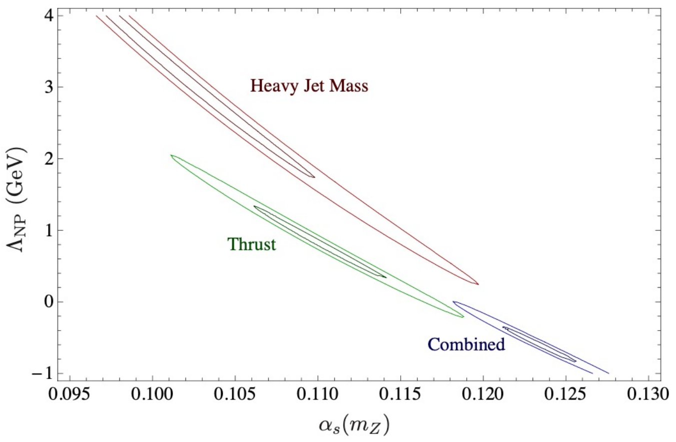
Perturbative prediction

Chien and Schwartz (2010)





Add power corrections:



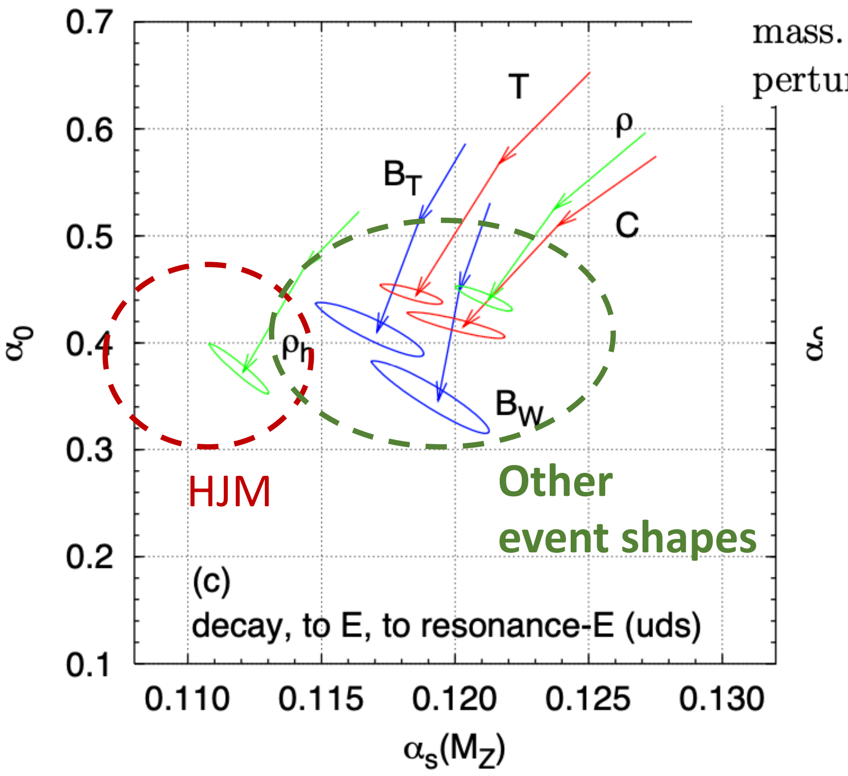
- Flat direction between α and Ω_1
  - Broken by shape and different Q's
- α_s seems very low

Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	χ^2/d.o.f.
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

- Need better NP modeling

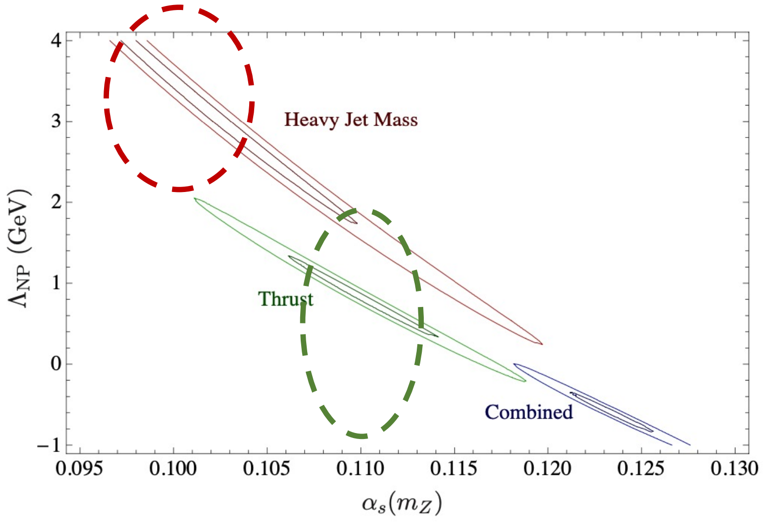
HJM has always been an outlier

Salam and Wicke 2001 (hep-ph/0102343)



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders. But in appendix D there is evidence from Monte Carlo

Chien and Schwartz 2010 (arXiv:1005.1644)
NNNLL resummation with NNLO matching



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What is different about thrust and HJM?

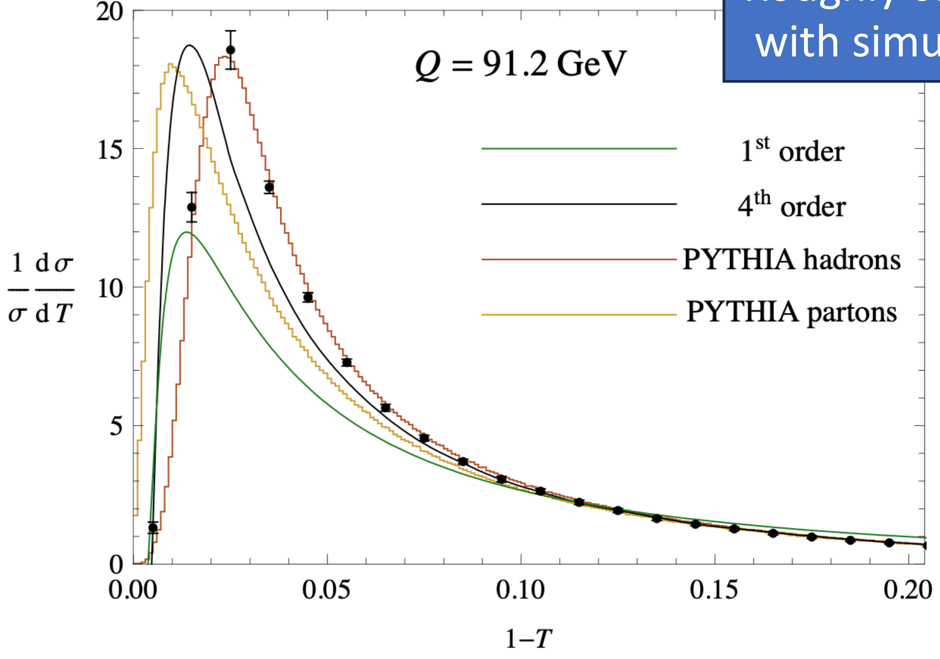
1. Different power corrections

2. Different perturbative behavior in the 3-jet region

Power corrections in dijet region

- Associated with hadronization effects, mass gap in QCD
- Soft contribution shifts

$$S(k) \to S(k+\Omega_1)$$



Roughly consistent with simulation

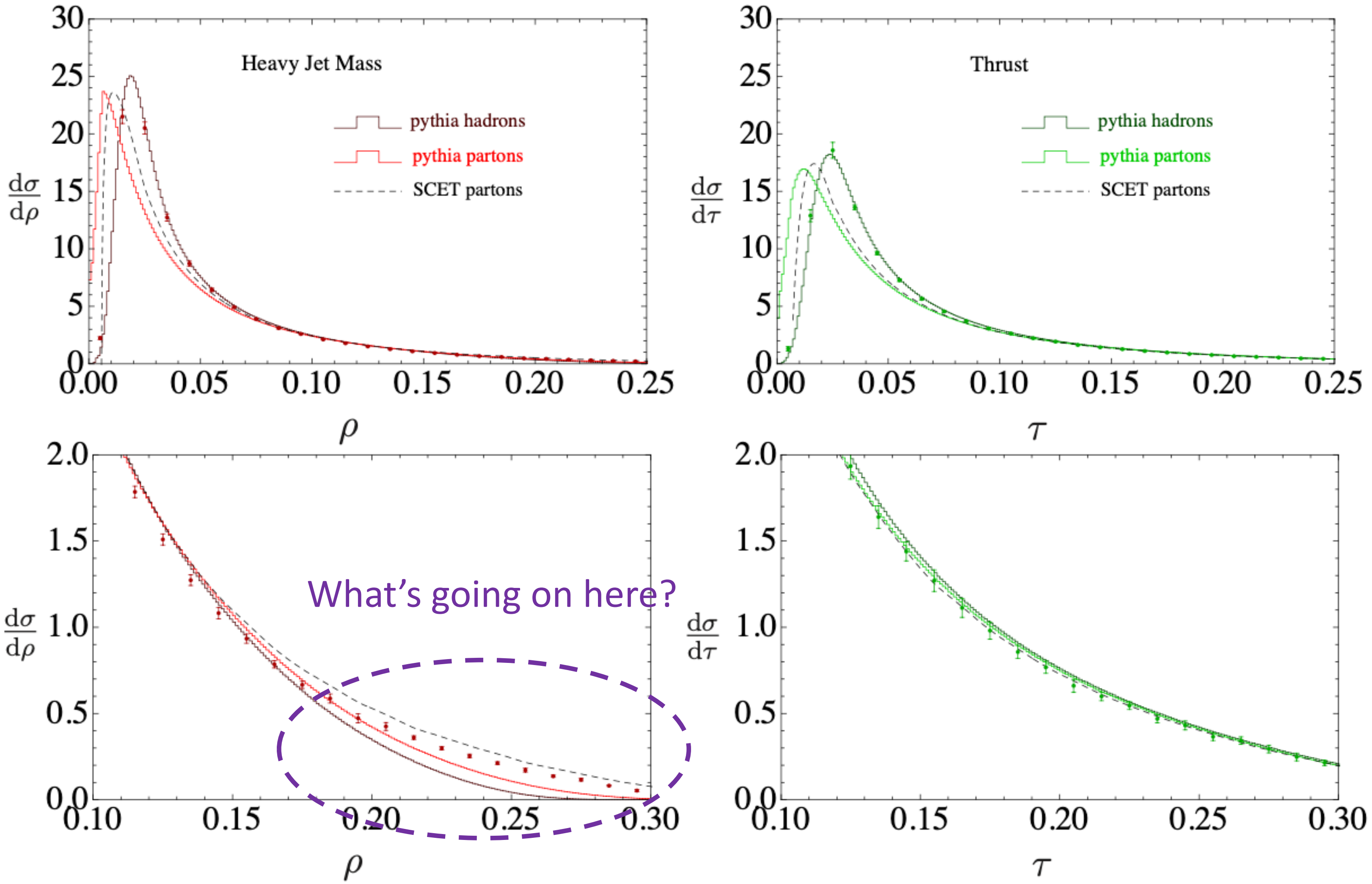
- Can rigorously derive power corrections as operator expectation value using SCET

$$\bar{\Omega}_1 = \frac{1}{2N_c}\langle 0|\mathrm{tr}\ \overline{Y}^T_{\bar{n}}(0)Y_n(0)\, i\hat{\partial}\, Y^\dagger_n(0)\overline{Y}^*_{\bar{n}}(0)|0\rangle\,.$$

- Can model higher corrections with a model function

$$S_\tau(k,\mu) = \int \mathrm{d}k'\ S^{\mathrm{part}}_\tau(k-k',\mu)\ S^{\mathrm{mod}}_\tau(k')\,,$$

Power Corrections from Pythia



Mass schemes

Thrust depends on 3-vectors

$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$

Heavy jet mass depends on 4-vectors

$$\rho \equiv \frac{1}{Q^2} \max(M_L^2, M_R^2)$$

E scheme: $E = \sqrt{m^2 + p^2}$

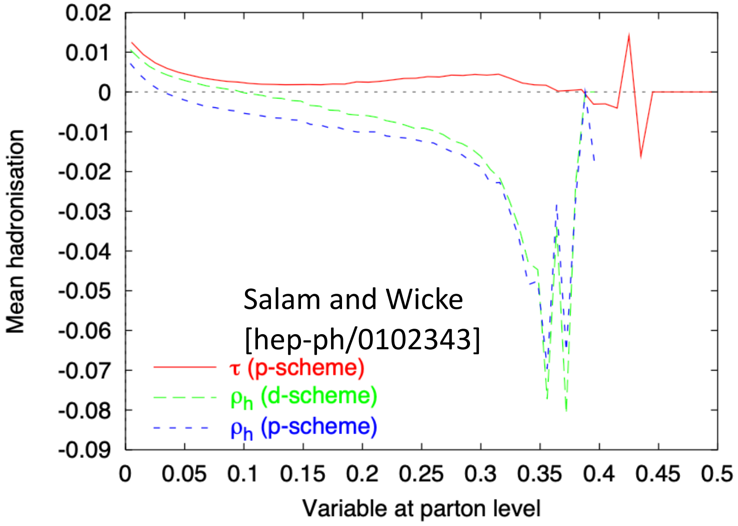
p scheme: $p = \sqrt{E^2 - m^2}$

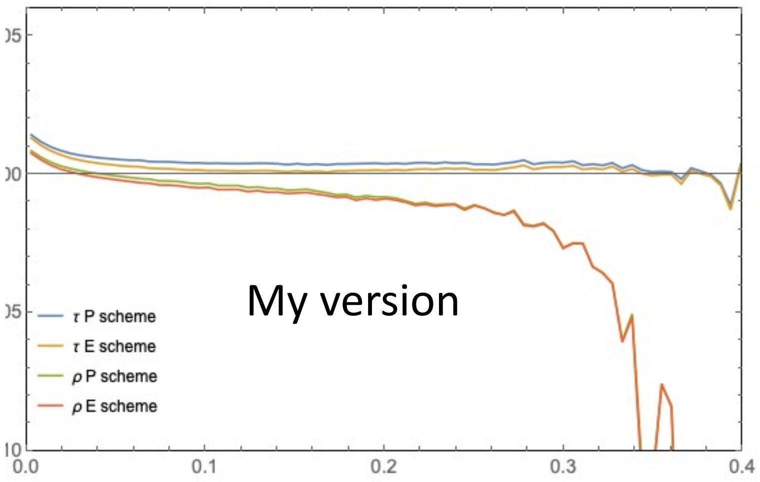
- Experiements measure energy and angle (E, θ,ϕ)
  - Or do they measure |p|, and angle (|p|,θ,ϕ)?
- Use particle mass to get (E,p, θ,ϕ)

