NON-GLOBAL LOGARITHMS TO 3, 4 AND 5 LOOPS, SYMBOLS AND THE POINCARE DISK

SCET XI

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Based on arXiv:1403.4949 with Hua Xing Zhu

Five things I learned:

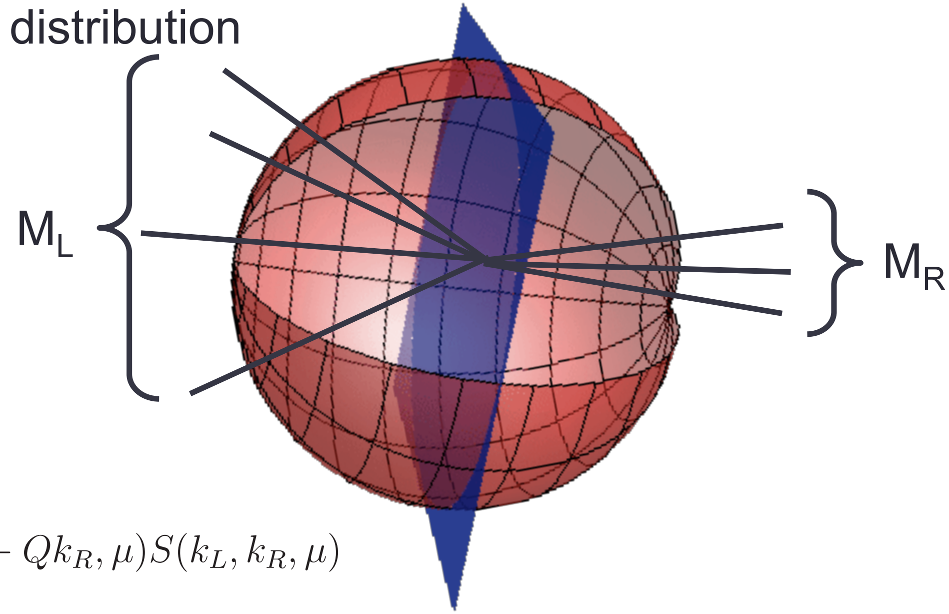
1. How **strong-energy ordering** works
2. Resummation which is (currently) beyond SCET
3. A hidden **symmetry** of soft functions
4. How to use **symbols** to compute integrals
5. That the hemisphere NGL to **5-loops** is

$$1 - \frac{\pi^2}{24}L^2 + \frac{\zeta(3)}{12}L^3 + \frac{\pi^4}{34560}L^4 + \left(-\frac{\pi^2\zeta(3)}{360} + \frac{17\zeta(5)}{480}\right)L^5 + \dots$$

What are non-global logarithms?

The doubly-differential hemisphere mass distribution

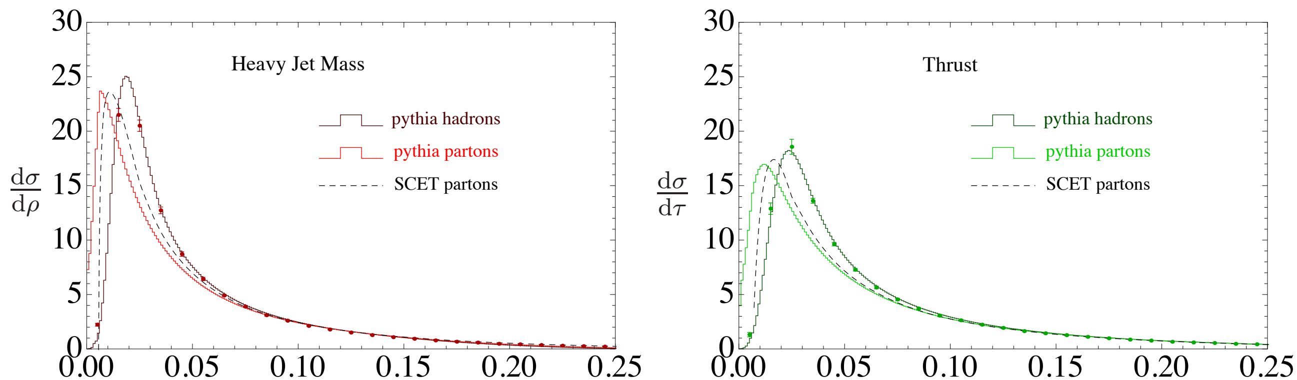
$$\frac{d^2\sigma}{dM_L^2 dM_R^2}$$



In SCET, factorization formula

$$\frac{d^2\sigma}{dM_L^2 dM_R^2} = H(Q^2,\mu)\int dk_L dk_R J(M_L^2 - Qk_L,\mu) J(M_R^2 - Qk_R,\mu) S(k_L,k_R,\mu)$$

- Valid as long as both M_L and M_R << Q
- Factorization formula use to resum thrust and heavy jet mass to NNLL



What are non-global logarithms?

