SCALE SETTING: LESSONS FROM SCET

Scales, Cambridge

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Two questions

1. What central value of the scales should we choose?

- Goal is to have best agreement with data

SCET helps with this

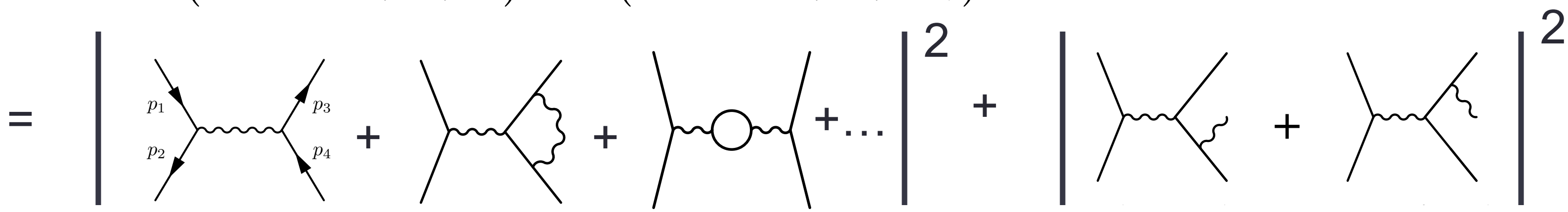
2. How much should we vary around that scale

- Goal is to produce uncertainty treatable like statistical uncertainty
- Want 95% (or 68%?) confidence that next order will be within uncertainty bands

Does SCET help with this?

Scale setting for inclusive observables

$$\sigma_{\text{tot}} = \sigma(e^+e^- \to \mu^+\mu^-) + \sigma(e^+e^- \to \mu^+\mu^-\gamma)$$



$$= \frac{4\pi\alpha(\mu)}{3Q^2}\left(1 + \frac{3\alpha(\mu)}{4\pi} + \beta_0 \ln\frac{Q}{\mu} + \cdots\right)$$

- Only one scale, so choosing $\mu = Q = E_{CM}$ turns σ_{tot} into a series in $\alpha(Q)$

$$\sigma_{\text{tot}} = \frac{4\pi\alpha(Q)}{3Q^2}\left[1 + 3\left(\frac{\alpha(Q)}{4\pi}\right) + (-\frac{3}{2} - 1.38 n_f)\left(\frac{\alpha(Q)}{4\pi)}\right)^2 + \cdots\right]$$

- Varying μ adds terms at higher order in α, example of higher order effects

$$\frac{4\pi\alpha(2Q)}{3Q^2} = \frac{4\pi\alpha(Q)}{3Q^2}\left[1 + \frac{1}{3}n_f \ln 2\left(\frac{\alpha(Q)}{4\pi}\right) + \cdots\right]$$

$$\frac{\ln 2}{3} n_f = 3 \text{ for } n_f \approx 13$$

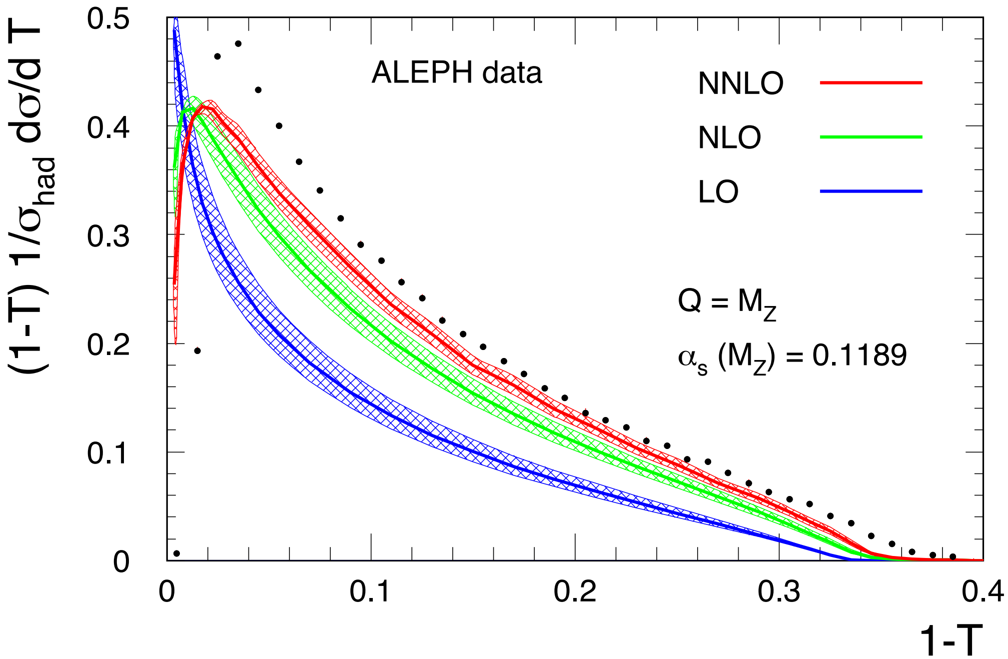
$$\frac{\ln x}{3} = 3 \text{ for } x = 8103.1$$

- Doesn't even have the right scaling with group factors
  - With one flavor, we need μ = 8000 Q to get NLO effect right

Thrust distribution: fixed order

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \delta(\tau) + C_F \frac{\alpha_s(\mu)}{2\pi}\left[\frac{-4\ln\tau - 3}{\tau} - 8 + 2\ln\tau + \cdots\right]$$
$$+ \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\left[\beta_0 \ln\frac{Q}{\mu} + \frac{C_F^2}{2}\frac{\ln^3\tau}{\tau} + \cdots\right]$$

[Gehrmann et al. 0711.4711, 2011]



- Choose $\mu = Q$
  - by dimensional analysis?
- Why not $\mu = \tau Q$ or $\mu^2 = \tau Q^2$?

- Clearly underestimating errors!
- Poor convergence

Thrust distribution: SCET

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_2}{\mathrm{d}\tau} = H(Q^2,\mu)\int \mathrm{d}p_L^2 \mathrm{d}p_R^2 \mathrm{d}k\, J(p_L^2,\mu)\, J(p_R^2,\mu)\, S_T(k,\mu)\delta(\tau - \frac{p_L^2+p_R^2}{Q^2} - \frac{k}{Q})$$

Each function has one scale

Hard function: Q
(hard scale, like COM energy)

Jet function: p^2
(mass of the jet)

Soft scale
(out-of-jet energy)

- Natural scales read off from factorization formula

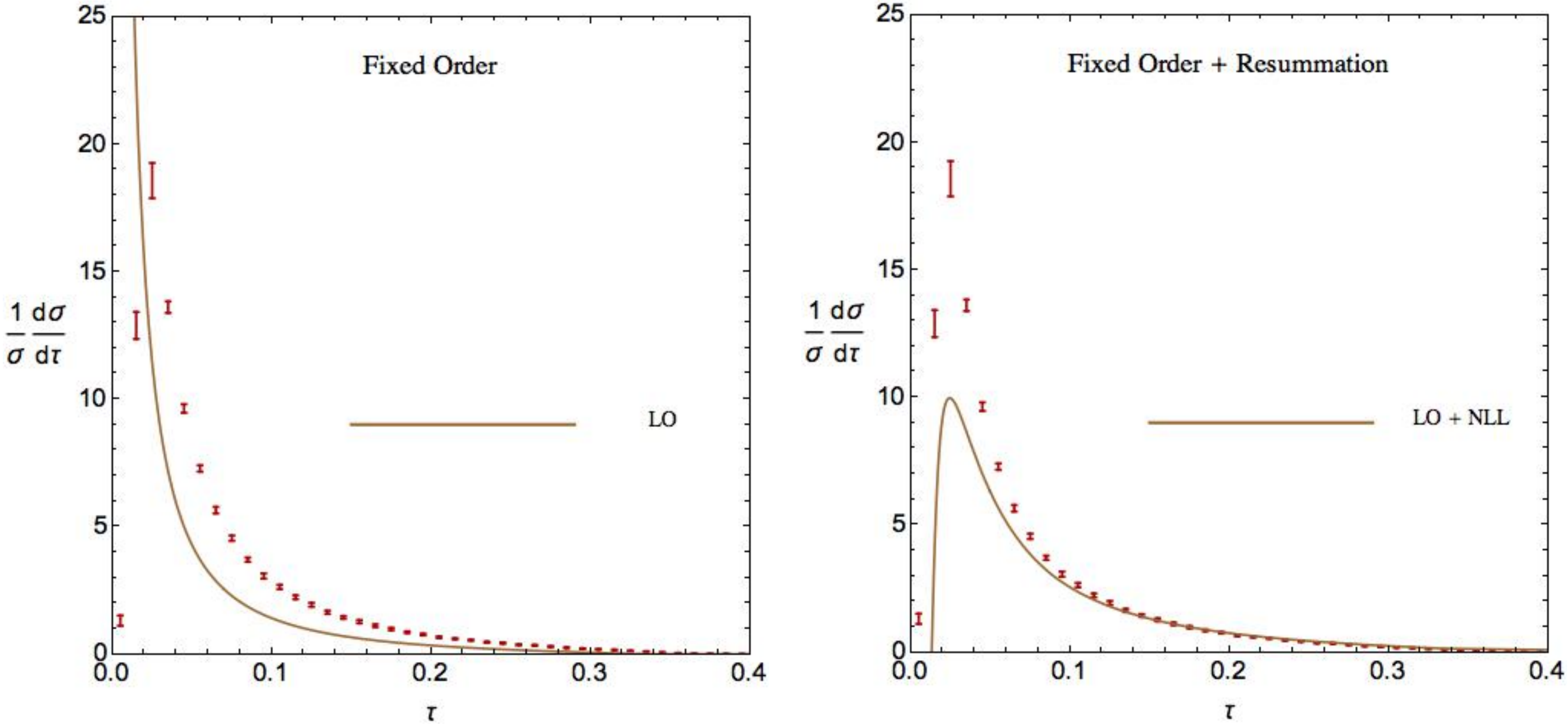
$$\mu_h = Q \qquad \mu_j = \sqrt{\tau}Q \qquad \mu_s = \tau Q$$