Look at **mean hadonization**:

- generate partons
- ρ_p = parton level thrust
- hadronize same partons
- ρ_h = hadron level thrust
- histogram difference $\rho_h - \rho_p$

for each ρ_p





- What's going on at large ρ?
- Turns out that scheme dependence doesn't affect fits much because it's largely absorbed into Ω_1

Phase space only

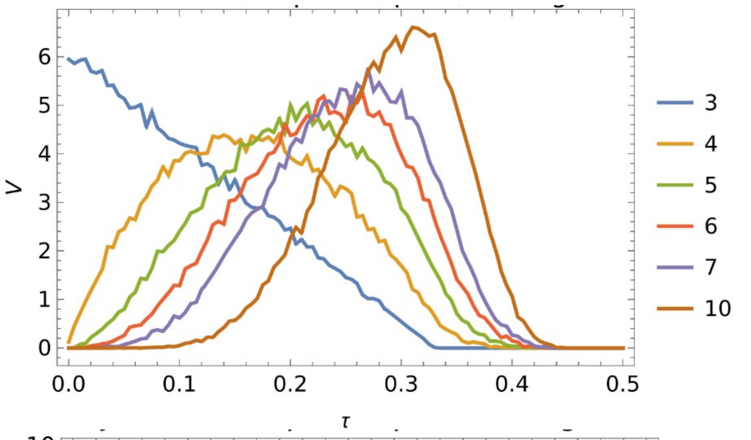
- Something else we can look at is thrust or heavy jet mass using **phase space only**
- Set M=1 and use RAMBO to generate events

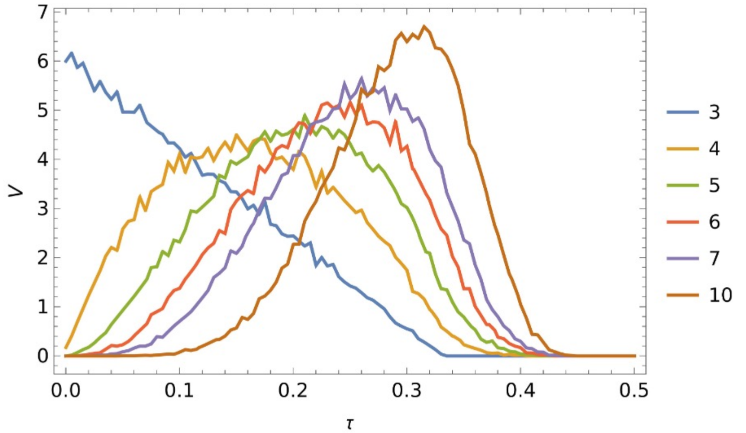
- corrections larger for ρ
- corrections positive even at large ρ

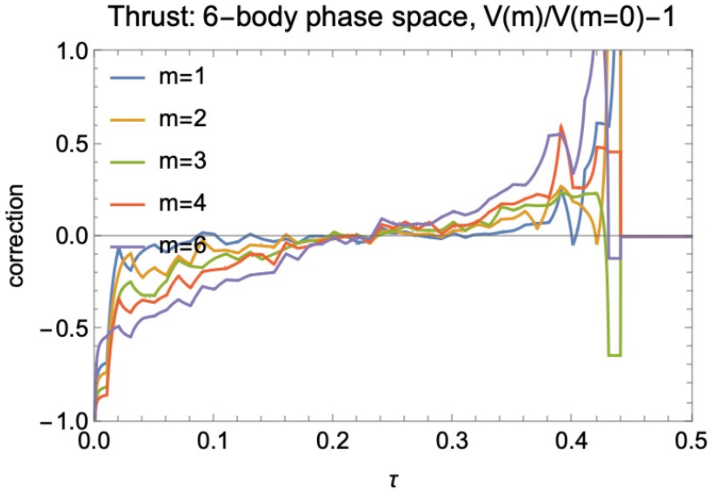
massless partons

massive partons m=5 GeV

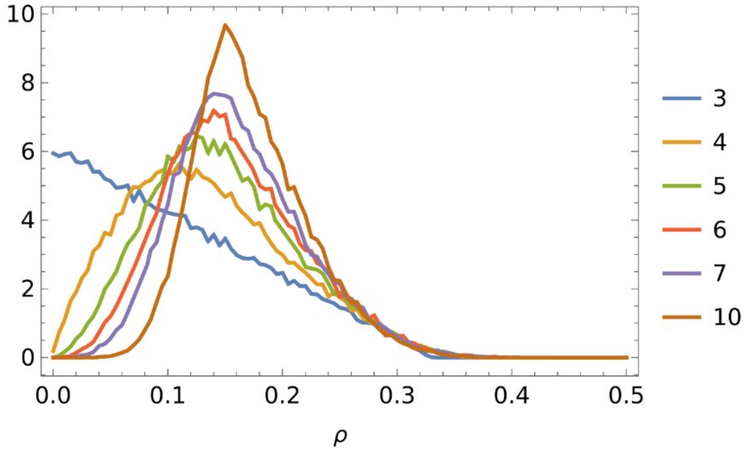
Thrust

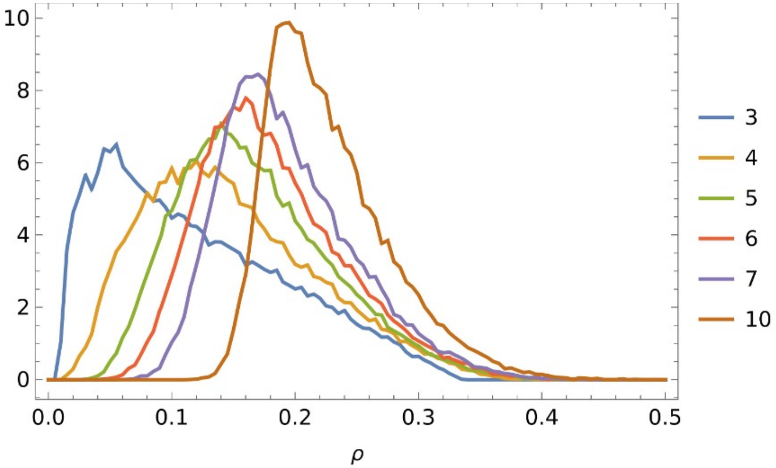


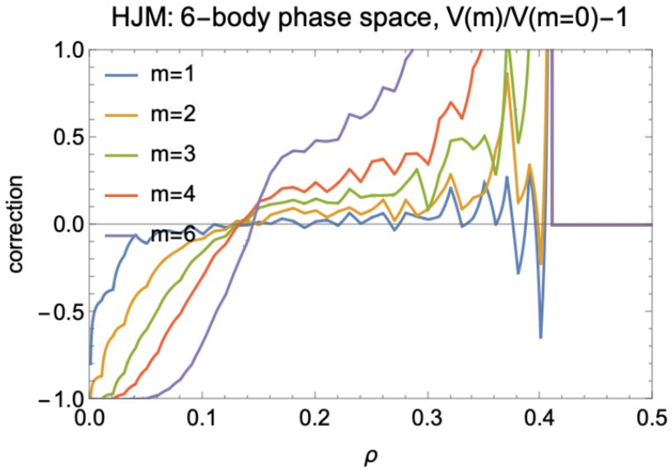




HJM



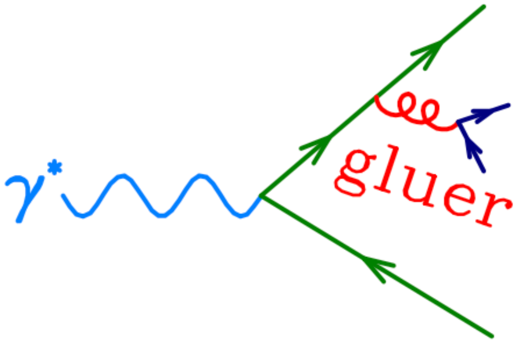




“Gluer” approach

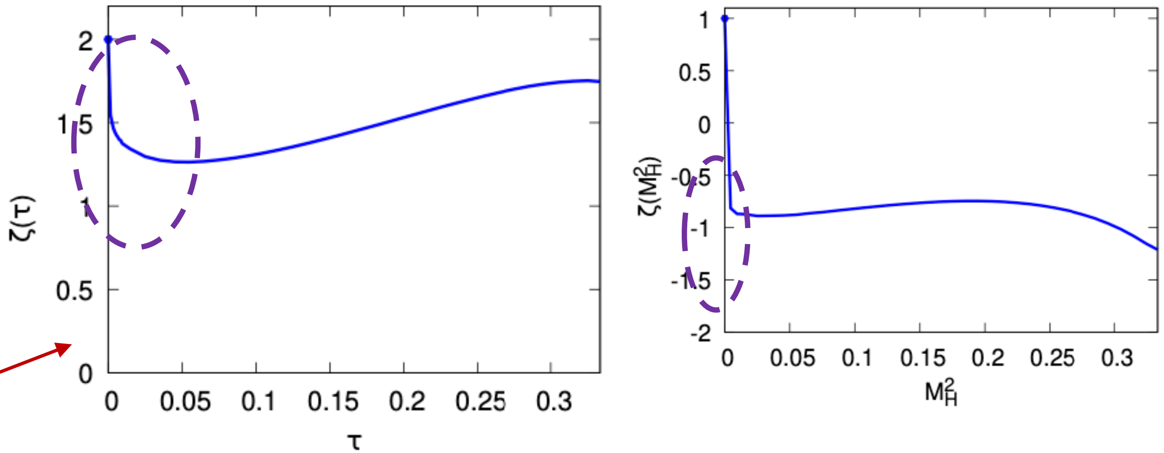
Luisoni, Monni, Salam: :2204.02247
Caola et al arXiv:2204.02247
Nason and Zanderighi: 2301.03607

Use a ”gluer” with offshellness m



- Determine the power correction from the dependence the offshellness regulator
- Map large Nf behavior to the leading IR renormalon

Nason & Zanderighi (2301.03607)



Model as $\Sigma(s) \longrightarrow \Sigma(s + H_{\rm NP}\zeta(s)),$

observable-dependent shift function

positive power for thrust

negative shift for heavy jet mass

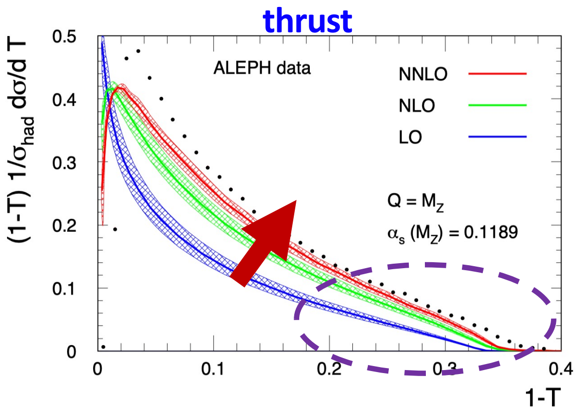
Universal parameter

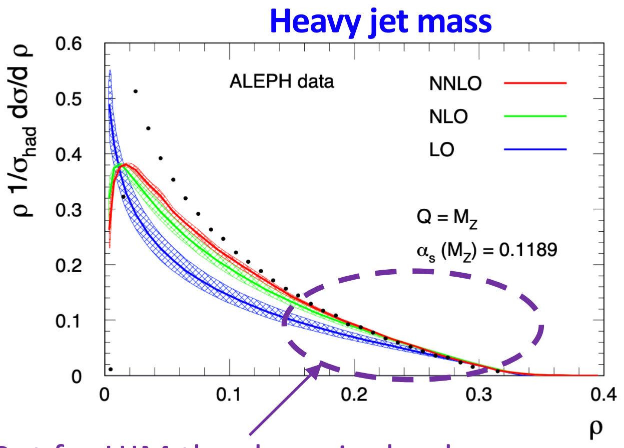
why is it universal?