Soft function can be written as:

$$S(k_L, k_R, \mu) = S_\mu(\ln \frac{k_L}{\mu}) S_\mu(\ln \frac{k_R}{\mu}) S_f(\ln \frac{k_L}{k_R})$$

Determined by RGE

Non-global piece
(no μ dependence)

- Thrust: depends only on $\widetilde{s}_f(0)$
- Heavy jet mass: depends only on some moment $\frac{2}{\pi}\int_0^\pi \widetilde{s}_{f2}(iL)\ln\left[2\cos(\frac{L}{2})\right]\mathrm{d}L$
- Structure of $S_f(L)$ in general **complicated**
    - Calculated at 2-loops in 1105.3767 and 1105.4628

$$\left(\frac{\alpha_s}{4\pi}\right)^2\left[-88\text{Li}_3(-z) - 16\text{Li}_4\left(\frac{1}{z+1}\right) - 16\text{Li}_4\left(\frac{z}{z+1}\right) + 16\text{Li}_3(-z)\ln(z+1)\right.$$
$$\left.+\frac{88\text{Li}_2(-z)\ln(z)}{3} - 8\text{Li}_3(-z)\ln(z) - 16\zeta(3)\ln(z+1) + 8\zeta(3)\ln(z) - \frac{4}{3}\ln^4(z+1)\right. + \dots$$

The logarithmic terms in $S_f(L)$ are called **non-global logarithms** (NGLs):

$$\tilde{s}_f(L) = \sum_i c_i(\alpha_s L)^i + d_i\alpha_s(\alpha_s L)^i + \cdots$$

Leading NGL

Subleading NGLs

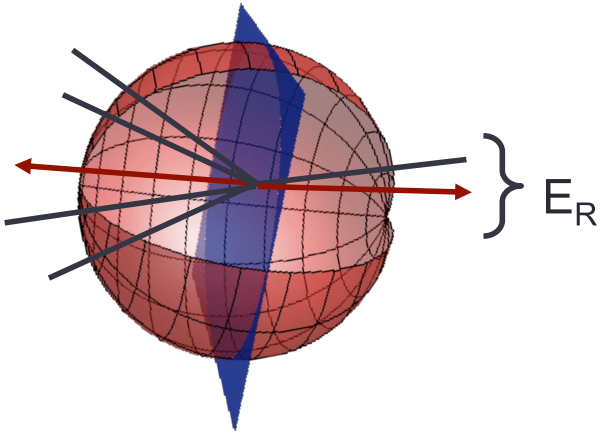
Hemisphere mass

- Non global logs are **single logs:** $(\alpha_s L)^n$
  - Collinear finite
- For ***leading*** NGL of hemisphere masses $L = \log(M_L/M_R)$
  - Can use $L = \log(Q/M_R)$ instead (integrate over M_L)
  - Can use energy instead of mass $L = \log(Q/E_R)$
    - Not kosher for collinear sensitive logs, but ok for NGLs
- **Large logs come from energy integrals**

$$\int_{E_2}^{Q} \frac{dE_1}{E_1} \int_{E_3}^{Q} \frac{dE_2}{E_2} \cdots \int_{E_n}^{Q} \frac{dE_{n-1}}{E_{n-1}} = \frac{1}{n!} \ln^n \frac{Q}{E_n}$$

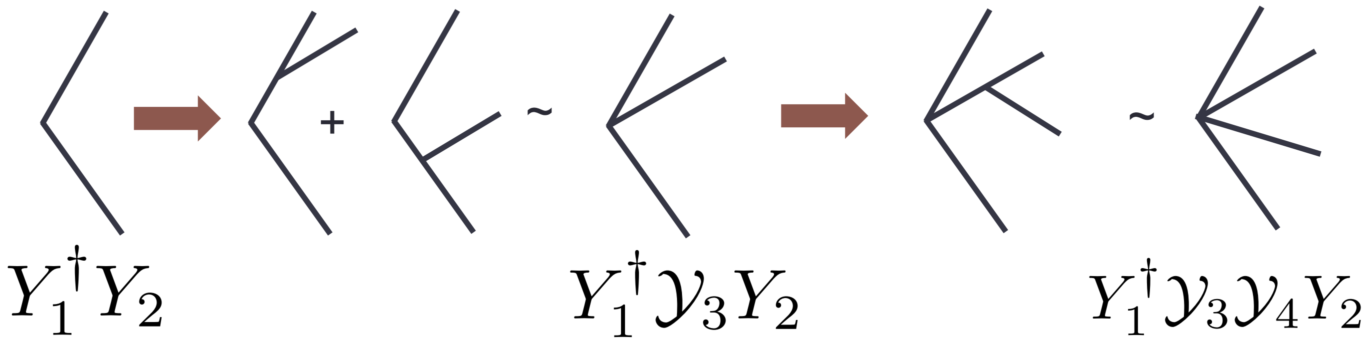
- Region where energies are not strongly ordered contributes finite part (subleading logs)

- E_R = energy of hardest gluon going right
  - n-1 emissions go left
  - **only softest 1 goes right**



Strong-energy ordering

- In SCET language: soft -> softer -> even softer … -> softest
- Evolution in Wilson line operators space?



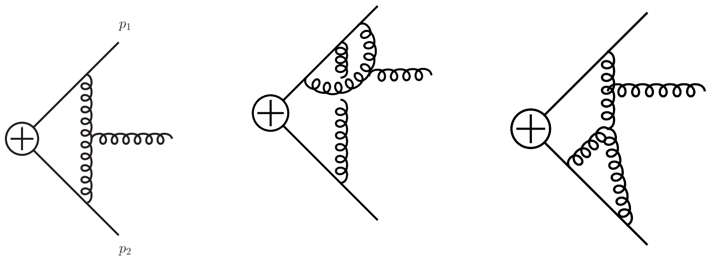
Real-emission matrix elements-squared simplify:

$$\left|\langle p_1 \cdots p_m \left| Y_a^\dagger Y_b \right| 0\rangle\right|^2 = N_c^m g^{2m} \sum_{\text{perms of } 1\cdots m} \frac{(p_a \cdot p_b)}{(p_a \cdot p_1)(p_1 \cdot p_2)\cdots(p_m \cdot p_b)}$$