- Evolve each function from its scale to common scale μ using RGE
- Logs of μ linked to logs of τ

$$\ln\frac{\mu_h}{\mu} = \ln\frac{Q}{\mu} \qquad \ln\frac{\mu_j}{\mu} = \ln\frac{Q}{\mu} + \frac{1}{2}\ln\tau \qquad \ln\frac{\mu_s}{\mu} = \ln\frac{Q}{\mu} + \ln\tau$$

- Reduces problem to the fixed-order inclusive calculation case
- Single scale at fixed order is misleading: multiple scale problem

Natural scales improve convergence



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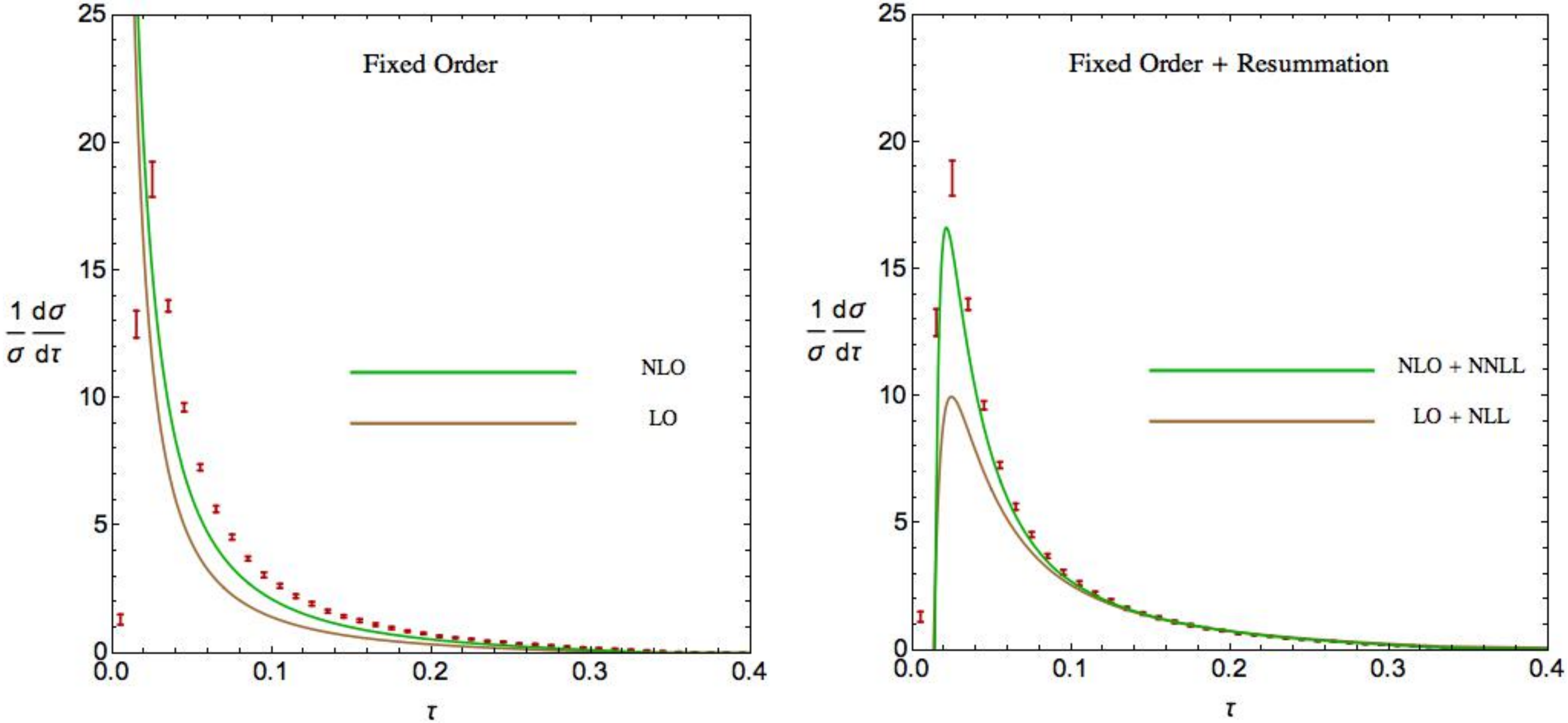