- **It’s not**
- 100% uncertainty on this function from the “gluer” approach
- perturbation theory not valid in this region anyway

Sudakov Shoulders

2. Perturbative difference

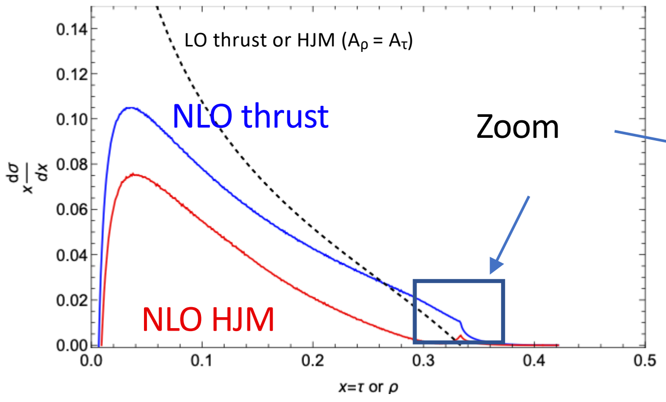




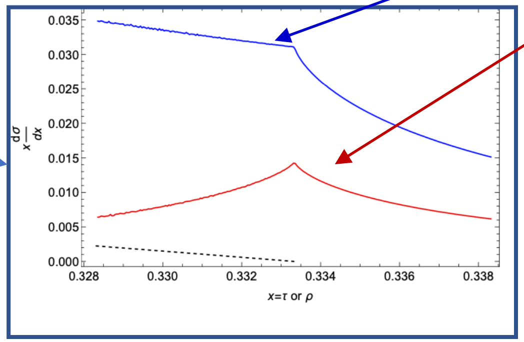
HJM shape much wrong at NNLO!

- Good agreement for larger α/larger NP
  - Shifting up or right would help
- But for HJM the shape is clearly wrong
  - Shifting up or right won't help

Zoom in on the far tail



Zoom

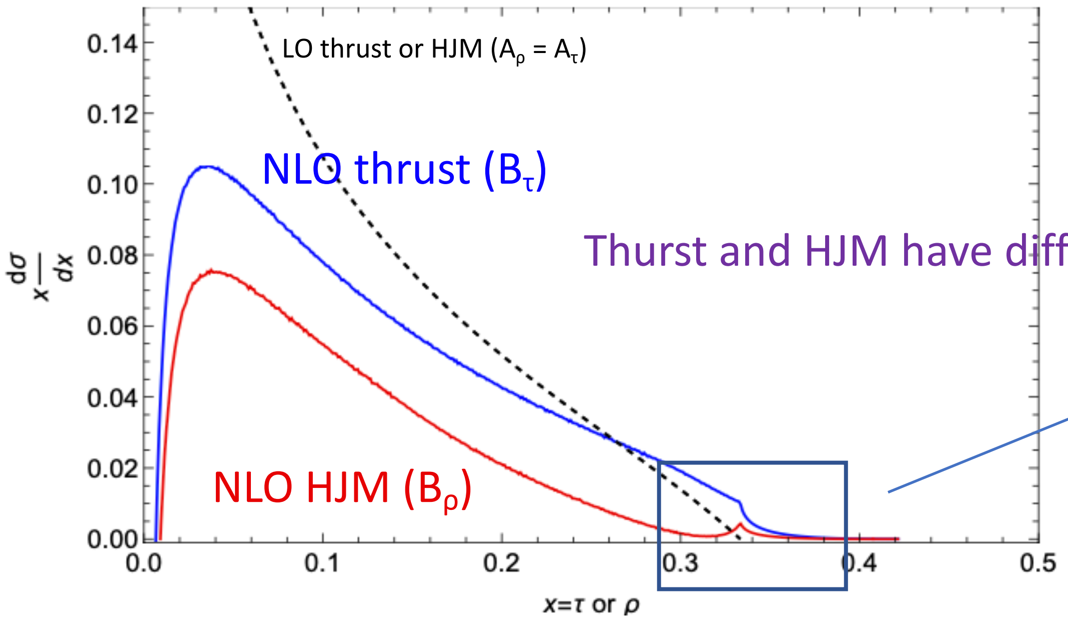


Sudakov shoulders

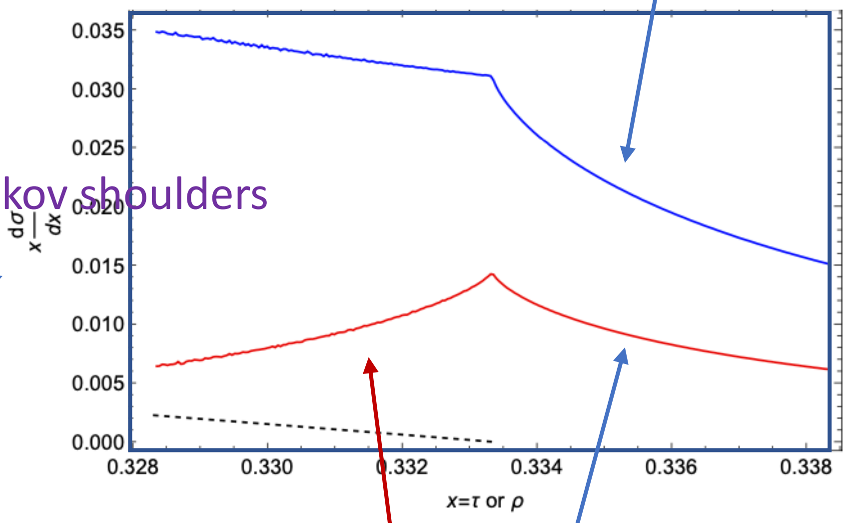
- Thrust has right shoulder only
- HJM has left and right shoulders

Could the Sudakov shoulder explain why HJM and Thrust are different?

Thrust and HJM have different Sudakov shoulders



Thurst and HJM have different Sudakov shoulders

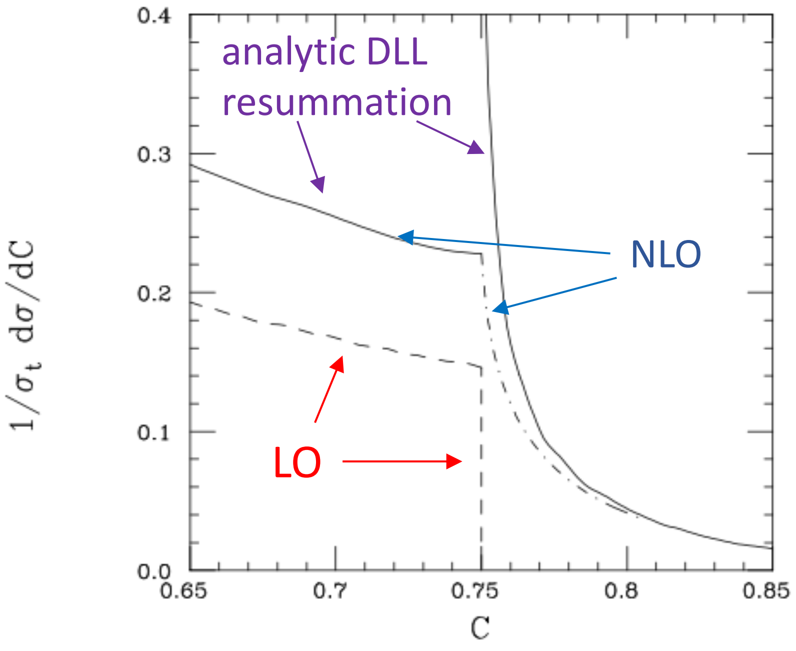


- Thrust has **right** shoulder only
- HJM has **left** and **right** shoulders

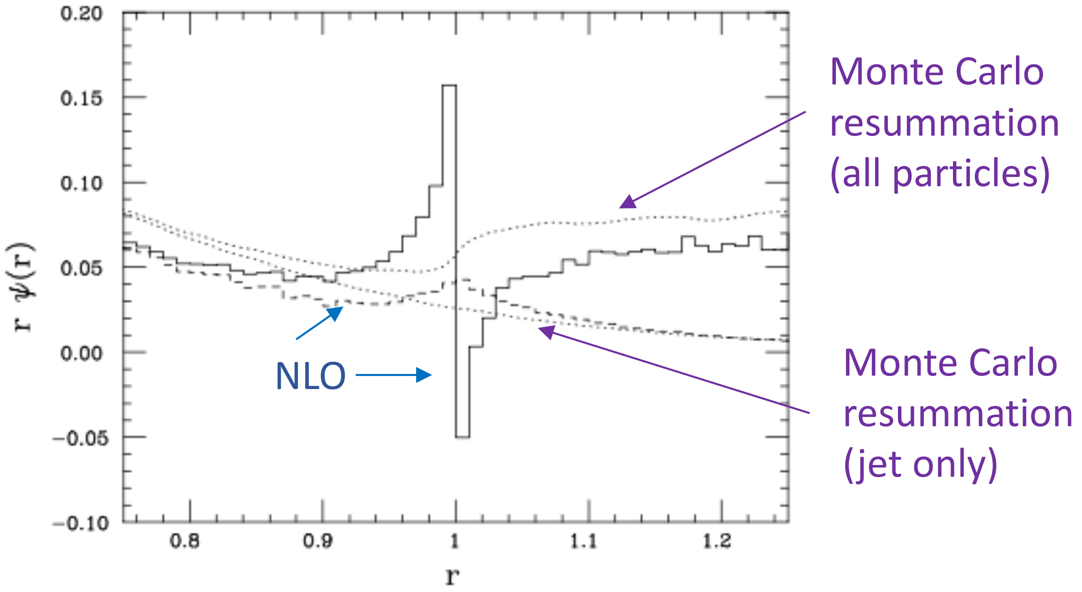
Could resummation of the Sudakov shoulder improve the theory prediction?

Sudakov Shoulders

- Catani and Webber (hep-ph/9710333)
  - C parameter has a right-shoulder
  - discontinuity at C=0.75



- Seymour (hep-ph/9707338)
  - Jet shape has a discontinuity at r=1

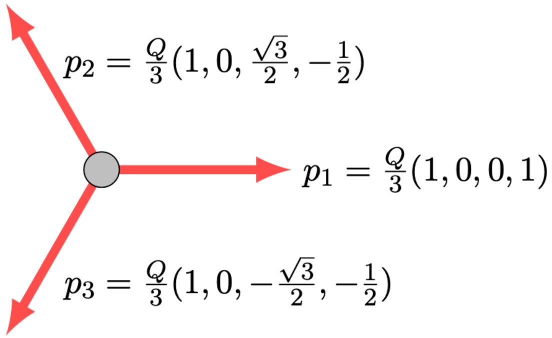


Sudakov Shoulders

Shoulders are a **generic feature** when range of observable changes order by order

LO: C < ¾, ρ,τ <⅓

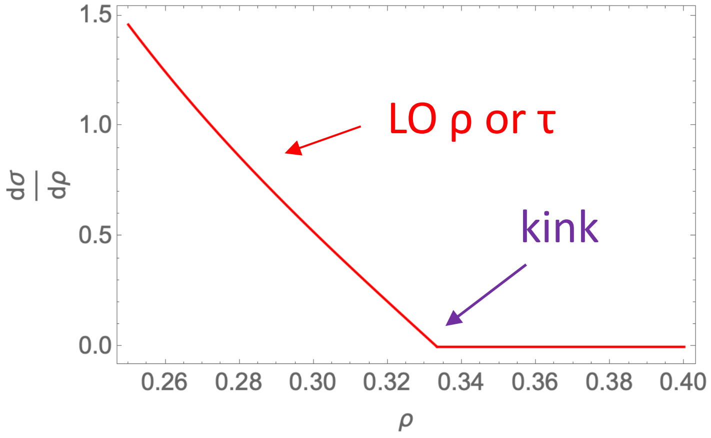
- Maximum value at unique trijet configuration



- Matrix element is finte at this endpoint
  - |M|~ constnat for ρ ~ ⅓

- Phase space zero volume at endpoint
  - Phase space volume vanishes linearly as ρ →⅓

- Cross section vanishes linearly as endpoint approached
  - Leads to kink (discontinuity in first derivative)

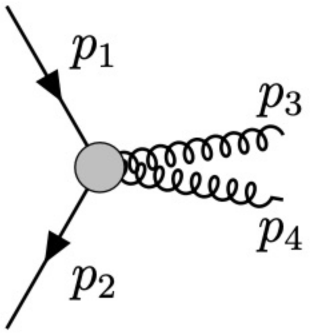


- at NLO get $\frac{d\sigma}{d\rho} \sim \alpha_s r \ln^2 r$ behavior

$$r \equiv \frac{1}{3} - \rho$$

Where do logs come from?