Extra simplification at large N_c

Strong-energy ordering

Virtual and real-virtual emissions also simplify



Basic idea

- Chop up virtual momenta into energy-ordered regions

$$\frac{1}{\sigma_0} d\sigma_m = \bar{\alpha}\frac{d\omega_1}{\omega_1}\frac{d\Omega_1}{4\pi}(\mathcal{W}_R+\mathcal{W}_V) + \frac{\bar{\alpha}^2}{2!}\frac{d\omega_1}{\omega_1}\frac{d\Omega_1}{4\pi}\frac{d\omega_2}{\omega_2}\frac{d\Omega_2}{4\pi}(\mathcal{W}_{RR}+\mathcal{W}_{RV}+\mathcal{W}_{VR}+\mathcal{W}_{VV}) + \frac{\bar{\alpha}^3}{3!}\frac{d\omega_1}{\omega_1}\frac{d\Omega_1}{4\pi}\frac{d\omega_2}{\omega_2}\frac{d\Omega_2}{4\pi}\frac{d\omega_3}{\omega_3}\frac{d\Omega_3}{4\pi}(\mathcal{W}_{RRR}+\mathcal{W}_{RRV}+\cdots)$$

- Virtual emissions have same form as real-emissions, with opposite sign
- Virtual emissions do **not** become new Wilson lines
- Virtual emissions are **not measured**

$$P_{ab}^1 = \frac{(ab)}{(a1)(1b)}$$

$$C_1 = \mathcal{P}_{ab}^{123} = \mathcal{W}_{RRR} = -\mathcal{W}_{RRV}$$

$$C_3 = \mathcal{P}_{ab}^{1}\mathcal{P}_{ab}^{23} = -\mathcal{W}_{VRR} = \mathcal{W}_{VRV}$$

$$C_2 = \mathcal{P}_{ab}^{12}\left(\mathcal{P}_{a1}^{3}+\mathcal{P}_{b1}^{3}\right) = \mathcal{W}_{RVV} = -\mathcal{W}_{RVR}$$

$$C_4 = \mathcal{P}_{ab}^{1}\mathcal{P}_{ab}^{2}\mathcal{P}_{ab}^{3} = \mathcal{W}_{VVR} = -\mathcal{W}_{VVV}$$

a1 and b1 dipoles

Independent emissions

Derive leading NGL integrand

Integrate $W_{RVR\ldots}$ against measurement function for right hemisphere mass

Two loops

$$S^{(2)}(\rho) = \bar{\alpha}^2 \int_{E_1>E_2} 1_R 2_R \theta_{\rho<1}\theta_{\rho<2}\left(\mathcal{P}^1_{ab}\mathcal{P}^2_{ab}\right) - \bar{\alpha}^2 \int_{E_1>E_2} 1_L 2_R \theta_{\rho<2}\left(\mathcal{P}^{12}_{ab} - \mathcal{P}^1_{ab}\mathcal{P}^2_{ab}\right)$$

Global part

Non-global part

Three loops

$$S^{(3)}(\rho) = \bar{\alpha}^3 \int_{E_1>E_2>E_3} 1_R 2_R 3_R \theta_{\rho<3}(-C_4) + \bar{\alpha}^3 \int_{E_1>E_2>E_3} 1_R 2_L 3_R \theta_{\rho<3}(C_3 - C_4)$$

$$+\, \bar{\alpha}^3 \int_{E_1>E_2>E_3} 1_L 2_R 3_R \theta_{\rho<3}(C_2 - C_4) + \bar{\alpha}^3 \int_{E_1>E_2>E_3} 1_L 2_L 3_R \theta_{\rho<3}(-C_1 + C_2 + C_3 - C_4)$$

- **Includes** collinear-divergent **global**-non-global cross terms
- Can subtract off using exponentiation
  - Need to use symmetries to know what to subtract

Four loops: a mess

- Hard to simplify

$$C_1 = \mathcal{P}^{123}_{ab} = \mathcal{W}_{RRR} = -\mathcal{W}_{RRV}$$

$$C_2 = \mathcal{P}^{12}_{ab}\left(\mathcal{P}^3_{a1} + \mathcal{P}^3_{b1}\right) = \mathcal{W}_{RVV} = -\mathcal{W}_{RV}$$

$$C_3 = \mathcal{P}^1_{ab}\mathcal{P}^{23}_{ab} = -\mathcal{W}_{VRR} = \mathcal{W}_{VRV}$$

$$C_4 = \mathcal{P}^1_{ab}\mathcal{P}^2_{ab}\mathcal{P}^3_{ab} = \mathcal{W}_{VVR} = -\mathcal{W}_{VVV}$$

Resummation of NGLs

1. Dasgupta-Salam (hep-ph/0104277, 2001)

- Large N_c monte carlo

$$\mathcal{S}(\alpha_s L) = \frac{1}{\sqrt{\Delta_{ab}(L)}} \sum_{\mathcal{C}|\mathcal{H}_R \text{ empty}} P_{\mathcal{C}}(L)$$

Probability of configuration C

Sudakov factor

Run Monte Carlo

$$\mathcal{S}(\alpha_s L) \simeq \exp\left(-C_F C_A \frac{\pi^2}{3}\left(\frac{1+(at)^2}{1+(bt)^c}\right) t^2\right)$$

2. Banfi-Marchesini-Smye (hep-ph/0206076, 2002)

$$E\partial_E\, G_{ab}(E) = \int \frac{d^2\Omega_k}{4\pi}\, \bar{\alpha}_s w_{ab}(k)\left[u(k)\, G_{ak}(E)\cdot G_{kb}(E) - G_{ab}(E)\right]$$