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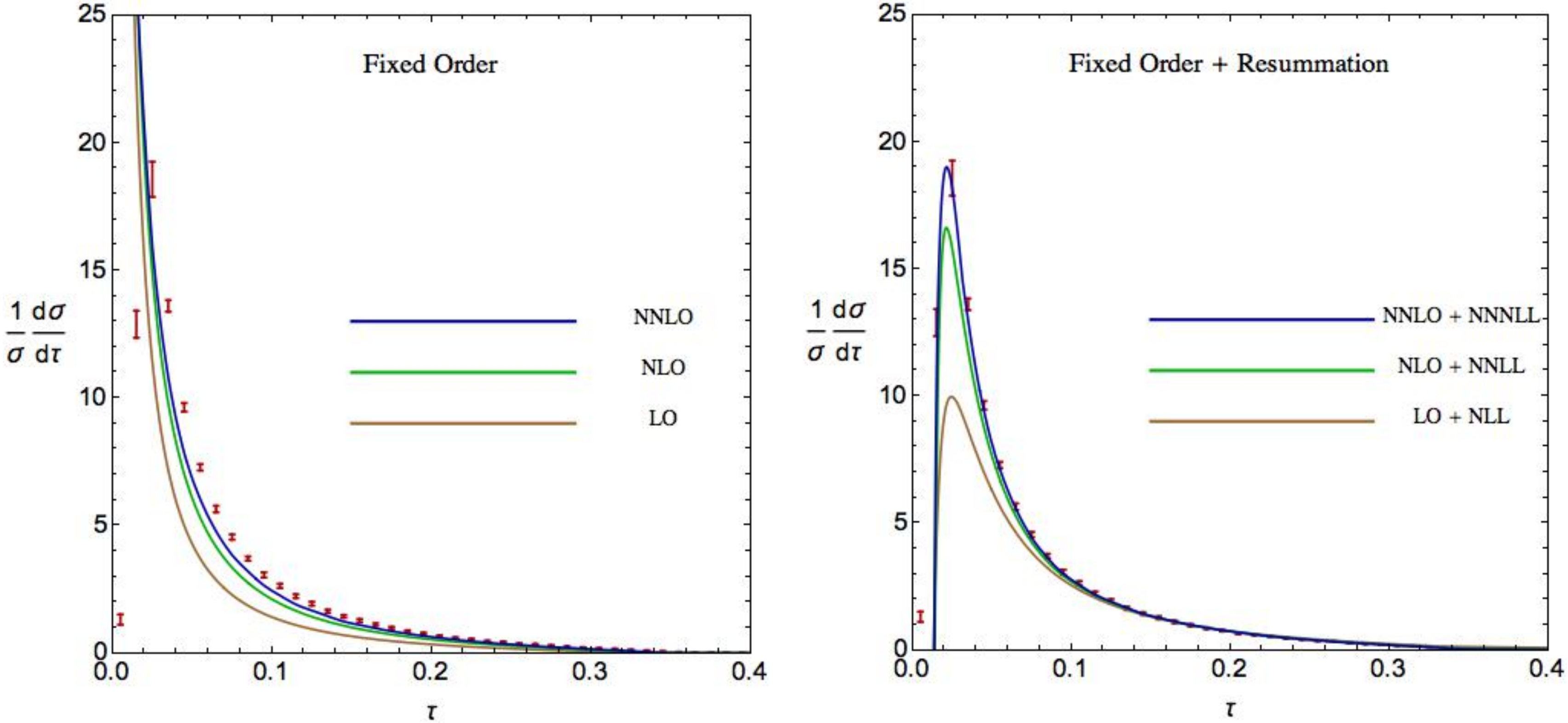
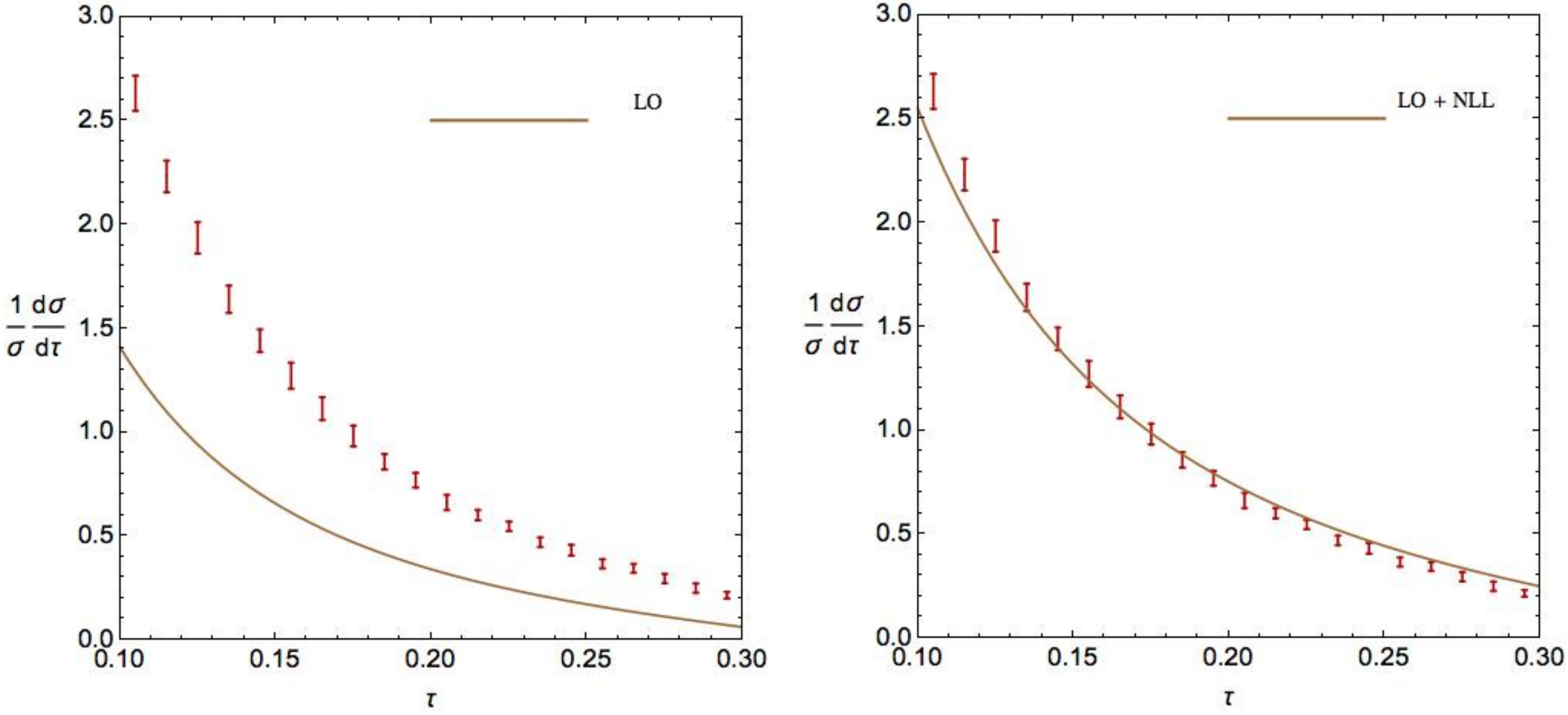


Natural scales improve convergence



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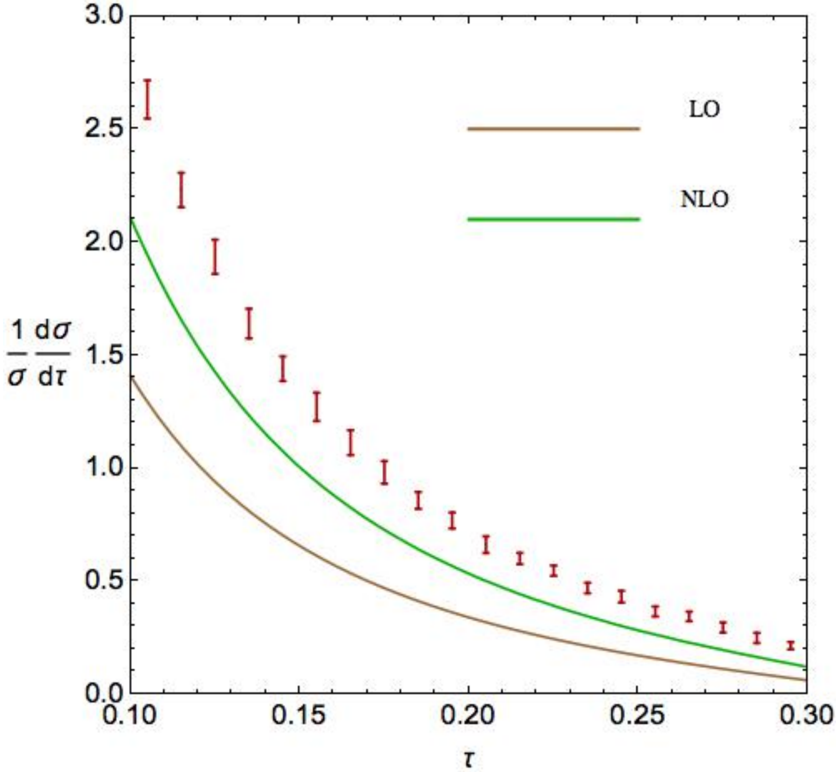