NLO: 4 partons

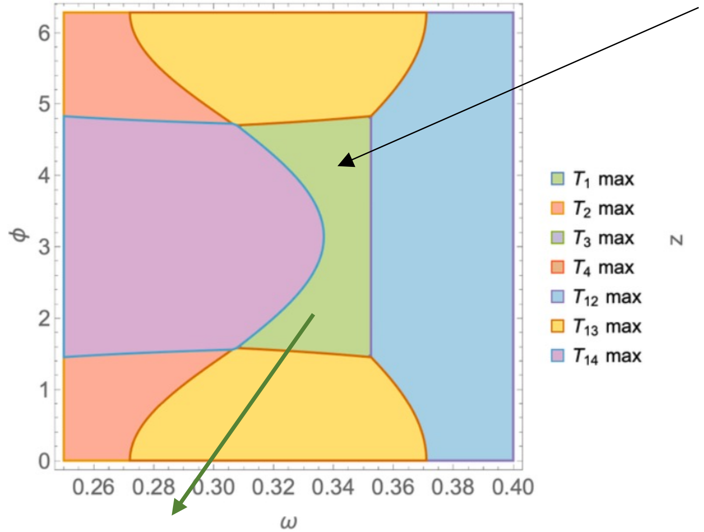


- 5 dimensional phse space
    - Choose 5 energies, angles, invariants

- Compute cross section in small r=1/3-ρ region

Thrust axis $T \equiv \max_{\vec{n}} \frac{\sum_j |\vec{p}_j \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$

- 7 possible options for thrust axis



only region that contributes for $r = \frac{1}{3} - \rho = 0.01$

$$I \sim |\mathcal{M}_0|^2 \frac{\alpha_s}{4\pi} C_F^2 \overbrace{\int_0^r \frac{ds_{34}}{s_{34}} \int_{\frac{9}{4}s_{34}}^1 \frac{dz}{z}}^{\text{cross section}} \overbrace{\int_{\frac{1}{3}-r}^{\frac{1}{3}+2r} ds_{234}}^{\text{Phase space closes off as r} \to 0}$$

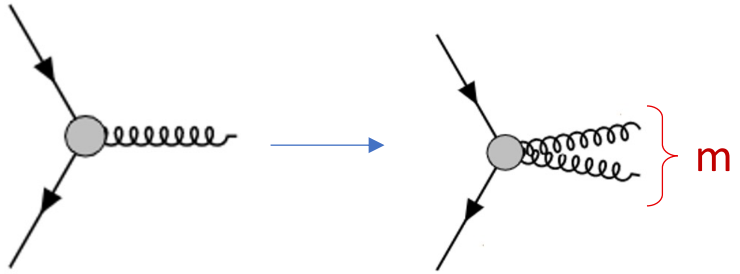
$$\cong |\mathcal{M}_0|^2 \frac{\alpha_s}{4\pi} C_F^2 r \ln^2 r$$

Large shoulder logs

Factorization

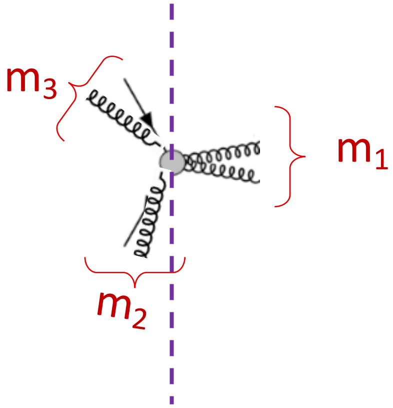
1. Soft and/or collinear emissions turn LO partons into jets

2. Derive constraint relating kinematics to ρ or τ



Consider the case of three massive partons (collinear radiation only)

- Phase space is 2d: described by s and t or $s_{12}=(p_1+p_2)^2$ and $r = ⅓-\rho$



Suppose thrust axis points in 1 direction (other cases by permuation)

- Two inequalities ($T_1 > T_2$ and $T_1 > T_3$), combined together into

$$T \equiv \max_{\vec{n}} \frac{\sum_j |\vec{p}_j \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$$

$$\frac{1}{3} - r + 2m_1^2 - 2m_3^2 < s_{12} < \frac{1}{3} + 2r - m_1^2 + 3m_2^2 + m_3^2$$

- For $m_1 = m_2 = m_3 = 0$ get $⅓-r < s_{12} < ⅓+ 2r$ so phase space cuts off as $r \to 0$

With masses,
phase space is zero unless

$$m_1^2 < r + m_2^2 + m_3^2$$

3. Constraint turns double logs for jet mass into shoulder logs

Factorization

1. Soft and/or collinear emissions turn LO partons into jets

2. Derive constraint relating kinematics to ρ or τ

- Leading power constraint for HJM is

$$\boxed{m_1^2 < r} + m_2^2 + m_3^2$$

  - Solutions for r > 0 and r < 0
  - Large logs in both left and right shoulder

- Leading power constraint for thrust is

$$t < m_1^2 + m_2^2 + m_3^2$$

$$t \equiv \tau - \frac{1}{3}$$

  - Only solutions for t > 0
  - t < 0 no phase space limits
    - not sensitive to emissions
  - Only right shoulder

3. Constraint turns double logs for jet mass into shoulder logs

e.g. One collinear emission (one nonzero mass) $\boxed{m_1^2 < r}$

$$\underbrace{\int_{\frac{1}{3}-r}^{\frac{1}{3}+2r} ds_{12}}_{\text{Phase space closes off as } r \to 0} \underbrace{\int_0^r dm_1^2 \frac{d\sigma}{dm^2}}_{\text{cross section}} \approx \int_{\frac{1}{3}-r}^{\frac{1}{3}+2r} ds_{12} \int_0^r dm_1^2 \frac{\ln m_1^2}{m_1^2} = r \ln^2 r$$

Phase space closes off as r → 0
- generates r prefactor

cross section
- generates sudakov logs

Soft radiation

Including soft and collienar radiation constraint is

HJM

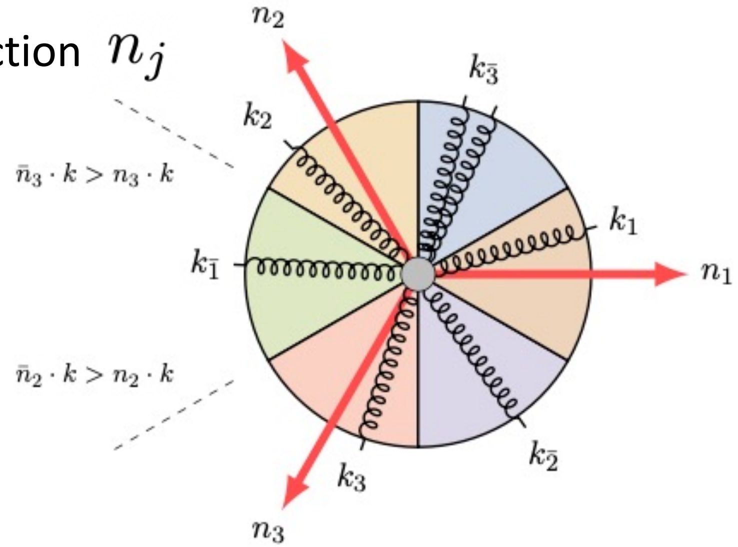
$$m_1^2 + 2p_1k_1 + 2v_{\bar{2}}k_{\bar{2}} + 2v_{\bar{3}}k_{\bar{3}} < r + m_2^2 + 2p_2k_2 + m_3^2 + 2p_3k_3 + 2v_{\bar{1}}k_{\bar{1}}$$

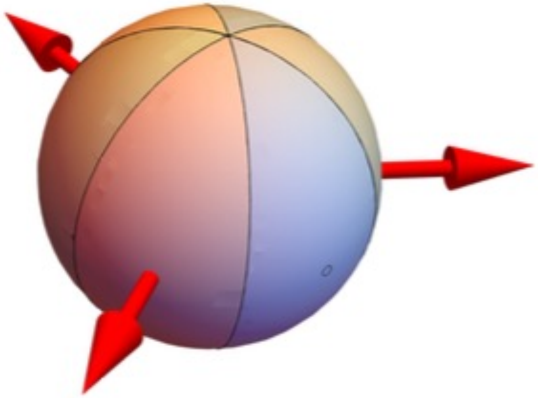
thrust

$$t < m_1^2 + m_2^2 + m_3^2 + 2p_1k_1 + 2p_2k_2 + 2p_3k_3 + 2v_{\bar{1}}k_{\bar{1}} + 2v'_{\bar{2}}k_{\bar{2}} + 2v'_{\bar{3}}k_{\bar{3}}$$

k_j = soft radiation in the sextant aligned with direction n_j

$k_{\bar{j}}$ = soft radiation in the sextant opposite n_j





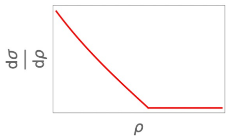
Compute 6-argument soft function

$$S(k_1, k_2, k_3, k_{\bar{1}}, k_{\bar{2}}, k_{\bar{3}})$$

Final factorization formula

$$\frac{1}{\sigma_1}\frac{d\sigma}{dr} = \underbrace{H(Q)}_{\text{Hard}} \int d^3m^2 d^6q\, \underbrace{J(m_1^2)}_{\text{Jet}}\underbrace{J(m_2^2)}_{\text{Jet}}\underbrace{J(m_3^2)}_{\text{Jet}}\overbrace{S_6(q_i)}^{\text{Soft}}\overbrace{W(m_j, q_i, r)\theta[W(m_j, q_i, r)]}^{\text{Measurement function}}$$

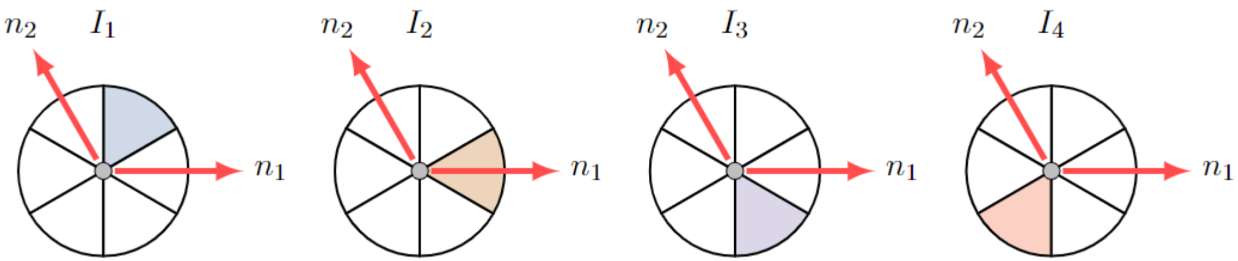
$$\xrightarrow{0^{\text{th}}\text{ order}} \quad \frac{1}{\sigma_0}\frac{d\sigma}{dr} = r\theta(r)$$



$$W(m_j, q_i, r) = r - m_1^2 + m_2^2 + m_3^2 + \frac{2Q}{3}(q_2 + q_3 + q_{\bar{1}} - q_1 - q_{\bar{3}} - q_{\bar{2}})$$

1-loop soft function

Need to compute 4 integrals



$$I_1 = \mathcal{N}\left[4\kappa - \frac{4}{3}\pi\ln 2 + \epsilon c_3\right], \quad I_2 = \mathcal{N}\left[-2\kappa + 2\pi\ln 2 + \epsilon c_4\right]$$

$$I_3 = \mathcal{N}\left[4\kappa + \frac{4}{3}\pi\ln 2 + \epsilon c_5\right], \quad I_4 = \mathcal{N}\left[-2\kappa + 2\pi\ln 2 + \epsilon c_6\right]$$

- Gieseking's constant (transcendality 2 number)

$$\kappa = \operatorname{Im}\operatorname{Li}_2\left(e^{\frac{i\pi}{3}}\right) = 1.014\ldots$$

- Like Catalan $C = \operatorname{Im}\operatorname{Li}_2\left(e^{\frac{i\pi}{2}}\right) = 0.915\ldots$
- Or $-\frac{\pi^2}{12} = \operatorname{Li}_2(e^{i\pi})$

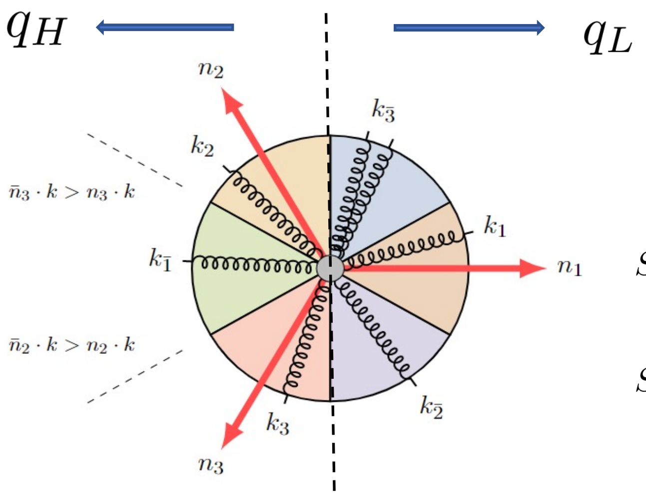
For NNLL resummation we need the constants.
Some of these we were only above to get numerically

$$\kappa = \Im\left[\operatorname{Li}_2\left(e^{\frac{i\pi}{3}}\right)\right], \quad c_1 = \Im\left[\operatorname{Li}_3\left(\frac{i}{\sqrt{3}}\right)\right], \quad c_2 = \Im\left[\operatorname{Li}_3\left(1+i\sqrt{3}\right)\right], \quad c_3 = -1.89958$$

$$c_4 = -3.83452, \quad c_5 = -12.3488, \quad c_6 = 5.56704, \quad c_7 = 15.9482, \quad c_8 = 3.52263$$