- Same assumptions as DS (strong-energy-ordering, leading NGL, large N)
- Supposedly equivalent
  - we give the first numerical check

BMS equation

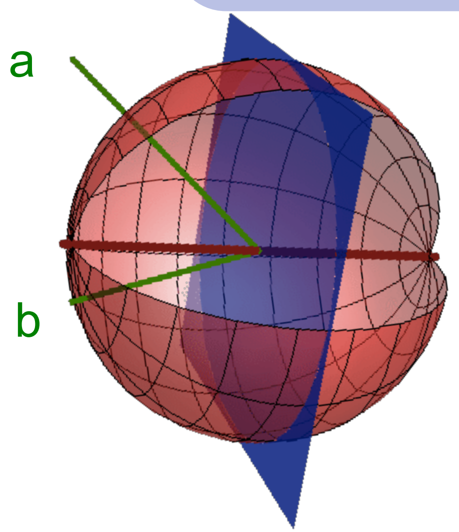
1. Start with strong-energy ordered integrand
2. Add measurement function
3. Take derivatives
4. Simplify with algebra
5. Add in virtual corrections
6. Divide by global term

$$\frac{(p_a\cdot p_b)}{(p_a\cdot p_1)\,(p_1\cdot p_2)\cdots(p_m\cdot p_b)}$$

$$u(\{p_i\}) = \Theta\Big(\rho Q - \sum_i 2(p_i\cdot n)\theta_R(p_i)\Big)$$

$$\partial_L g_{ab}(L) = \int_{\text{left}} \frac{d\Omega_j}{4\pi}\mathcal{W}_{ab}^j\left[U_{abj}(L)g_{aj}(L)g_{jb}(L) - g_{ab}(L)\right]$$

$$U_{abj}(L) = \exp\left[L\int_{\text{right}}\frac{d\Omega_1}{4\pi}\left(\mathcal{W}_{ab}^1 - \mathcal{W}_{aj}^1 - \mathcal{W}_{jb}^1\right)\right]$$



$g_{ab}(L)$ gives the leading NGL
for the right hemisphere mass
from an ab dipole

We checked that this agrees with the SEO picture + global subtractions to 4 loops

Symmetries of BMS equation

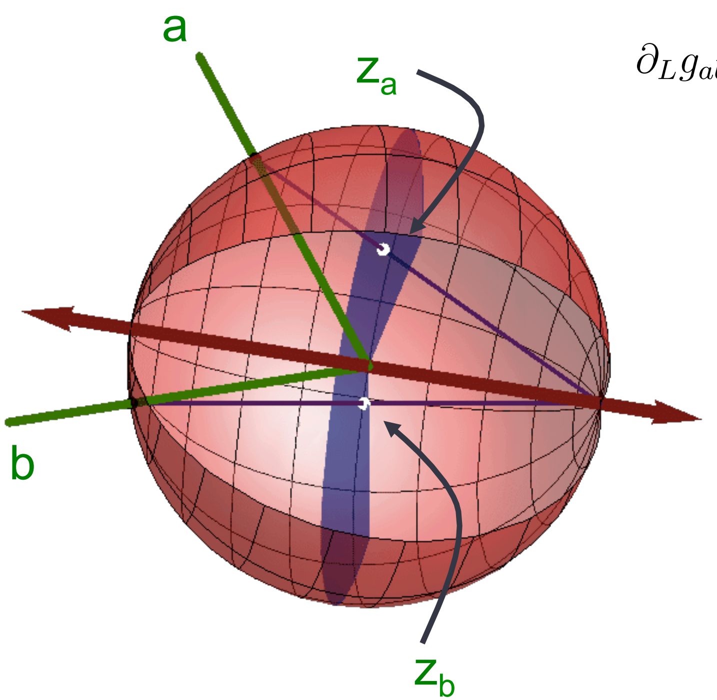
Performing the right-hemisphere integral (softest emission) exactly, BMS becomes

$$\partial_L g_{ab}(L) = \frac{1}{4\pi}\int_0^1 d\cos\theta_j \int_0^{2\pi} d\phi_j \frac{(ab)}{(aj)(jb)}\left[2^{L/2}\cos^L\theta_j \left\{\frac{[ab]}{[aj][jb]}\right\}^{L/2} g_{aj}(L)g_{jb}(L) - g_{ab}(L)\right]$$