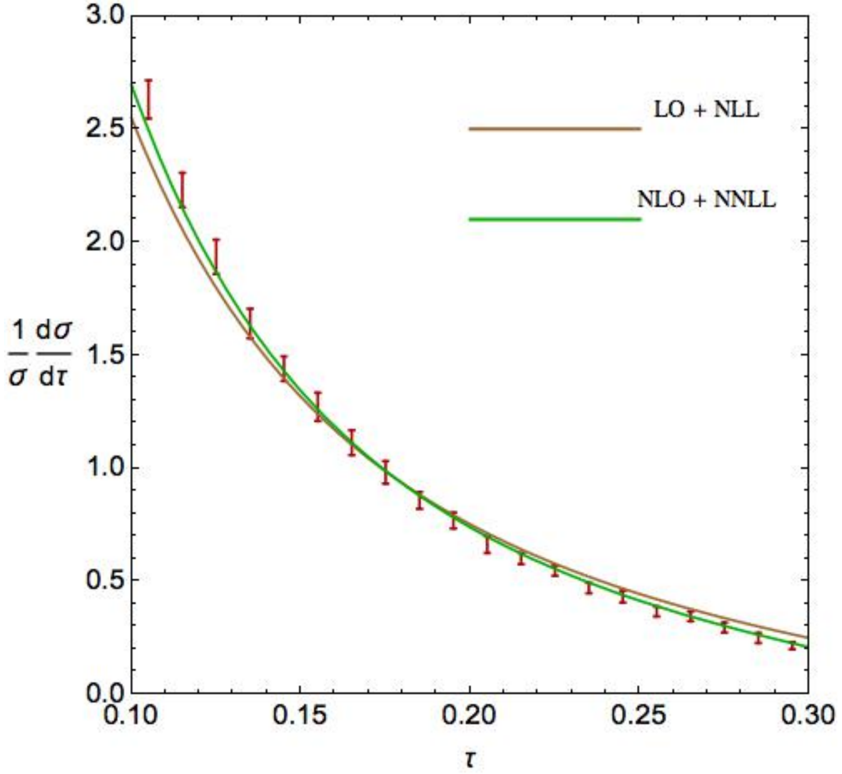

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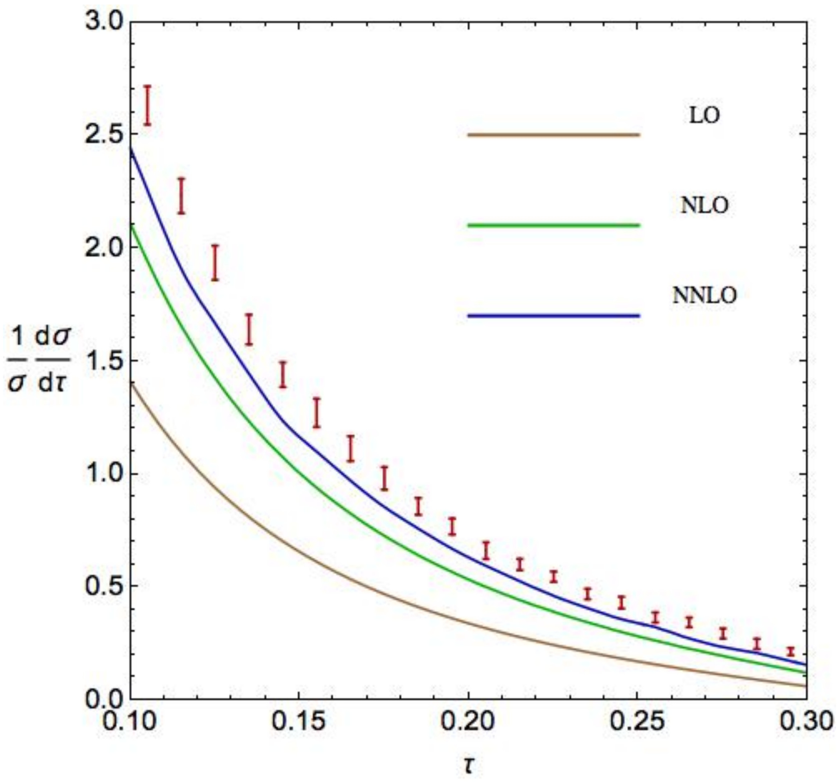
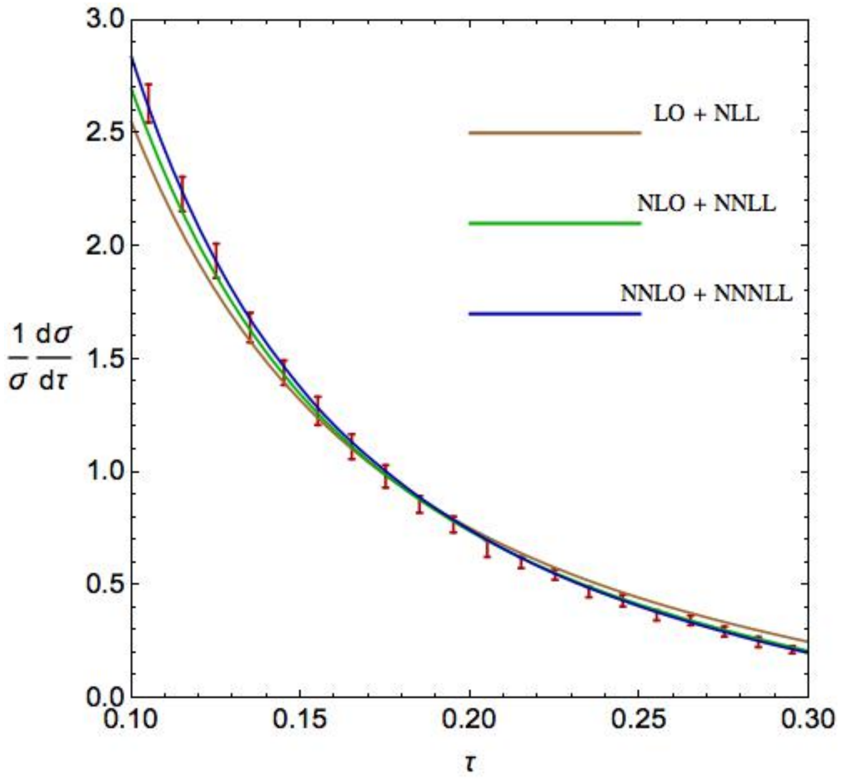
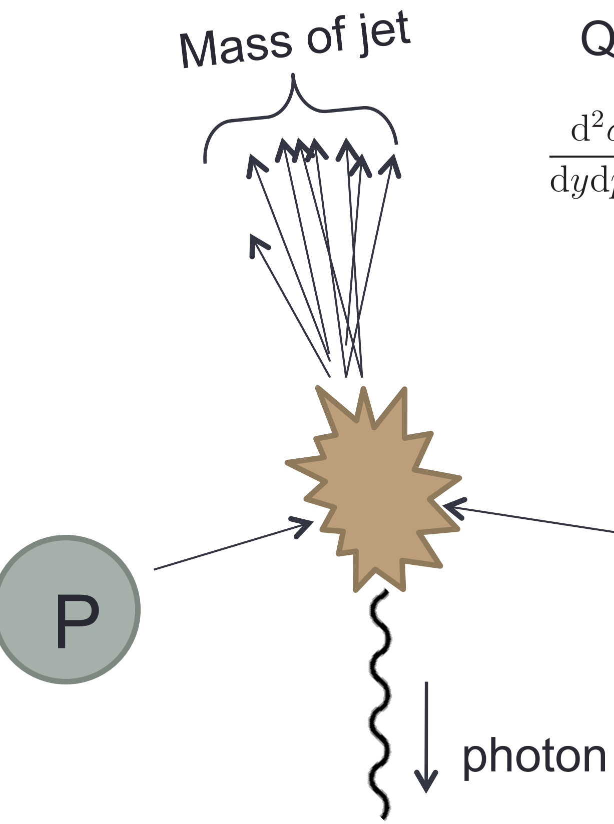




Hadron collisions more complicated



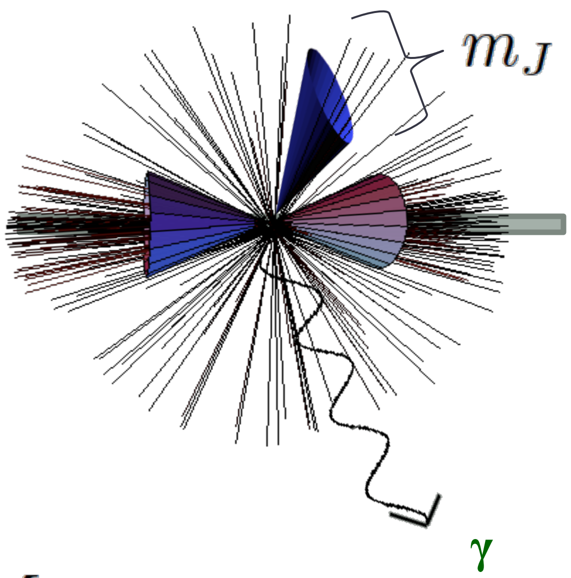
Qualitatively similar in threshold expansion:

$$\frac{d^2\sigma}{dy\,dp_T} = \frac{2}{p_T}\sum_{ab}\int_{\frac{p_T}{E_{CM}}e^{y}}^{1-\frac{p_T}{E_{CM}}e^{-y}} dv \int_{\frac{p_T}{E_{CM}}\frac{1}{v}e^{y}}^{1} dw \left[x_1 f_{a/N_1}(x_1,\mu)\right]\left[x_2 f_{b/N_2}(x_2,\mu)\right]\frac{d^2\hat{\sigma}_{ab}}{dw\,dv}$$

$$\frac{d^2\hat{\sigma}}{dw\,dv} = w\,\widetilde{\sigma}(v)\,H(p_T,v,\mu)\int dk\,J(m_X^2-(2E_J)k,\mu)\,S(k,\mu)\,.$$

$$M_X^2 = m_X^2 + \frac{p_T^2}{v\bar{v}}\left[(1-x_1)v + (1-x_2)\bar{v}\right]$$

Jet mass in direct photon

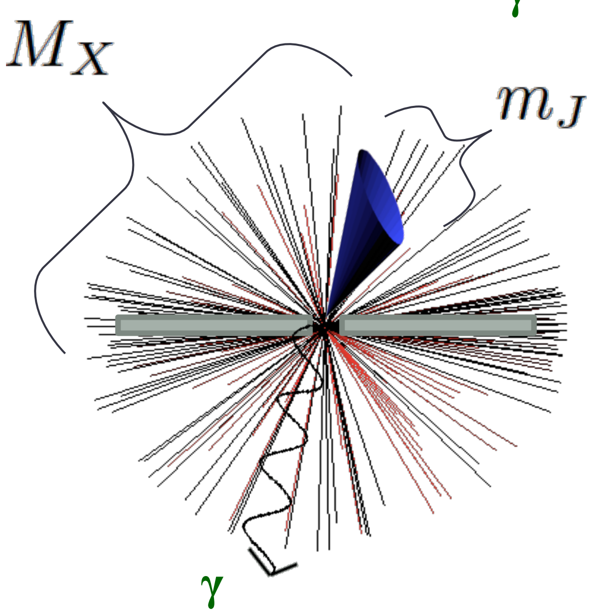


- We want to measure the **jet mass** m_J^2
- We expect **resummation** to be **important** as $m_J^2 \to 0$