1-loop soft function

Combine integrals together to produces trijet hemisphere soft function



$$S(q_L, q_H) = \int d^6 q_i \; S_6(q_i) \delta(q_L - q_1 - q_{\bar{2}} - q_{\bar{3}}) \delta(q_H - q_{\bar{1}} - q_2 - q_3)$$

$$S_g^{\text{one-loop , hemi}}(q_L, q_H, \mu) = \delta(q_L)\delta(q_H) + \frac{\alpha_s(\mu)}{4\pi}\delta(q_L)\left[\frac{-4C_F\Gamma_0 \ln\frac{k_H}{\mu} + \gamma_{sqq}}{k_H}\right]_\star + \frac{\alpha_s(\mu)}{4\pi}\delta(q_H)\left[\frac{-2C_A\Gamma_0 \ln\frac{k_L}{\mu} + \gamma_{sg}}{k_L}\right]_\star + \cdots$$

$$S_q^{\text{one-loop , hemi}}(q_L, q_H, \mu) = \delta(q_L)\delta(q_H) + \frac{\alpha_s(\mu)}{4\pi}\delta(q_L)\left[\frac{-2(C_F + C_A)\Gamma_0 \ln\frac{k_H}{\mu} + \gamma_{sqg}}{k_H}\right]_\star + \frac{\alpha_s(\mu)}{4\pi}\delta(q_H)\left[\frac{-2C_F\Gamma_0 \ln\frac{k_L}{\mu} + \gamma_{sq}}{k_L}\right]_\star$$

$\gamma_{sqq} = -4C_F \ln 6,$ $\gamma_{sg} = -2C_A \ln 3 + 4C_F \ln 2$

$\gamma_{sqg} = -2(C_A + C_F)\ln 6,$ $\gamma_{sq} = -2C_F \ln\frac{3}{2} + 2C_A \ln 2$

- RGE Consistency check - $-\gamma_h = \gamma_{jg} + 2\gamma_{jq} + \gamma_{sqq} + \gamma_{sg} = \gamma_{jg} + 2\gamma_{jq} + \gamma_{sqg} + \gamma_{sq}$ ✓

Previously computed in literature

Resummed distribution

HJM, ρ< ⅓

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dr} = \Pi_g(\partial_{\eta_\ell},\partial_{\eta_h})\, r\left(\frac{rQ}{\mu_s}\right)^{\eta_\ell}\left(\frac{rQ}{\mu_s}\right)^{\eta_h}\frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)}\frac{\sin(\pi\eta_\ell)}{\sin(\pi(\eta_\ell+\eta_h))}$$

HJM, ρ > ⅓

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{ds} = \Pi_g(\partial_{\eta_\ell},\partial_{\eta_h})\, s\left(\frac{sQ}{\mu_s}\right)^{\eta_\ell}\left(\frac{sQ}{\mu_s}\right)^{\eta_h}\frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)}\frac{\sin(\pi\eta_h)}{\sin(\pi(\eta_\ell+\eta_h))}$$

Thrust, τ > ⅓

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dt} = \Pi_g(\partial_{\eta_\ell},\partial_{\eta_h})\, t\left(\frac{tQ}{\mu_s}\right)^{\eta_\ell}\left(\frac{tQ}{\mu_s}\right)^{\eta_h}\frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)}$$

$$\eta_\ell = 2C_A\; A_\Gamma(\mu_j,\mu_s) \\ -2C_A\left(\frac{\alpha_s}{4\pi}\right)\ln r$$

$$\eta_h = 4C_F\; A_\Gamma(\mu_j,\mu_s) \\ \sim -4C_F\left(\frac{\alpha_s}{4\pi}\right)\ln r$$

$$\begin{aligned}\Pi_g(\partial_{\eta_\ell},\partial_{\eta_h}) &= \exp\left[4C_F S(\mu_h,\mu_j)+4C_F S(\mu_s,\mu_j)+2C_A S(\mu_h,\mu_j)+2C_A S(\mu_s,\mu_j)\right] \\ &\times\exp\left[2A_{\gamma sg}(\mu_s,\mu_h)+2A_{\gamma sqq}(\mu_s,\mu_h)+2A_{\gamma jg}(\mu_j,\mu_h)+4A_{\gamma jq}(\mu_j,\mu_h)\right] \\ &\times H(Q,\mu_h)\widetilde{j}_q\left(\partial_{\eta_h}+\ln\frac{Q\mu_s}{\mu_j^2}\right)\widetilde{j}_{\bar q}\left(\partial_{\eta_h}+\ln\frac{Q\mu_s}{\mu_j^2}\right)\widetilde{j}_g\left(\partial_{\eta_\ell}+\ln\frac{Q\mu_s}{\mu_j^2}\right)\widetilde{s}_{qq}(\partial_{\eta_h})\widetilde{s}_g(\partial_{\eta_\ell})\end{aligned}$$

Check fixed order expansion

$$\begin{aligned}\frac{1}{\sigma_1}\frac{d\sigma^{\rm sub}}{dr} &= \frac{\alpha_s}{4\pi}\left\{-\frac{1}{2}(2C_F+C_A)\Gamma_0 r\ln^2 r+\Big[(C_A+2C_F)\Gamma_0+\gamma_{jg}+2\gamma_{jq}+2\gamma_{sg}+4\gamma_{sq}\Big]r\ln r\right\} \\ &= \frac{\alpha_s}{4\pi}\left\{-2(2C_F+C_A)r\ln^2 r+\left[2C_F\left(1+4\ln\frac{4}{3}\right)+C_A\left(\frac{1}{3}+4\ln\frac{4}{3}\right)+\frac{4}{3}n_fT_F\right]r\ln r\right\}+c_0(\alpha_s)+c_1(\alpha_s)r\end{aligned}$$

Check fixed order expansion

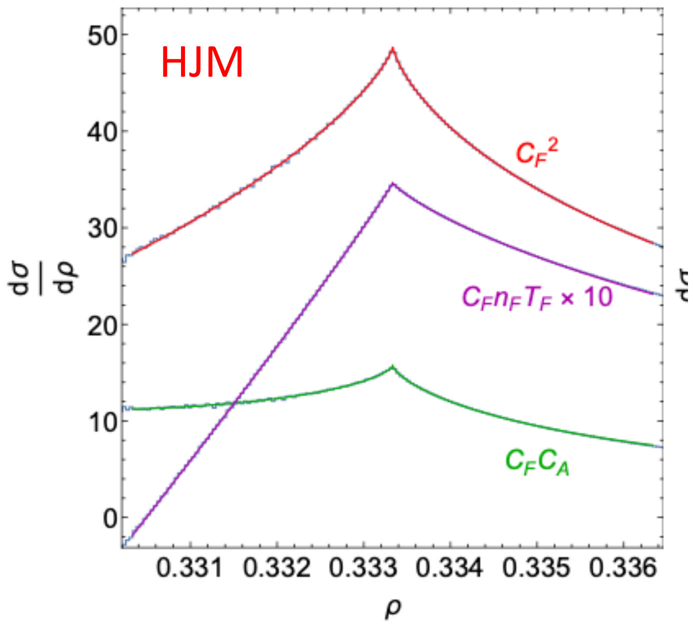
Expanding to order α_s for the HJM left shoulder

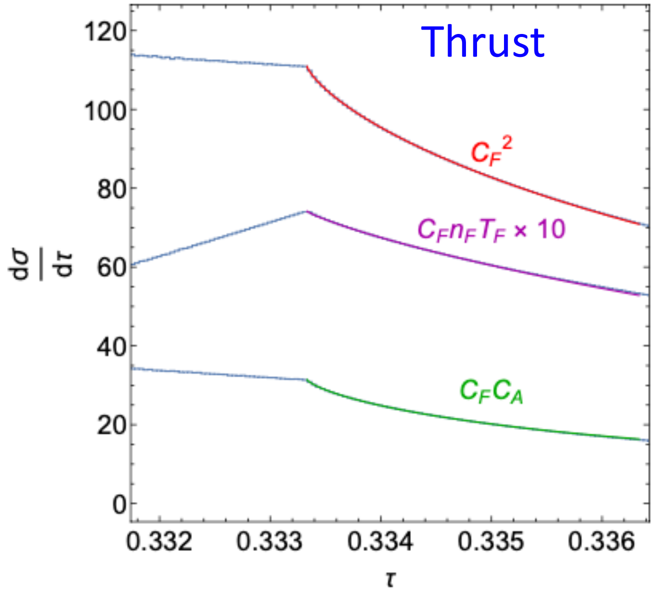
$$\frac{1}{\sigma_1}\frac{d\sigma^{\rm sub}}{dr} = \frac{\alpha_s}{4\pi}\left\{-\frac{1}{2}(2C_F+C_A)\Gamma_0 r\ln^2 r + \Big[(C_A+2C_F)\Gamma_0+\gamma_{jg}+2\gamma_{jq}+2\gamma_{sg}+4\gamma_{sq}\Big]r\ln r\right\}$$
$$= \frac{\alpha_s}{4\pi}\left\{-2(2C_F+C_A)r\ln^2 r+\left[2C_F\left(1+4\ln\frac{4}{3}\right)+C_A\left(\frac{1}{3}+4\ln\frac{4}{3}\right)+\frac{4}{3}n_fT_F\right]r\ln r\right\} + c_0(\alpha_s)+c_1(\alpha_s)r$$

And similarly (with differnet constants) for the HJM right shoulder and thrust

constant and linear term not predicted
(more on this soon)

Excellent agreement with event 2 (NLO fixed order)





- used cutoff = 10^{-12}, 12 trillion events for event2

Resummed distribution

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dr} = \Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h}) r \left(\frac{rQ}{\mu_s}\right)^{\eta_\ell} \left(\frac{rQ}{\mu_s}\right)^{\eta_h} \frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)} \frac{\sin(\pi\eta_\ell)}{\sin(\pi(\eta_\ell+\eta_h))}$$

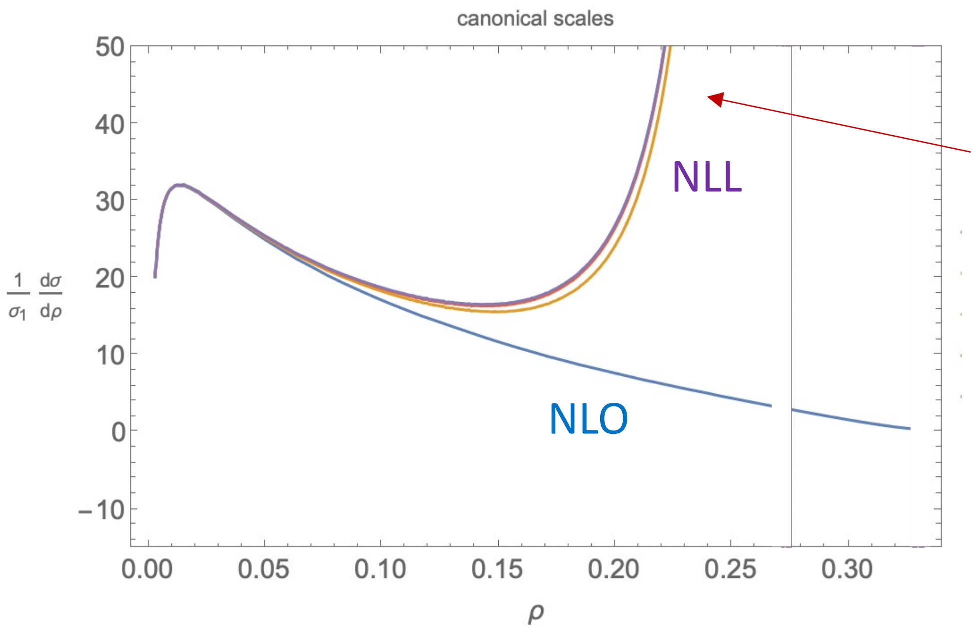
$$\eta_\ell = 2C_A A_\Gamma(\mu_j, \mu_s) \qquad \eta_h = 4C_F A_\Gamma(\mu_j, \mu_s)$$

$$\mu_h = Q$$
$$\mu_j = \sqrt{r}Q$$
$$\mu_s = rQ$$

canonical scales

$$\eta_\ell = -C_A \frac{\Gamma_0}{2}\frac{\alpha_s}{4\pi}\ln r \qquad \eta_h = -2C_F\frac{\Gamma_0}{2}\frac{\alpha_s}{4\pi}\ln r$$

Plot it



Blows up at ρ∼0.25

- Arises when

$$\sin(\pi(\eta_\ell+\eta_h)) = 0 \quad \Leftrightarrow \quad \eta_\ell + \eta_h = (C_A + 2C_F)\frac{\Gamma_0}{2}\frac{\alpha_s}{4\pi}\ln r = 1, 2, 3, \cdots$$

- Comes from running associated with cusp anomalous dimension
- We call this a "**Sudakov Landau pole**"
  - Present even if β=0
  - Similar effects seen in q_T resummation for Drell-Yan
    - resolved by doing resummation in impact-parmaeter space

UV divergences

Factorization formula is of the form

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma_g}{d\rho} = \Pi_g \int_0^\infty dm_\ell^2 \int_0^\infty dm_h^2\ \widetilde{sj}_g \cdot \left(\frac{Qm_l^2}{\mu_{s\ell}}\right)^a \left(\frac{Qm_h^2}{\mu_{sh}}\right)^b \frac{1}{m_l^2 m_h^2}\frac{e^{-\gamma_E(a+b)}}{\Gamma(a)\Gamma(b)}\Bigg|_{\substack{a=\eta_\ell \\ b=\eta_h}} \times (r+m_h^2-m_\ell^2)\theta(r+m_h^2-m_\ell^2)$$

- can have $r \ll 1, \quad m_h^2 \gg 1, \quad m_\ell^2 \gg 1$
- integral is linearly UV divergent in this region for 0 < a,b <1

Simplest to **take two more derivatives** and consider

$$f(r) \equiv \frac{1}{\Gamma(a)}\frac{1}{\Gamma(b)}\int_0^\infty dx \int_0^\infty dy\ x^{a-1}y^{b-1}\delta(r+y-x)$$

$$= \frac{1}{\Gamma(a+b)}\left[r^{a+b-1}\frac{\sin(\pi a)}{\sin(\pi(a+b))}\theta(r) + (-r)^{a+b-1}\frac{\sin(\pi b)}{\sin(\pi(a+b))}\theta(-r)\right]$$

- Now UV finite
  - Will need to integrate twice and set integration constants $c_0 + c_1 r$
- Formula is valid for ρ< ⅓ (r>0) as well as ρ > ⅓ (r<0)
- Still has Sudakov Landau pole at a+b=1,2,3,...