$$(ij) = 1-\cos\theta_{ij}$$

$$[ij] = (i\bar{j}) = 1-\cos\theta_{\bar{i}j}$$

Project onto unit disk $z = \frac{\sin\theta}{1+\cos\theta}e^{i\phi}$



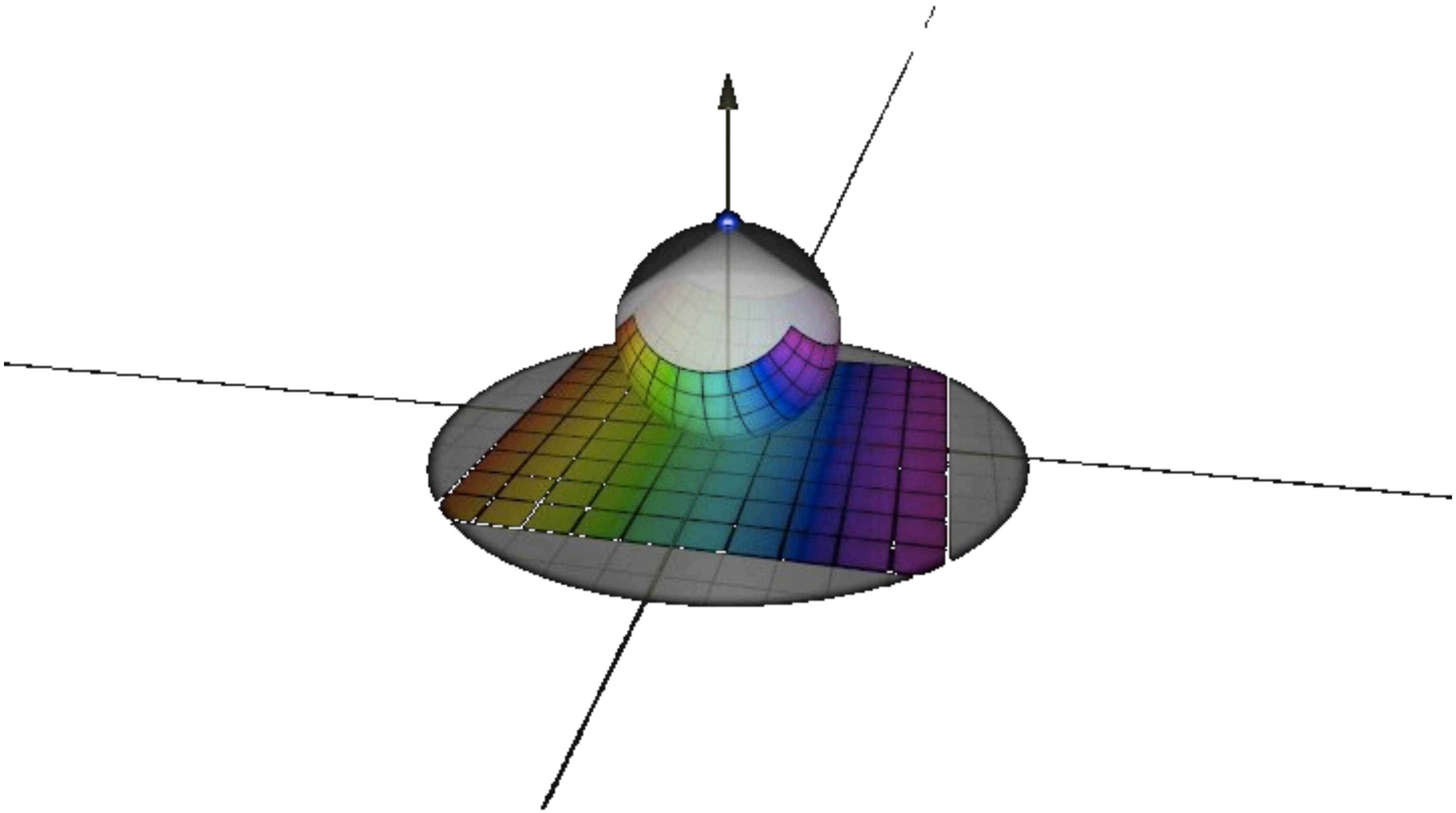
$$\partial_L g_{ab}(L) = \int_{|z|<1} \frac{dz_j\, d\bar{z}_j}{2\pi}\frac{|z_a-z_b|^2}{|z_a-z_j|^2|z_j-z_b|^2}$$
$$\times\left\{\left[\frac{1+\langle ab\rangle}{(1+\langle aj\rangle)(1+\langle jb\rangle)}\right]^{L/2} g_{aj}(L)g_{jb}(L) - g_{ab}(L)\right\}.$$

BMS equation respects hyperbolic metric on Poincare disk

$$\langle ij\rangle = \frac{|z_i-z_j|^2}{(1-|z_i|^2)(1-|z_j|^2)}$$

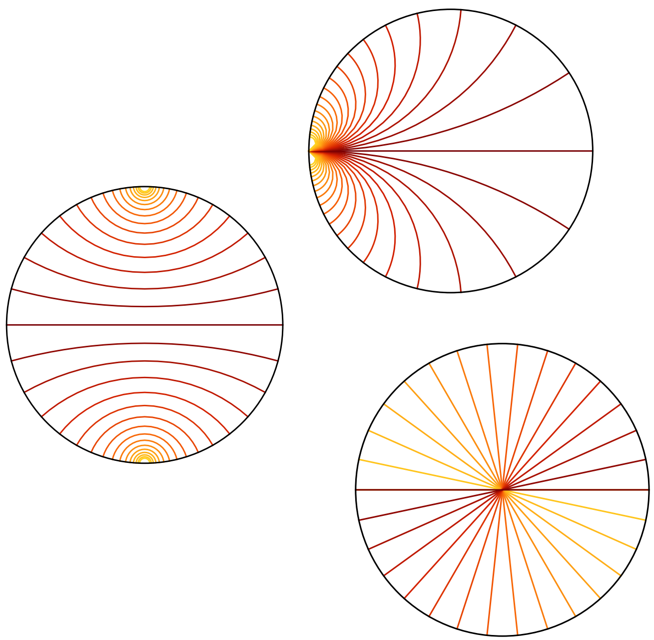
Isometry group is PSL(2,R)