Assumption for
SCET factorization theorem

Machine Threshold limit

- **Initial state**: 2 protons
- **Final state:** 1 **jet** + 1 photon+ **soft radiation only** (no jet-like proton remnants)



Observable is photon p_T and rapidity (y)

- **Inclusive** measurement -- no jet definition necessary

Factorization derived at small M_X

M_X = mass of everything-but-the-photon

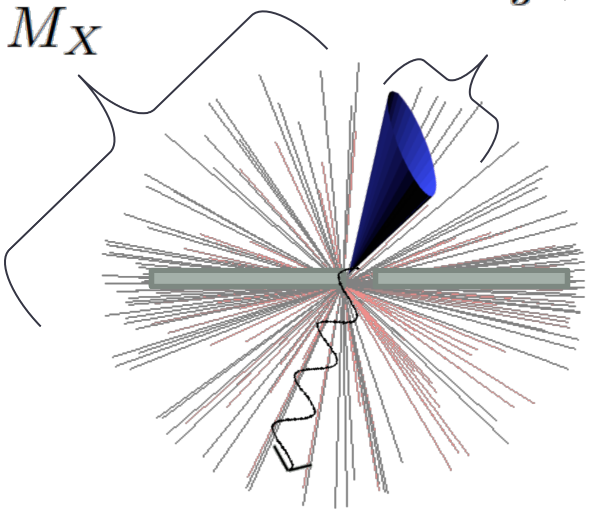
$$M_X^2 = {E_{\rm CM}}^2 - 2 p_T E_{\rm CM} \cosh y$$

- M_X typically large – so **why is this regime interesting**?

Threshold Enhancement

(mass of everything but the photon) M_X

m_J (mass of **jet**)



Machine threshold

$$M_X \to 0$$

- Assumed for SCET calculation

Partonic threshold

$$m_J \to 0$$

- Where partonic logs are large

Typical event

large | | large | large

$$M_X^2 = m_J^2 + (1-x_1)\frac{t}{s} + (1-x_2)\frac{u}{s}$$

small

logs = logs + logs + logs

- typical x << 1
- Most of large M_X comes from proton remnants

•**jet masses** are typically small (as we know) → expect some logs still large

Direct photon in SCET

PDF $\langle p_1|\bar{\chi}_1\chi_1|p_1\rangle$

PDF $\langle p_1|\bar{\chi}_3\chi_3|p_1\rangle$

$$\frac{d^2\sigma_{q\bar{q}}}{dy\,dp_T} = \frac{2}{p_T}\int_{\frac{p_T}{E_{CM}}e^{y}}^{1-\frac{p_T}{E_{CM}}e^{-y}} dv \int_{\frac{p_T}{E_{CM}}\frac{1}{v}e^{y}}^{1} dw \left[(wx_1)f_{q/N_1}(x_1,\mu)\right]\left[x_2 f_{\bar{q}/N_2}(x_2,\mu)\right]$$

$$\times\, \tilde{\sigma}_{q\bar{q}}(v) H_{q\bar{q}}(p_T,v,\mu)\int dk\, J_g(m_X^2-(2E_J)k,\mu)\, S_{q\bar{q}}(k,\mu)$$

Hard function

- NLO (from QCD)
- SCET: γ_H to 3-loops

Jet function

$\langle 0|\mathcal{A}_2^{\nu}\mathcal{A}_2^{\nu}|0\rangle$

- Quark jet to NNLO
- Gluon jet to NLO
- γ_{Jq} and γ_{Jq} to 3-loops

Soft function

- both channels to NLO
- γ_S to 3-loops (from RG and Casimir scaling)

Direct photon distribution with
NNLL resummation + NLO fixed order

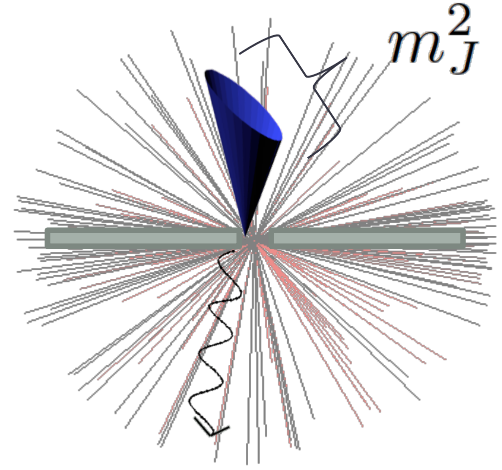
What are the matching scales?

Matching scales appear as:

$$\frac{\mu_h^2}{p_T^2}, \quad \frac{\mu_j^2}{m_J^2}, \quad \frac{\mu_s}{\mu_j^2/\mu_h}$$

Hard scale = p_T

Jet scale = m_J?



m_J^2 = mass of **jet**

- Works for thrust $\frac{d\sigma}{dm_J^2} \sim \exp\left[\alpha_s \log\frac{m_J^2}{E_{\rm CM}^2}\right]$

- Problematic for direct photon
  - m_J is integrated over, including $m_J = 0$

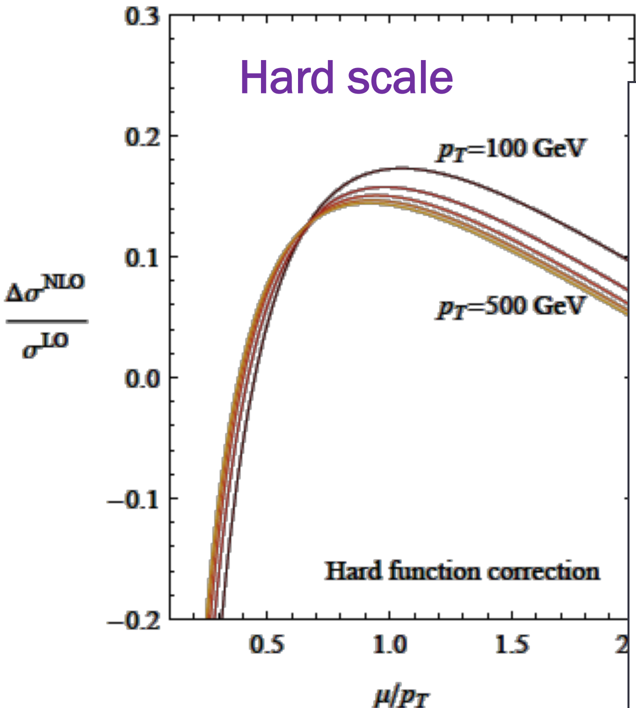
$$\frac{d\sigma}{dM_X^2} = \int dm_J^2 \,\delta\left(M_X^2 - m_J^2 - (1-x_1)\frac{t}{s} - (1-x_2)\frac{u}{s}\right) f(m_J^2, \cdots)$$