Position vs momentum space

In momentum space, distribution is complicated and non-analytic

$$f(r) \equiv \frac{1}{\Gamma(a)}\frac{1}{\Gamma(b)}\int_0^\infty dx \int_0^\infty dy\ x^{a-1}y^{b-1}\delta(r+y-x)$$
$$= \frac{1}{\Gamma(a+b)}\left[r^{a+b-1}\frac{\sin(\pi a)}{\sin(\pi(a+b))}\theta(r) + (-r)^{a+b-1}\frac{\sin(\pi b)}{\sin(\pi(a+b))}\theta(-r)\right]$$

In **position space**, distribution is remarkably simple

$$\tilde{f}(z) = \int_{-\infty}^{\infty} dr\, f(r) e^{izr} = (-iz)^a (iz)^b$$

- difficult to compute
- must carefully track analytic continuation

No longer has Sudakov landau poles at a+b = 1,2,3...

- Note: must be position space not Laplace space
- In **Laplace space** distribution is **not simple**
  - For 1-sided Laplace transform, need to flip sign in exponent to make integral convergent

$$\mathcal{L}[f](\nu) = \int_0^\infty dr\ e^{-\nu r} f(r) + \int_{-\infty}^0 dr\ e^{\nu r} f(r) = \nu^{-a-b}\frac{\sin(\pi a)+\sin(\pi b)}{\sin(\pi(a+b))}\,.$$

  - Still has Sudakov Landau pole

Γ_0 approximation

To undestand better what is happening, consider the "Γ_0 approximation"

- set all γ_j=0 and β=0
- keep only leading cusp anomalous dimension Γ_0

Momentum space

$$\frac{1}{\sigma_{\rm LO}}\frac{d^3\sigma_g}{d\rho^3} = e^{-2\hat\alpha C_F \ln^2\frac{\mu_h}{\mu_{jh}} - 2\hat\alpha C_F\ln^2\frac{\mu_{sh}}{\mu_{jh}} - \hat\alpha C_A \ln^2\frac{\mu_h}{\mu_{j\ell}} - \hat\alpha C_A\ln^2\frac{\mu_{s\ell}}{\mu_{j\ell}}} \times \frac{e^{-\gamma_E(a+b)}}{\Gamma(a+b)}$$

$$\times\left[\frac{1}{r}\left(\frac{rQ}{\mu_{s\ell}}\right)^a\left(\frac{rQ}{\mu_{sh}}\right)^b\frac{\sin(\pi a)}{\sin(\pi(a+b))}\theta(r) + \frac{1}{(-r)}\left(\frac{-rQ}{\mu_{s\ell}}\right)^a\left(\frac{-rQ}{\mu_{sh}}\right)^b\frac{\sin(\pi b)}{\sin(\pi(a+b))}\theta(-r)\right]$$

$$\mu_{s\ell} = \mu_{sh} = |r|Q$$
$$\mu_{j\ell}^2 = \mu_{jh}^2 = Q\mu_{s\ell} = Q\mu_{sh}$$

canonical scales

$$\left(\frac{1}{\sigma_{\rm LO}}\frac{d^3\sigma}{d\rho^3}\right) = \frac{1}{r}e^{-\Gamma_0\frac{1}{2}C_A\hat\alpha\ln^2|r| - \Gamma_0 C_F\ln^2|r|}\frac{e^{-\gamma_E(a+b)}}{\Gamma(a+b)}\left[\frac{\sin(\pi a)}{\sin(\pi(a+b))}\theta(r) - \frac{\sin(\pi b)}{\sin(\pi(a+b))}\theta(-r)\right]$$

$$a = -C_A\hat\alpha\Gamma_0\ln|r|$$
$$b = -2C_F\hat\alpha\Gamma_0\ln|r|$$

- Sudakov Landau poles are there
- Not of the double-logarithmic form exp[L g_1(α L)]
  - Includes problematic subleading terms

Position space

$$\widetilde\sigma(z) = \int_{-\infty}^{\infty} dr\, e^{izr}\,\frac{1}{\sigma_{\rm LO}}\frac{d^3\sigma}{d\rho^3}$$

$$= e^{-2\hat\alpha C_F \ln^2\frac{\mu_h}{\mu_{jh}} - 2\hat\alpha C_F\ln^2\frac{\mu_{sh}}{\mu_{jh}} - \hat\alpha C_A \ln^2\frac{\mu_h}{\mu_{j\ell}} - \hat\alpha C_A\ln^2\frac{\mu_{s\ell}}{\mu_{j\ell}}}\left(-iz\frac{\mu_{s\ell}e^{\gamma_E}}{Q}\right)^{-a}\left(iz\frac{\mu_{sh}e^{\gamma_E}}{Q}\right)^{-b}$$

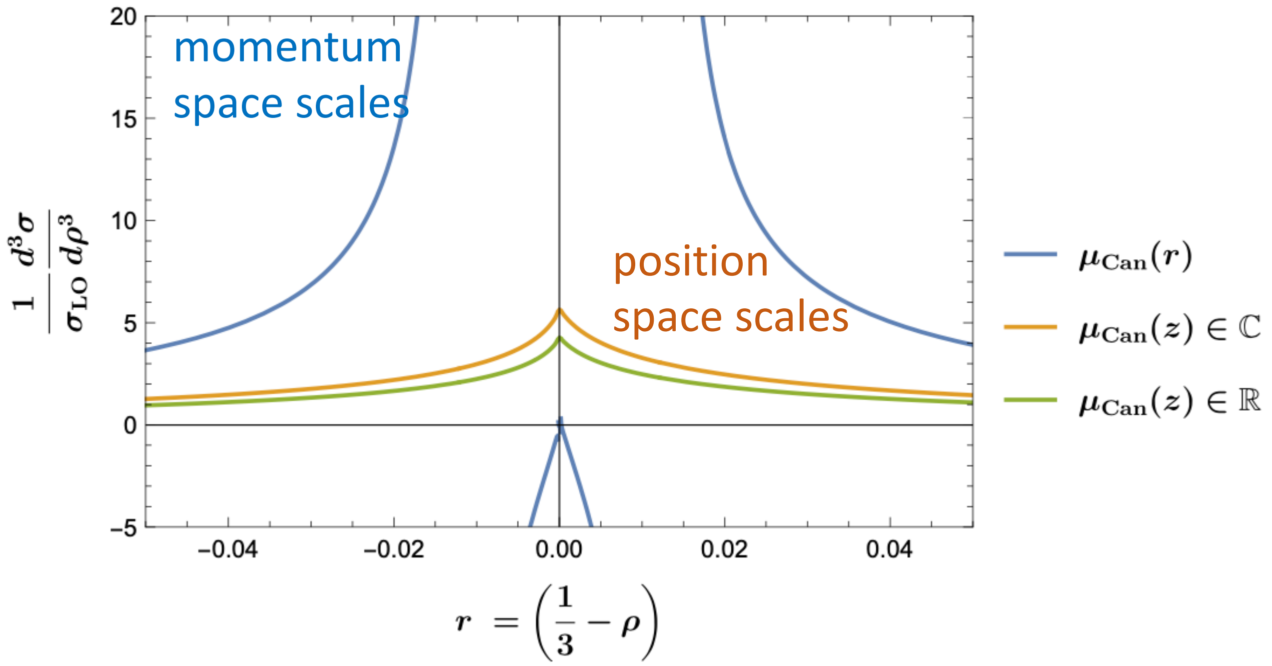
canonical complex scales

$$\mu_{s\ell} = i\frac{Qe^{-\gamma_E}}{z}, \quad \mu_{sh} = -i\frac{Qe^{-\gamma_E}}{z}$$

$$\widetilde\sigma(z) = \exp\left[-\frac{1}{2}\hat\alpha C_A\Gamma_0\ln^2(-ize^{\gamma_E}) - \hat\alpha C_F\Gamma_0\ln^2(ize^{\gamma_E})\right]$$

- Sudakov Landau poles are gone
- Exactly of the double-logarithmic form exp[L g_1(α L)]
  - Problematic subleading terms are absent

Γ_0 approximation



With position space scale setting

- Sudakov Landau poles are gone
- Distribution is continuous across r=0
- With running coupling wil be smooth across r=0

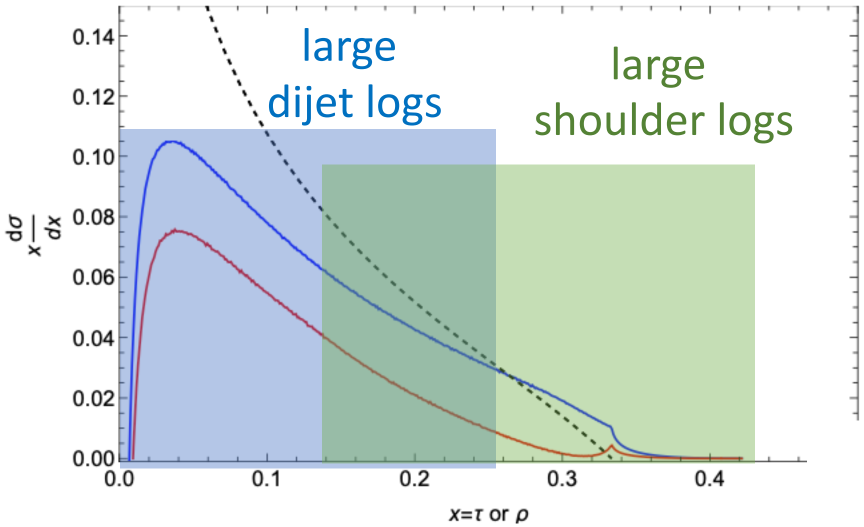
Can show that

$$\left(\frac{1}{\sigma_{\rm LO}}\frac{d^3\sigma}{d\rho^3}\right)_{\rm pos} = e^{-\hat{\alpha}\Gamma_0 \frac{1}{2} C_A \partial_a^2 - \hat{\alpha}\Gamma_0 C_F \partial_b^2} \left(\frac{1}{\sigma_{\rm LO}}\frac{d^3\sigma}{d\rho^3}\right)_{\rm mom}$$

- derivatives blow up at Sudakov Landau pole
- exp(-∂^2) supresses the divergence

Matching to the dijet region

We need to match bewteen the shoulder region and the dijet region



Look at fixed order

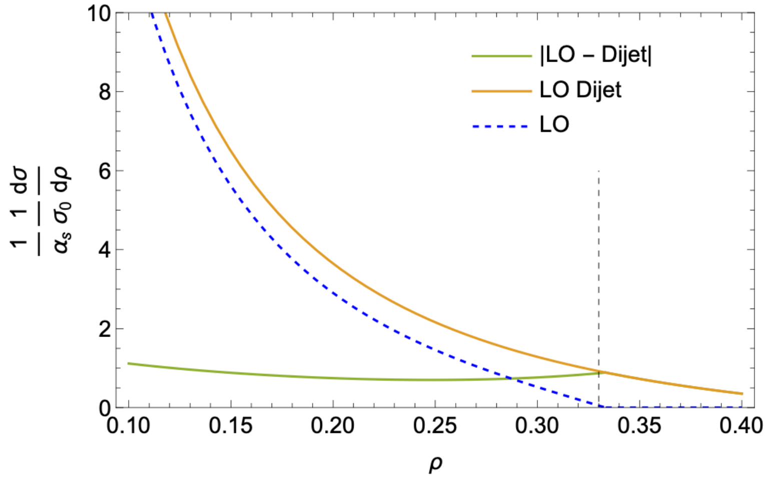
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\rho} = C_F\frac{\alpha_s}{2\pi}\left\{\frac{3(1+\rho)(3\rho-1)}{\rho} + \frac{[4+6\rho(\rho-1)]\ln\frac{1-2\rho}{\rho}}{\rho(1-\rho}\right]\theta\left(\frac{1}{3}-\rho\right)$$