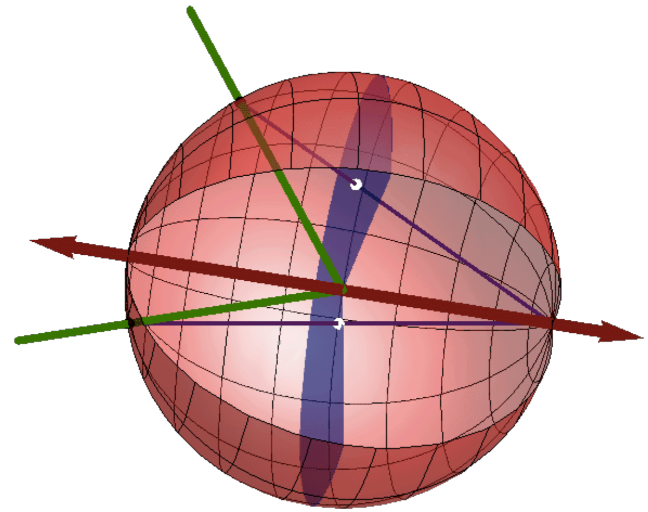
Mobius transformations



Mobius symmetries

- Geodesics are circular arcs perpendicular to boundary
- Isometry group maps **geodesics** to **geodesics**





Azimuthal rotations
(only obvious symmetry)

Instead of 4 angles for a,b: only 1 independent variable

$$g_{ab}(L) = g(\langle ab \rangle, L) =: g\left(\frac{1-\cos\theta_{ij}}{2\cos\theta_i\cos\theta_j}, L\right)$$

Enormous simplification!

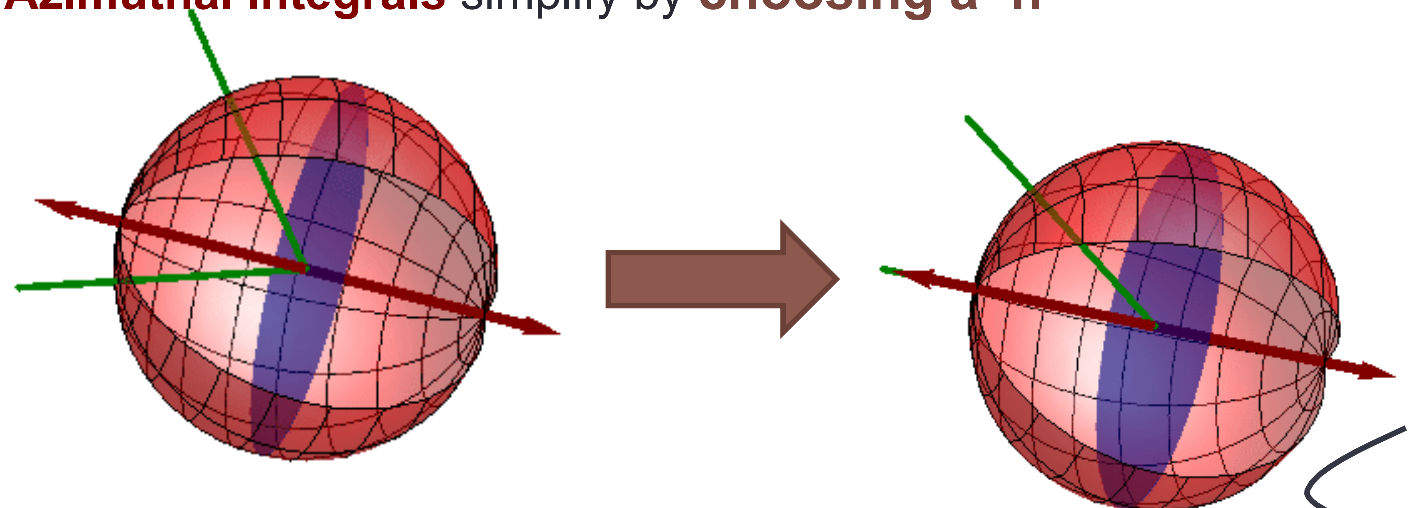
Now integrate

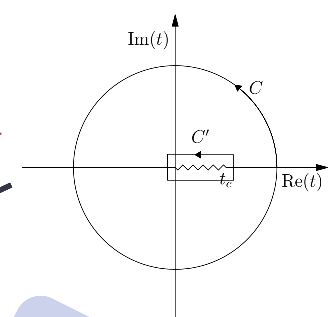
Iterative structure:

- 3-loop integrates over 2-loop, etc

$$\partial_L g^{(3)}_{\bar{n}b}(L) = \frac{1}{4\pi}\int_0^1 d\cos\theta_j \int_0^{2\pi} d\phi_j \frac{(\bar{n}b)}{(\bar{n}j)(jb)}\left[\frac{L^2}{2}(r_{\bar{n}b} - r_{\bar{n}j} - r_{jb})^2 + g^{(2)}_{\bar{n}j} + g^{(2)}_{jb} - g^{(2)}_{\bar{n}b}\right]$$

- At each order, 1 polar angle and 1 azimuthal angle integral
- **Azimuthal integrals** simplify by **choosing a=n**





Azimuthal integrals can be done by contour integration:

3-loops

$$\Phi_3 = \frac{1}{\cos\theta_b - \cos\theta_j}\left[\ln\frac{1+\langle j\bar{n}\rangle}{1+\langle b\bar{n}\rangle}\ln\frac{\langle j\bar{n}\rangle + \langle b\bar{n}\rangle + |\langle j\bar{n}\rangle - \langle b\bar{n}\rangle|}{2}\mathrm{Li}_2(-\langle j\bar{n}\rangle) - \mathrm{Li}_2(-\langle b\bar{n}\rangle)\right]$$