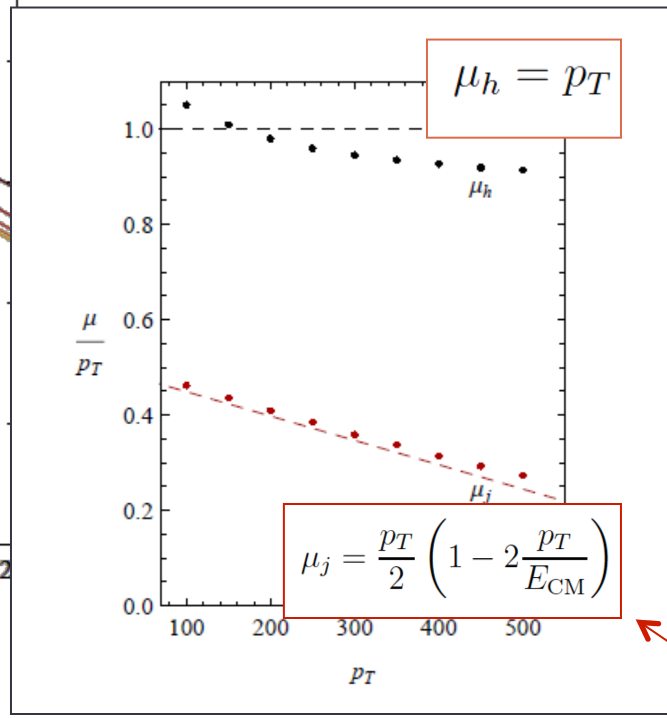
$$f \sim \exp\left[\alpha_s(\mu_J)\log\frac{\mu_J^2}{\mu_h^2}\right] \times \cdots \;\rightarrow \exp\left[\alpha_s(m_J)\log\frac{m_J^2}{p_T^2}\right] \times \cdots$$

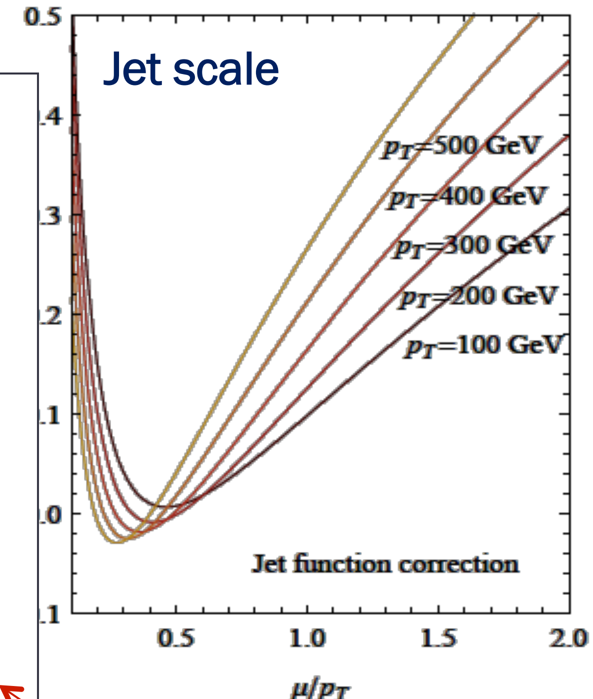
  - probes Landau pole of QCD → unphysical power corerctions

All matching scales should depend only physical, observable scales –*i.e.* p_T

Natural scales







note that

$$\mu_J = \langle m_J \rangle \lesssim p_T^\gamma$$

So we take:

$$\mu_h = p_T$$

$$\mu_j = \frac{p_T}{2}\left(1 - 2\frac{p_T}{E_{CM}}\right)$$

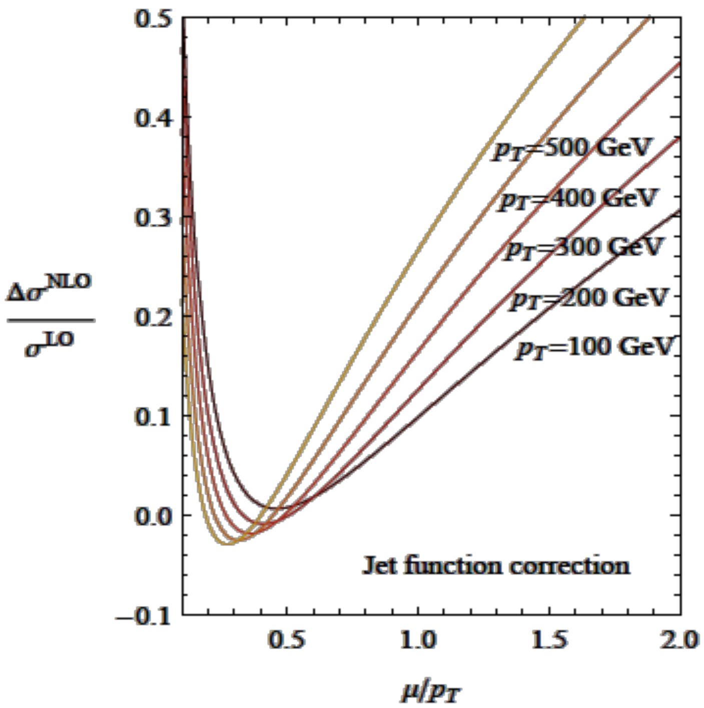
$$\mu_s = \mu_j^2/\mu_h$$

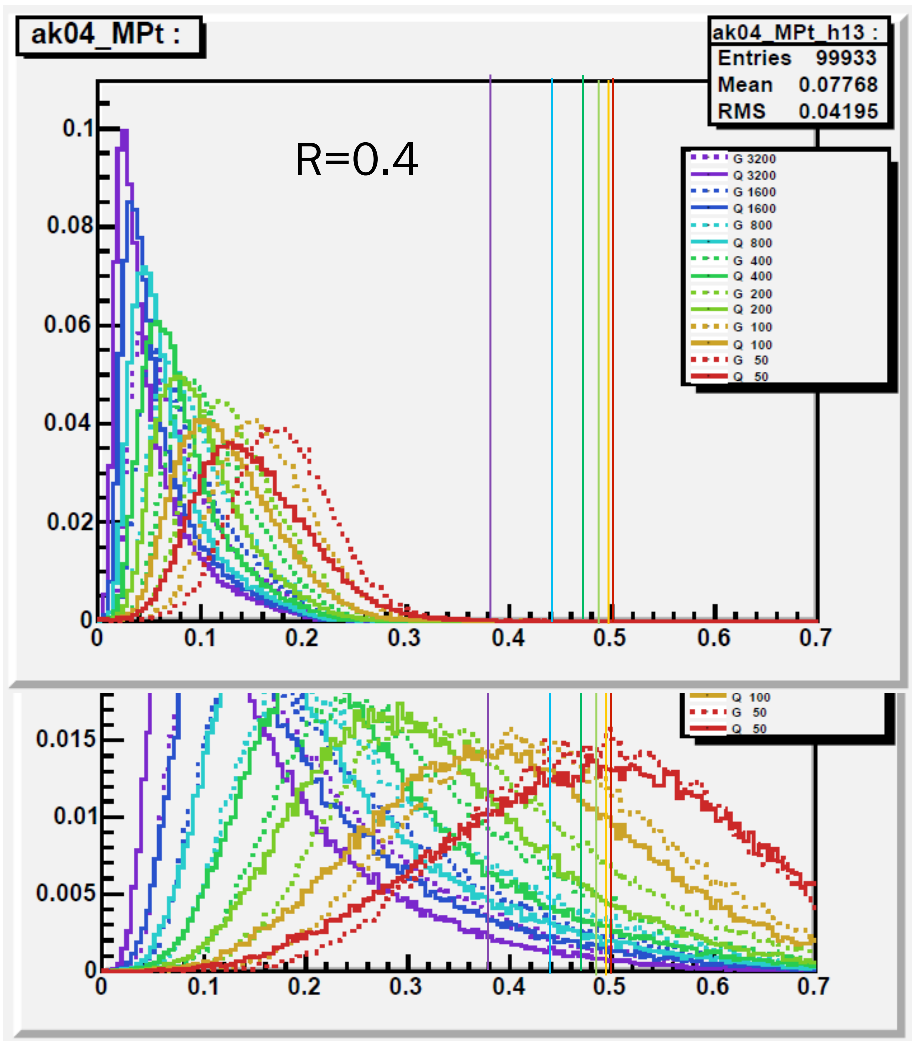
Always well above L_{QCD}

- avoids unphysical region

Jet masses

Rule of thumb “m = 0.2 pt”