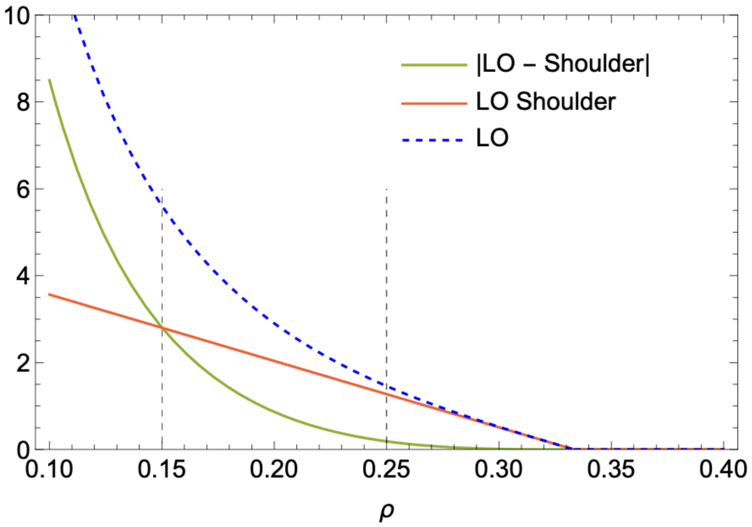
expand near ρ=0

$$\frac{1}{\sigma_0}\frac{d\sigma^{\rm dij}}{d\rho} = C_F\frac{\alpha_s}{2\pi}\left(-\frac{3}{\rho}-\frac{4\ln\rho}{\rho}\right)$$

expand near ρ=⅓

$$\frac{1}{\sigma_0}\frac{d\sigma^{\rm shoulder}}{d\rho} = C_F\frac{\alpha_s}{2\pi}72\left(\frac{1}{3}-\rho\right)\theta\left(\frac{1}{3}-\rho\right)$$



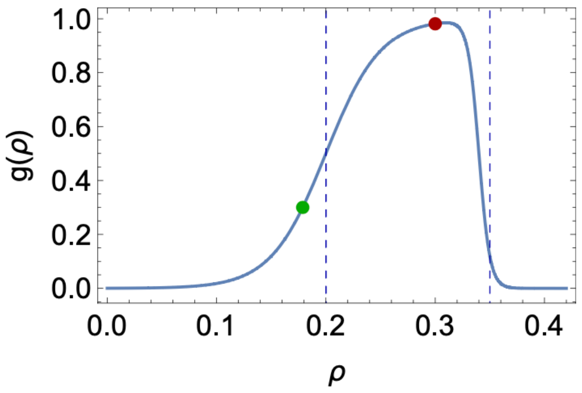


- want pure shoulder for ρ > 0.25
- fade to dijet by ρ < 0.15

Matching to the dijet region

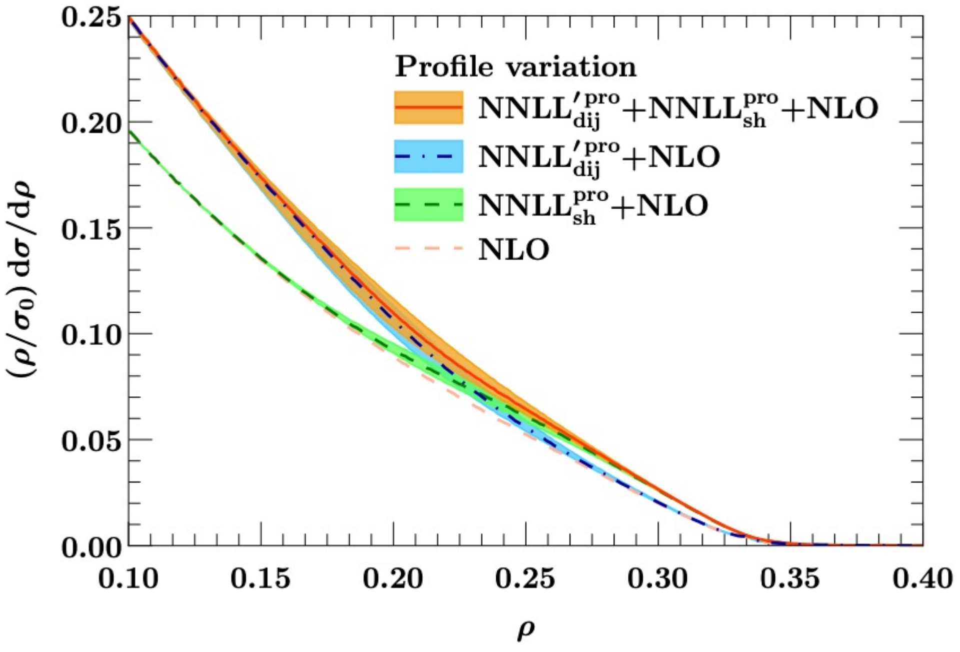
Turn off resummation in shoulder and dijet region by using ρ dependent profile functions for soft and jet scales:

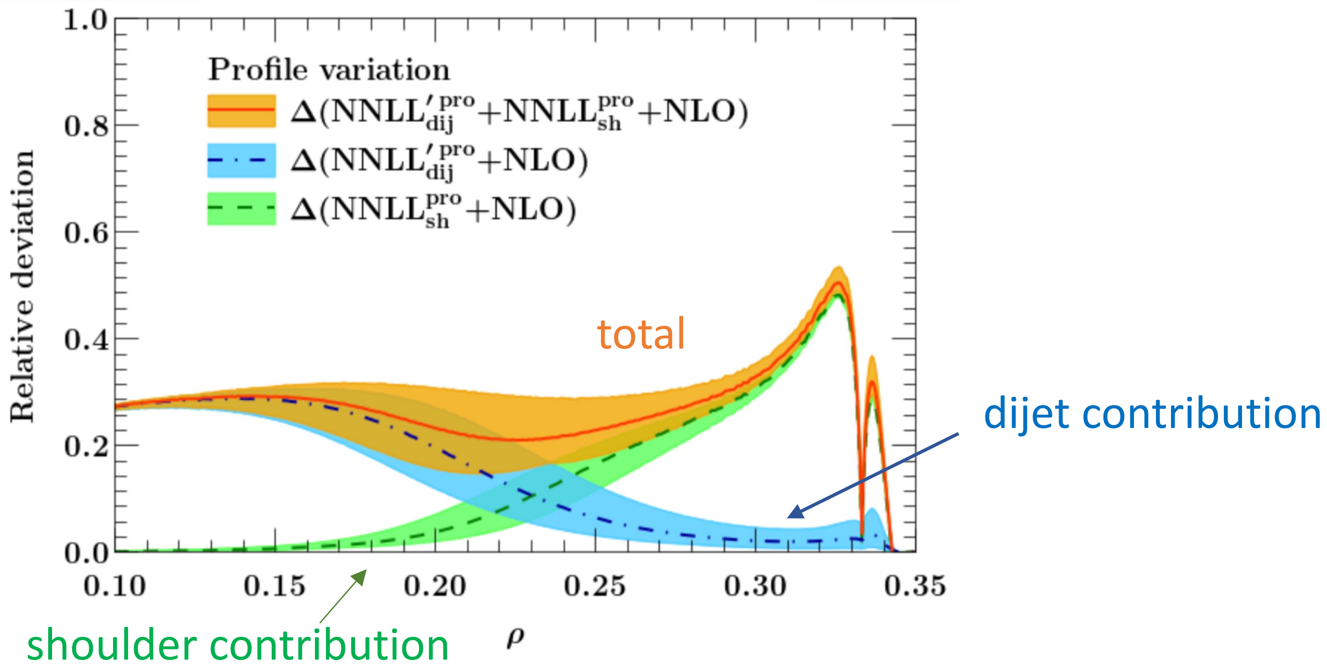
$$\mu_{j,s}^{\text{pro}}(z,\rho) = \mu_h^{1-g(\rho)}[\mu_{j,s}^{\text{can}}(z)]^{g(\rho)}$$



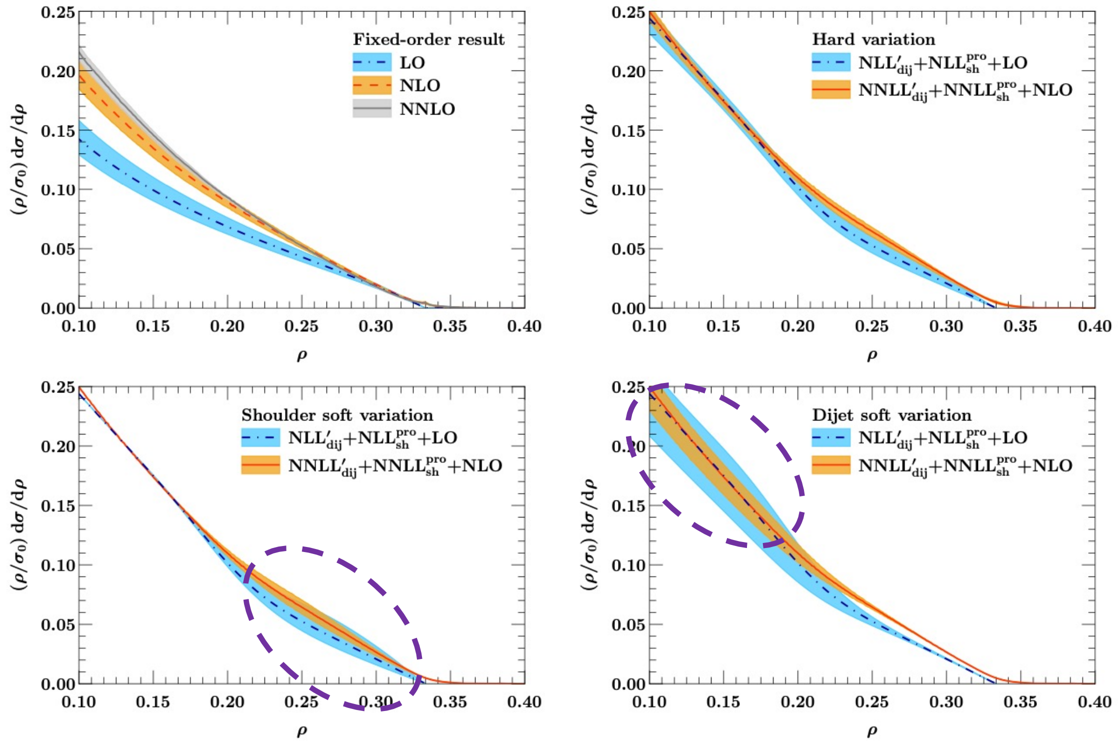
We use sigmoid functions to

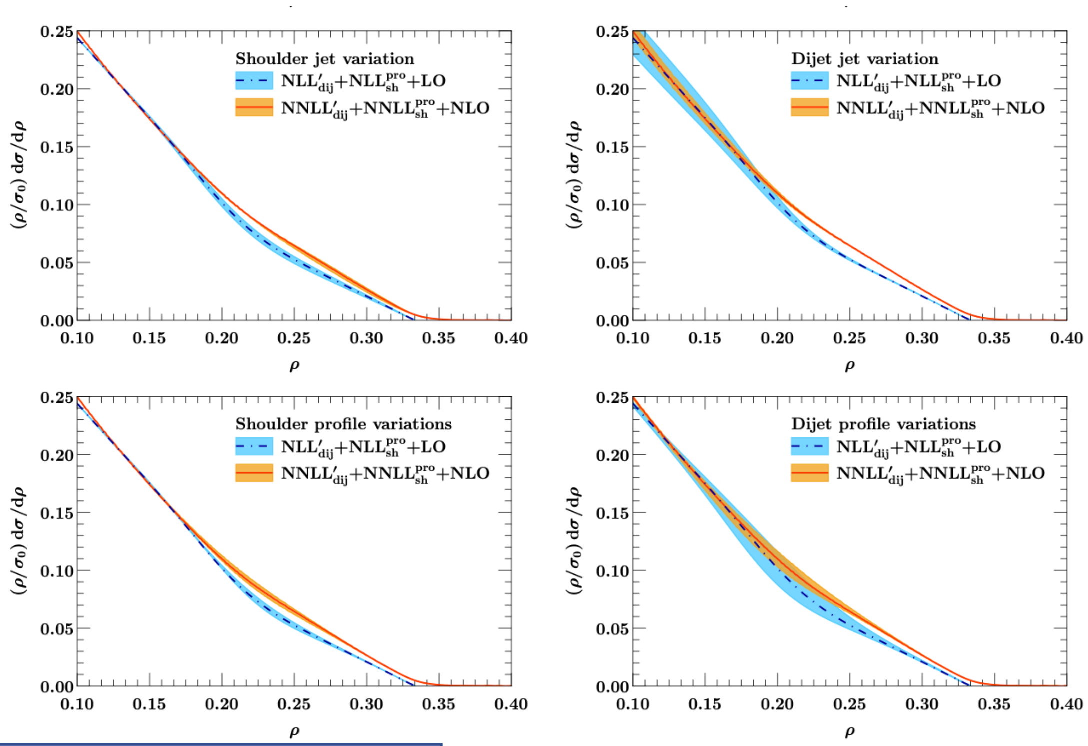
- fade between shoulder and dijet
- fade out resummation in the right shoulder.