Polar integrals

$$\Phi_3 = \frac{1}{\cos\theta_b - \cos\theta_j}\left[\ln\frac{1+\langle j\bar{n}\rangle}{1+\langle b\bar{n}\rangle}\ln\frac{\langle j\bar{n}\rangle+\langle b\bar{n}\rangle+|\langle j\bar{n}\rangle-\langle b\bar{n}\rangle|}{2}\mathrm{Li}_2(-\langle j\bar{n}\rangle)-\mathrm{Li}_2(-\langle b\bar{n}\rangle)\right]$$

Transcendentality weight 2

Hard to integrate over $d\cos\theta_j$

- Mathematica can do 3-loop, but not easily

$$g_{\bar{n}b}^{(3)}(L) = L^3\left[\frac{\pi^2}{72}\ln(1+u_1) - \frac{1}{24}\ln^2 u_1\ln(1+u_1) + \frac{1}{12}\ln u_1\ln^2(1+u_1) - \frac{1}{36}\ln^3(1+u_1)\right.$$
$$\left. - \frac{1}{12}\ln u_1\mathrm{Li}_2(-u_1) + \frac{1}{12}\ln(1+u_1)\mathrm{Li}_2(-u_1) + \frac{1}{12}\mathrm{Li}_3(-u_1) - \frac{1}{12}\mathrm{Li}_3\left(\frac{1}{1+u_1}\right) + \frac{\zeta(3)}{12}\right],$$

Transcendentality weight 3

- Mathematica **cannot** do 4-loop azimuth or 4-loop polar

Goncharov Polylogarithms

Classical Polylogarithms (CPLs)

$$\mathrm{Li}_k(x) = \int_0^x \frac{dt}{t}\,\mathrm{Li}_{k-1}(t)$$

Goncharov Polylogarithms (GPLs)

$$G(w_1,\ldots,w_n;x) = \int_0^x \frac{dt}{t-w_1} G(\underbrace{w_2,\ldots,w_n}_{\text{index}};\,t),$$

argument

$$G(\underbrace{0,\ldots,0}_{n};x) = \frac{1}{n!}\ln^n x$$

$$G(\underbrace{a,\cdots,a}_{n};x) = \frac{1}{n!}\ln^n\left(1-\frac{x}{a}\right)$$

$$G(\underbrace{0,\cdots,0}_{n-1},a;x) = -\mathrm{Li}_n\left(\frac{x}{a}\right)$$

3-loop polar integral:

$$\frac{1}{L^3} g_{\bar{n}b}^{(3)}(L) = -\frac{1}{24}\int_0^\infty d\langle j\bar{n}\rangle \left(\frac{1}{\langle j\bar{n}\rangle} - \frac{1}{\langle j\bar{n}\rangle - \langle b\bar{n}\rangle}\right)\Phi_3(\langle b\bar{n}\rangle, \langle j\bar{n}\rangle)$$

Easy to integrate if we can write as GPLs with $\langle j\bar{n}\rangle$ in argument only

Symbols

- Introduced to physics by Goncharov, Spradlin, Vergu and Volovich
  - Dramatically simplified 2-loop 6 particle MHV amplitude in N=4 from 17 pages to 2 lines

$$R_6^{(2)}(u_1,u_2,u_3) = \sum_{i=1}^{3}\left(L_4(x_i^+,x_i^-) - \frac{1}{2}\mathrm{Li}_4(1-1/u_i)\right) - \frac{1}{8}\left(\sum_{i=1}^{3}\mathrm{Li}_2(1-1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72}$$

Symbol is a mapping to a **tensor algebra** over the group of **rational functions**

- Acts nicely on polylogarithms and iterated integrals

$$\mathrm{Li}_k(x) = \int_0^x \frac{dt}{t}\,\mathrm{Li}_{k-1}(t) \qquad G(w_1,\dots,w_n;x) = \int_0^x \frac{dt}{t-w_1}G(w_2,\dots,w_n;t),$$

$$\mathcal{S}[\mathrm{Li}_k(x)] = -(1-x)\otimes\underbrace{x\otimes\cdots\otimes x}_{k-1} \qquad \mathcal{S}\left[G\left(a_1,\dots,a_n;x\right)\right] = \left(1-\frac{x}{a_n}\right)\otimes\cdots\otimes\left(1-\frac{x}{a_1}\right)$$

- Maps constants to zero
- Two functions with the same symbol agree up to constants
  - For functions of uniform transcendentality, constants must be transcendental:

Transcendentality weight:	1	2	3	4	5
Constant:	π	$\zeta(2)$	$\pi^3,\zeta(3)$	$\pi\zeta(3),\zeta(4)$	$\zeta(5),\zeta(2)\zeta(3)$

Symbols

1. Original function: $\Phi_3 = \frac{1}{\cos\theta_b - \cos\theta_j}\left[\ln\frac{1+\langle j\bar{n}\rangle}{1+\langle b\bar{n}\rangle}\ln\frac{\langle j\bar{n}\rangle + \langle b\bar{n}\rangle + |\langle j\bar{n}\rangle - \langle b\bar{n}\rangle|}{2}\mathrm{Li}_2(-\langle j\bar{n}\rangle) - \mathrm{Li}_2(-\langle b\bar{n}\rangle)\right]$

2. Calculate its symbol: $\mathcal{S}\left[\Phi_3(u_1,u_2)|_{u_1>u_2}\right] = u_1\otimes(1+u_2) - (1+u_1)\otimes(1+u_2) - u_2\otimes(1+u_2) + (1+u_2)\otimes u_1$

$$- (1+u_2)\otimes(1+u_1) - (1+u_2)\otimes u_2 + 2\left[(1+u_2)\otimes(1+u_2)\right]$$