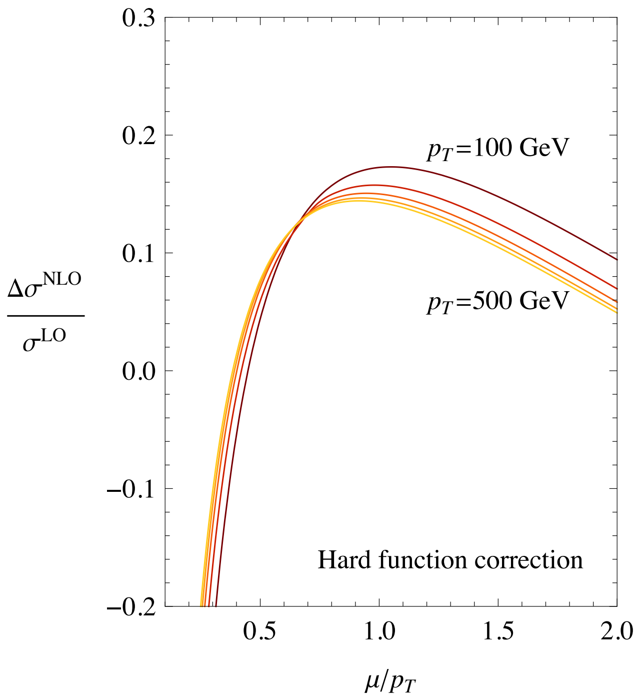




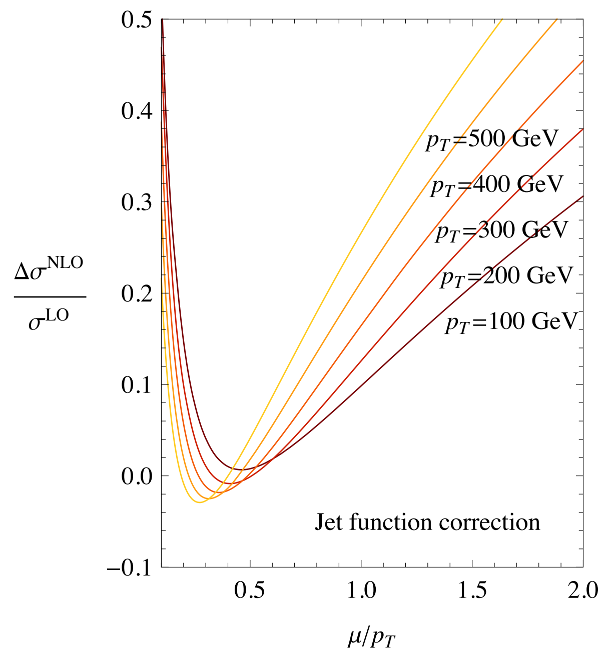
m_J really is close to the mass of the partonic jet

Direct Photon scales

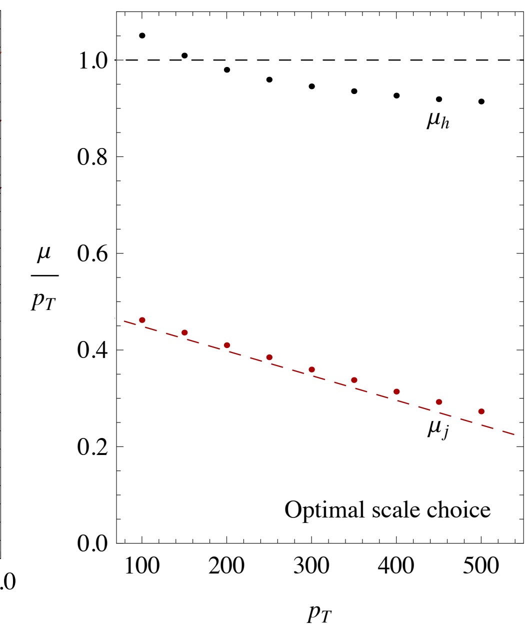
Vary hard scale only



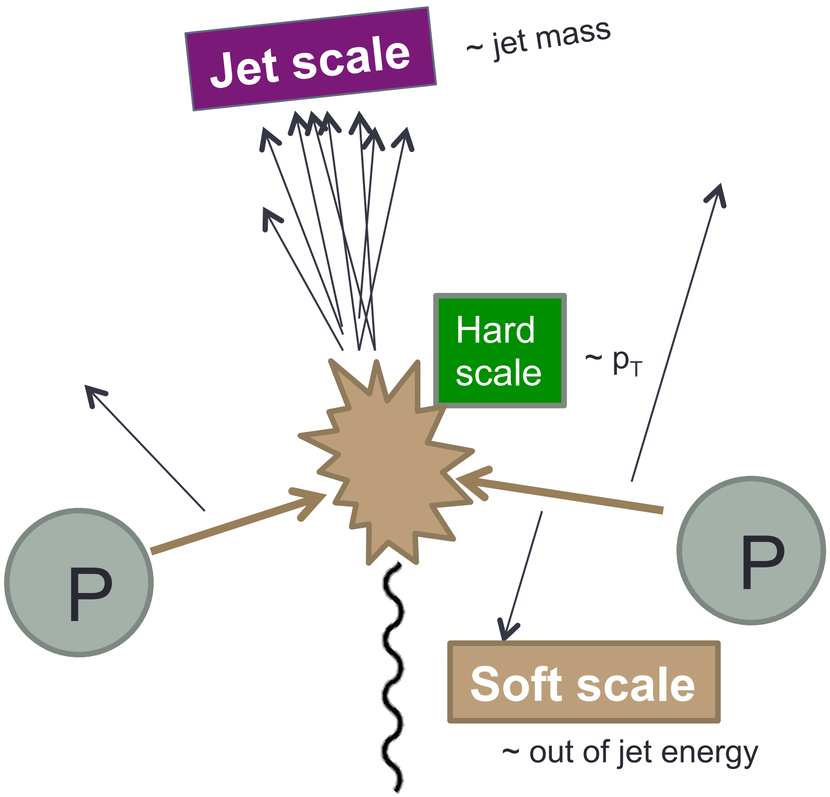
Vary jet scale only



Optimal scale choice (minimal scale variation)



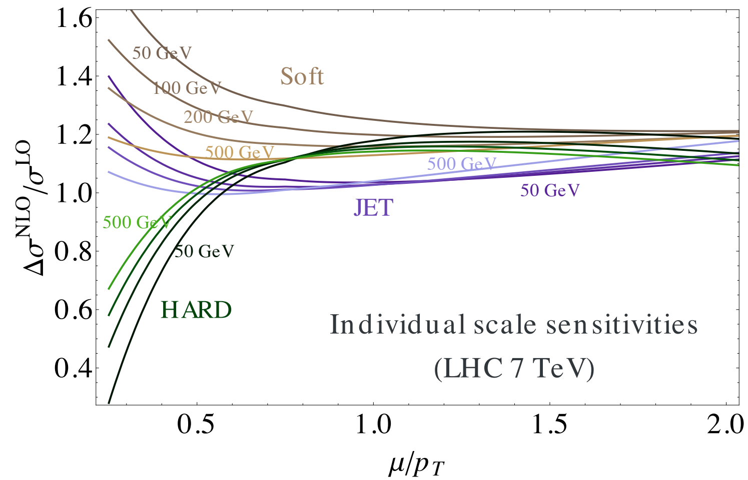
EFTs reveal the relevant scales:



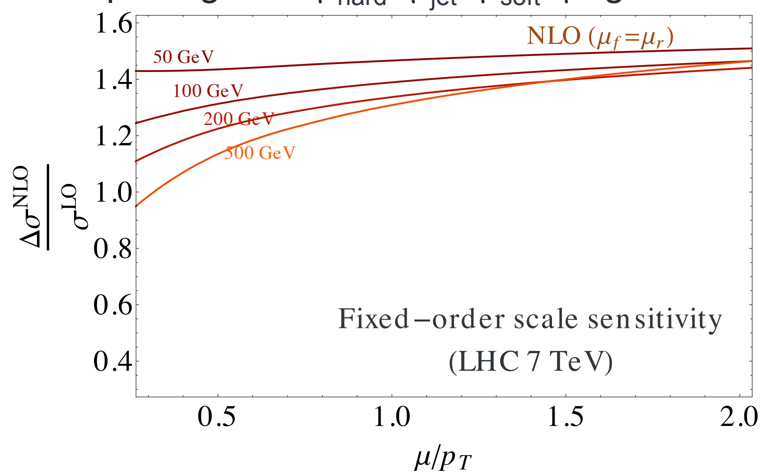
High p_T
W boson

Becher, Lorentzen and **MDS**, Phys.Rev. D 86 (2012)

Individual variation show extrema
(natural μ_{hard}, μ_{jet}, μ_{soft} scales, like Q)



When put together $\mu_{hard}=\mu_{jet}=\mu_{soft}=\mu$ gives NLO



No natural μ at NLO (or N^nLO). **Cannot set all scales equal**.