Scale variations

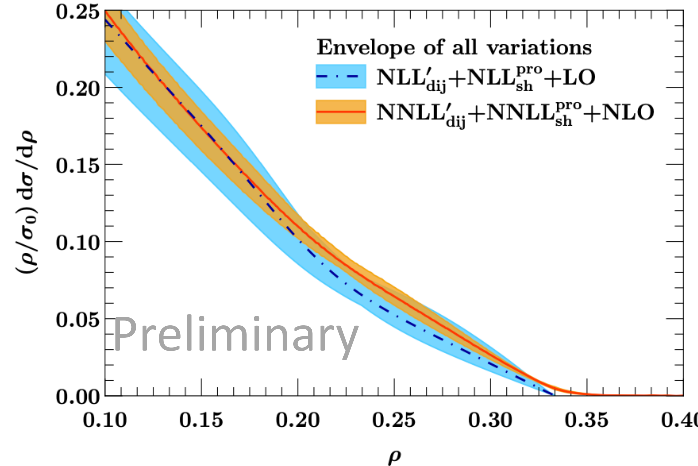




$$\mu_h = 2^{v_h} Q$$

$$\mu_{s,\mathrm{sh}}(z) = \sqrt{\left(2^{v_h} 2^{v_s} \frac{Q e^{-\gamma_E}}{|z|}\right)^2 + \left(\mu_s^{\min}\right)^2}$$

$$\mu_{j,\mathrm{sh}}(z) = \Big(\mu_h \mu_{s,\mathrm{sh}}(z)\Big)^{v_j} Q^{1-2v_j}$$

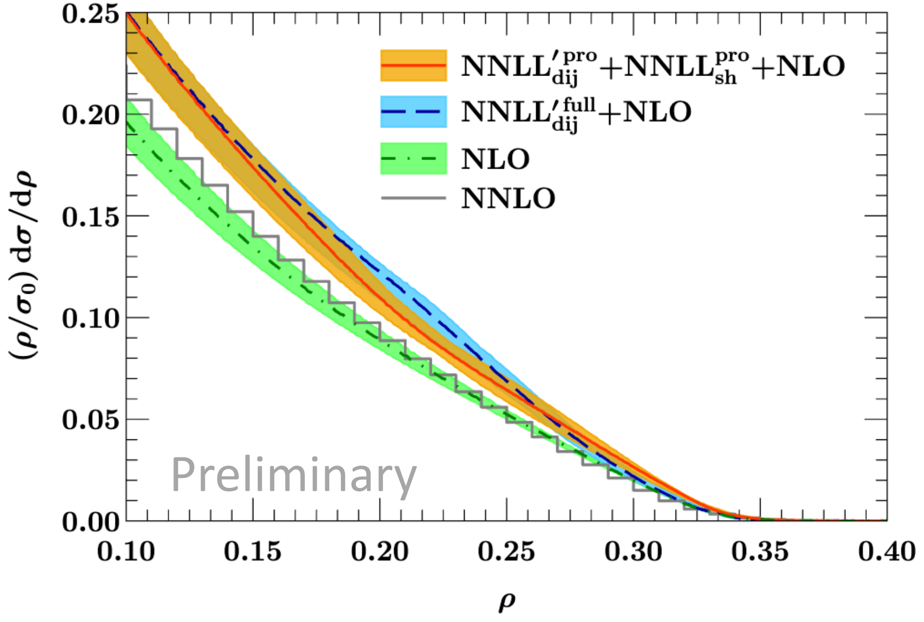


Envelope of all variations

- dominated by soft variation
- will go down a bit with N^3LL dijet resummation

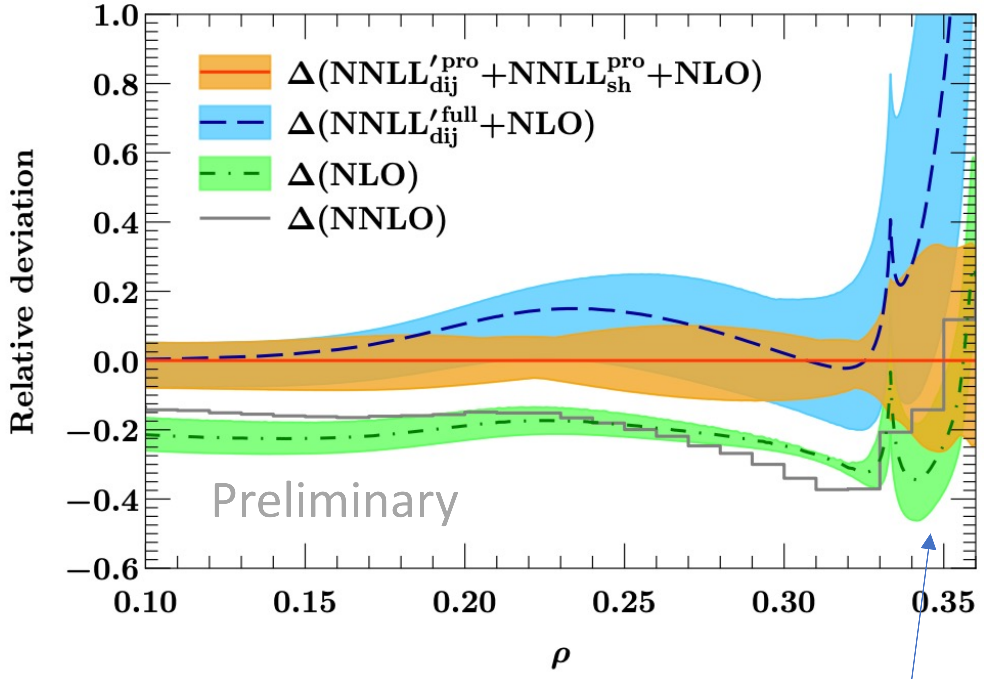
Final result

Compare to NLO and just dijet resummation



ratio to best

$$\Delta(d\sigma) = \frac{d\sigma - d\sigma_{\mathrm{sh+dij+NLO}}}{d\sigma_{\mathrm{sh+dij+NLO}}}$$

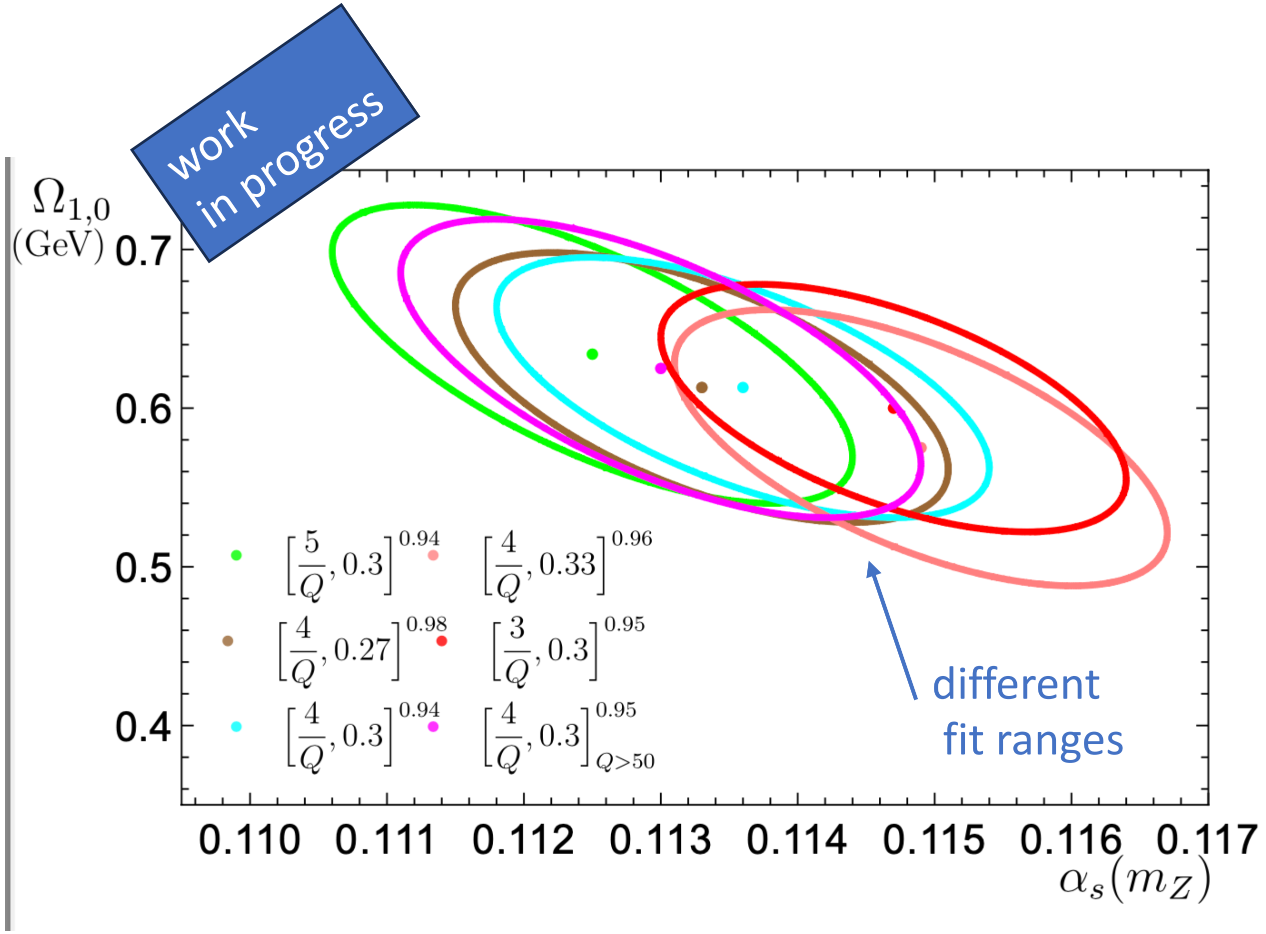


- Matched result is
  - 20% higher than NLO
  - More important than NNLO correction
- Pure dijet + NLO is 5% to 20% larger for $0.15 < \rho < 0.3$

things go nuts because denominator is zero

New fits for HJM

Work in progress with Vicent Mateu,
Andre Hoang, Iain Stewart, Miguel Benitez-Rathgeb,
Xioayuan Zhang, Arindam Bhattacharya

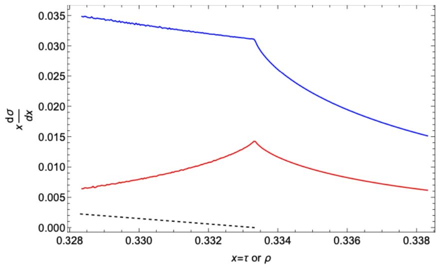


- NP model function
- Renormalon subtraction
- m_b effects
- QED corrections
  - α_s still coming out very low

$\alpha_s \sim 0.114$

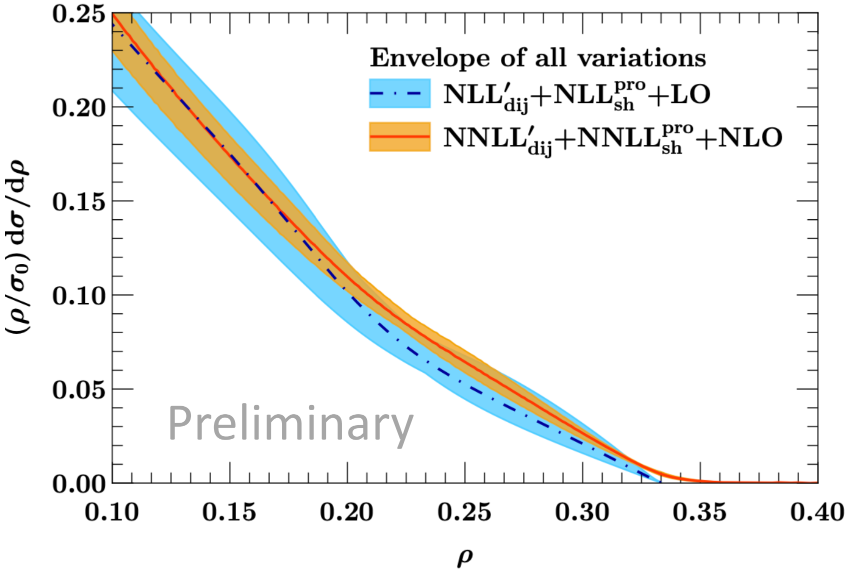
- Compatible with thrust
- Still low compared to world average
- fairly large dependence on fit range

Conclusions



- Sudakov shoulders are large logarithms associated with perturbative phase space boundaries
  - Heavy jet mass has a left Sudakov shoulder ($\rho < ⅓$) and a right sudakov shoulder ($\rho > ⅓$)
    - large logs extend down to $\rho = 0.15$ where α_s fits are done
  - Thrust only has a right shoulder ($\tau > ⅓$)
    - shoulder resummation is not critical for α_s fits for thrust

- Factorization theorem for the Sudakov shoulder allows for systematic resummation
  - Scales must be set in **position space** before Fourier transforming to avoid spurious Sudakov Landau poles
  - We resummed the HJM shoulder to NNLL, matching to NNLL dijet resummation and NLO fixed order



- **Effect is signifcant**
  - 5% to 100% larger than dijet+NLO for $0.15 < \rho < 0.3$

Next steps

- Extend matching to NNLO and N^{3}LL dijet
- Study **power corrections**
  - Factorization formula valid in the trijet region gives operator definition
  - How to interpolate power corrections from dijet to shoulder region?
  - How many non-perturbative parameters are needed?
- Extract α_s from e^+e^- data