Challenges in SCET

- Have many scale variations

Thrust distribution: SCET

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_2}{\mathrm{d}\tau} = H(Q^2,\mu)\int \mathrm{d}p_L^2\mathrm{d}p_R^2\mathrm{d}k\, J(p_L^2,\mu)\, J(p_R^2,\mu)\, S_T(k,\mu)\delta(\tau - \frac{p_L^2+p_R^2}{Q^2} - \frac{k}{Q})$$

- Compute each to fixed-order at its natural scale

$$\mu_h = Q\,, \quad \mu_j = \sqrt{\tau}Q\,, \quad \mu_s = \tau Q$$

$$\ln\frac{\mu_h}{\mu} = \ln\frac{Q}{\mu} \qquad \ln\frac{\mu_j}{\mu} = \ln\frac{Q}{\mu} + \frac{1}{2}\ln\tau \qquad \ln\frac{\mu_s}{\mu} = \ln\frac{Q}{\mu} + \ln\tau$$

e.g. $J(p^2,\mu) = \exp\left[-4S(\mu_j,\mu) + 2A_J(\mu_j,\mu)\right]\widetilde{j}(\partial_{\eta_j},\mu_j)\frac{1}{p^2}\left(\frac{p^2}{\mu_j^2}\right)^{\eta_j}\frac{e^{-\gamma_E\eta_j}}{\Gamma(\eta_j)}$

1. Central values for scale choices are not arbitrary
2. Multiple different scales are relevant to minimize all logs

Scale setting

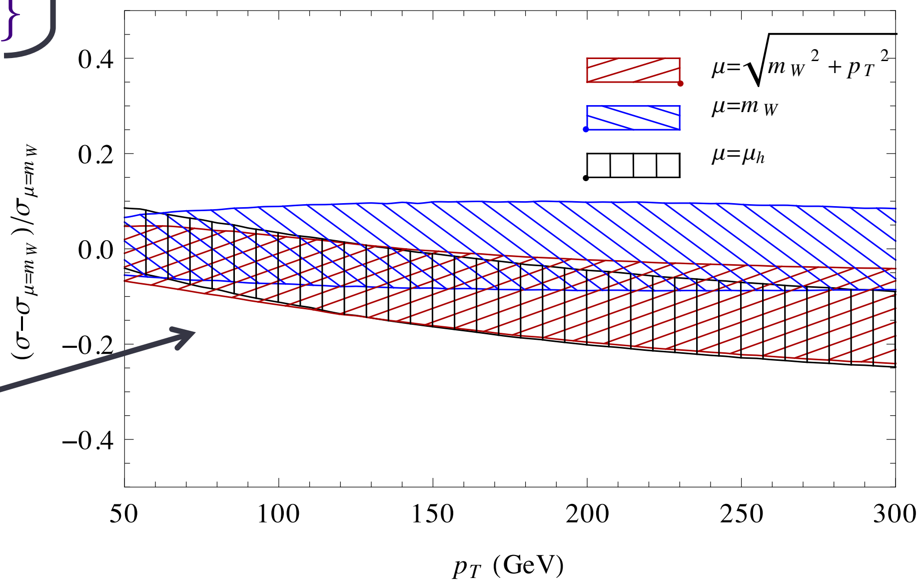
- Fixed order calculations have one scale μ to choose
- Choice only clear for **completely inclusive** cross sections
- p_T vetos, jet energy cuts, triggers, etc. introduce **new scales**

Example: Inclusive W production, differential in p_T of the W

Many reasonable scale choices:

$$\mu = H_T$$
$$\mu = \sqrt{p_T^2 + m_W^2}$$
$$\mu = \max\{m_W, E_{\rm jet}\}$$

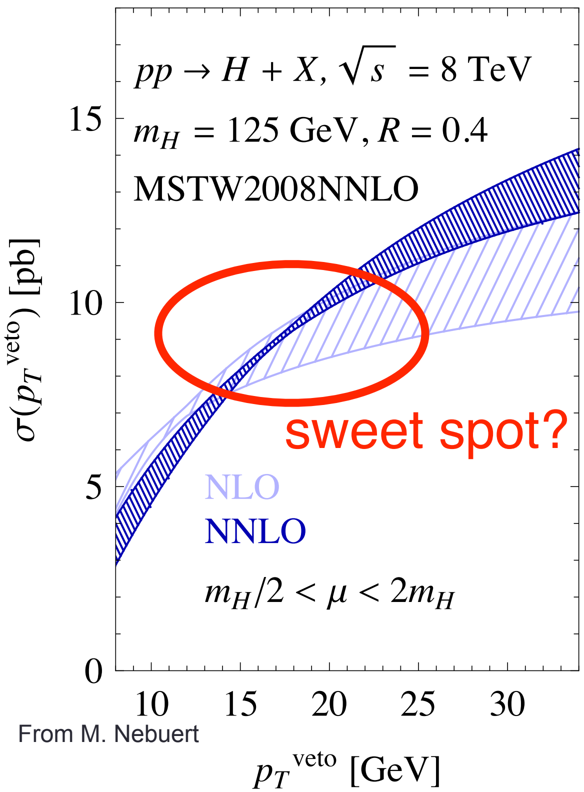
Pick one and vary by a factor of 2 or 4 or 100



Differences between parameterizations are larger than the individual variations

Higgs cross section with p_T veto

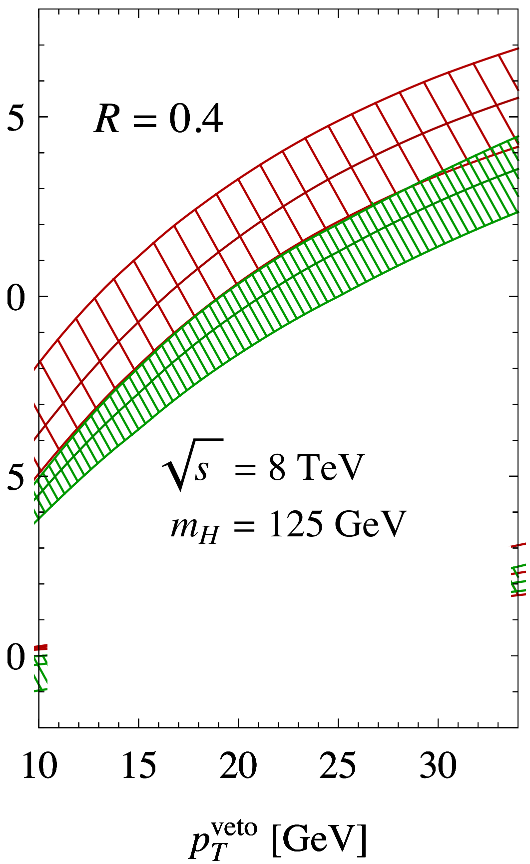
Fixed order (NNLO)



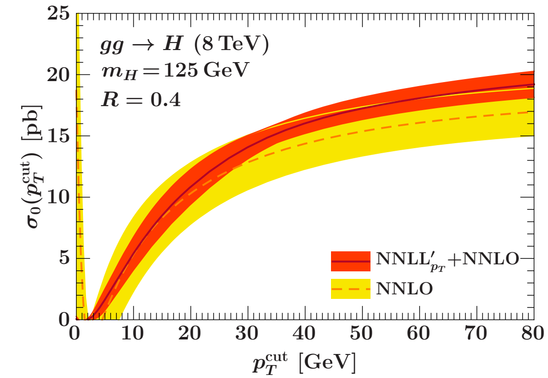
From M. Nebuert

Resummed (3 different groups)

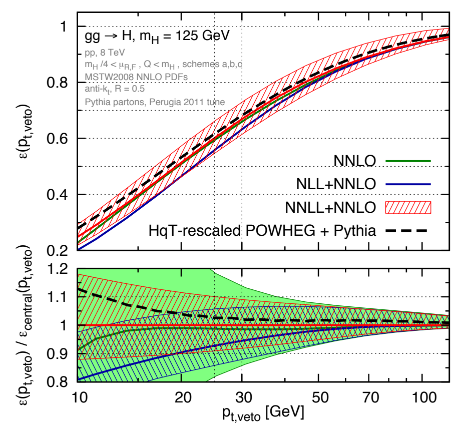
Becher, Neubert, Rothen



Stewart, Tackmann, Walsh, Zuberi



Banfi, Salam, Monni, Zanderighi



- **NNLO** has cancellation which **underestimates uncertainty** (Anastasiou, Dissertori, Stockli)
- Resumming logs of mH/pTveto changes cross section by 10-20% vs NNLO.