3. Find GPLs in canonical form with same symbol

$$\Phi_3^G(u_1,u_2) = G(0;u_1)G(-1;u_2) - G(-1;u_1)G(-1;u_2) - G(-1,0;u_2) - G(0,-1;u_2) + 2G(-1,-1,u_2)$$

4. Check numerically or with coproduct for missing transcendental numbers

$$\Phi_3(u_1,u_2)|_{u_1>u_2} = \Phi_3^G(u_1,u_2) + c\,\pi^2$$

5. Integrate $\frac{1}{L^3}g^{(3)}_{\bar{n}b}(L) = \frac{\pi^2}{36}G(-1;u_1) - \frac{1}{4}G(-1,-1,-1;u_1) + \frac{1}{4}G(-1,-1,0;u_1) + \frac{1}{12}G(-1,0,-1;u_1) - \frac{1}{12}G(-1,0,0;u_1)$

6. Convert back to classical polylogarithms:

$$x = \langle ab\rangle$$

$$\frac{1}{L^3}g^{(3)}_{ab}(L) = \frac{\pi^2}{72}\ln(1+x) - \frac{1}{24}\ln^2 x\ln(1+x) + \frac{1}{12}\ln x\ln^2(1+x) - \frac{1}{36}\ln^3(1+x) - \frac{1}{12}\ln x\mathrm{Li}_2(-x) + \frac{1}{12}\ln(1+x)\mathrm{Li}_2(-x) + \frac{1}{12}\mathrm{Li}_3(-x) - \frac{1}{12}\mathrm{Li}_3\left(\frac{1}{1+x}\right) + \frac{\zeta(3)}{12},$$

4-loops

$$x = \langle ab \rangle$$

$$\frac{1}{L^4} g_{ab}^{(4)}(L) = \frac{\pi^2}{36} G(-1,-1;x) - \frac{\pi^2}{144} G(-1,0;x) - \frac{3}{16} G(-1,-1,-1,-1;x) + \frac{3}{16} G(-1,-1,-1,0;x) + \frac{1}{12} G(-1,-1,0,-1;x) - \frac{1}{12} G(-1,-1,0,0;x) + \frac{1}{48} G(-1,0,-1,-1;x) - \frac{1}{96} G(-1,0,-1,0;x) - \frac{1}{32} G(-1,0,0,-1;x) + \frac{1}{48} G(-1,0,0,0;x) - \frac{\zeta(3)}{16} G(-1;x)$$

We stopped at $g_{ab}(L)$ at 4-loops and $g_{n\bar{n}}(L)$ at 5-loops:

$$g_{n\bar{n}}(L) = 1 - \frac{\pi^2}{24} L^2 + \frac{\zeta(3)}{12} L^3 + \frac{\pi^4}{34560} L^4 + \left(-\frac{\pi^2 \zeta(3)}{360} + \frac{17\zeta(5)}{480} \right) L^5 + \dots$$

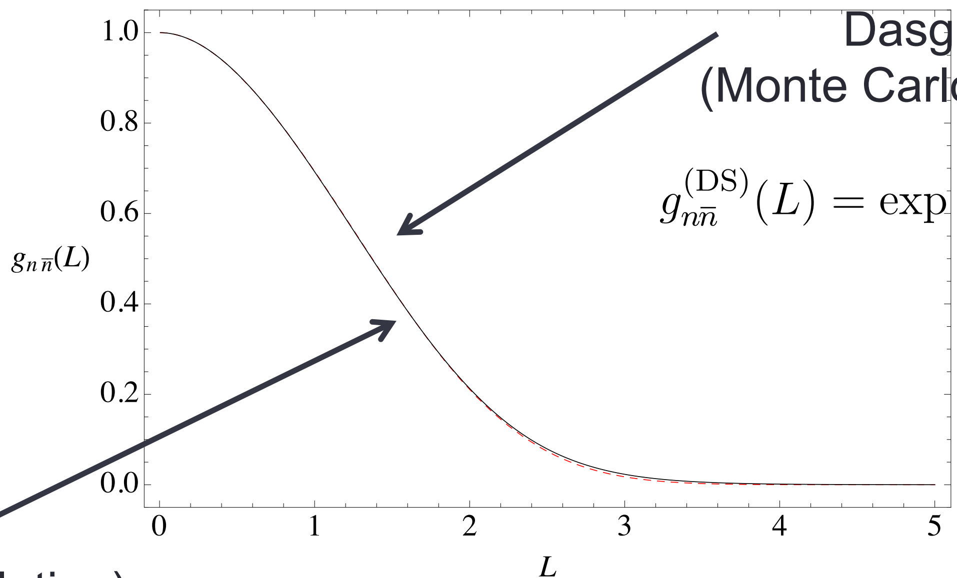
Resummation

BMS equation

$$\partial_L g_{ab}(L) = \frac{1}{4\pi}\int_0^1 d\cos\theta_j \int_0^{2\pi} d\phi_j \frac{(ab)}{(aj)(jb)}\left[2^{L/2}\cos^L\theta_j \left\{\frac{[ab]}{[aj][jb]}\right\}^{L/2} g_{aj}(L)g_{jb}(L) - g_{ab}(L)\right]$$

Boundary condition is simple: $g_{ab}(0) = 1$

Can solve numerically by discretizing and integrating from L=0



Dasgupta-Salam
(Monte Carlo event generator)

$$g^{(\mathrm{DS})}_{n\bar{n}}(L) = \exp\left[-\frac{\pi^2}{24}L^2\frac{1+0.180625L^2}{1+0.325472L^{1.33}}\right]$$

DS fit to MC output

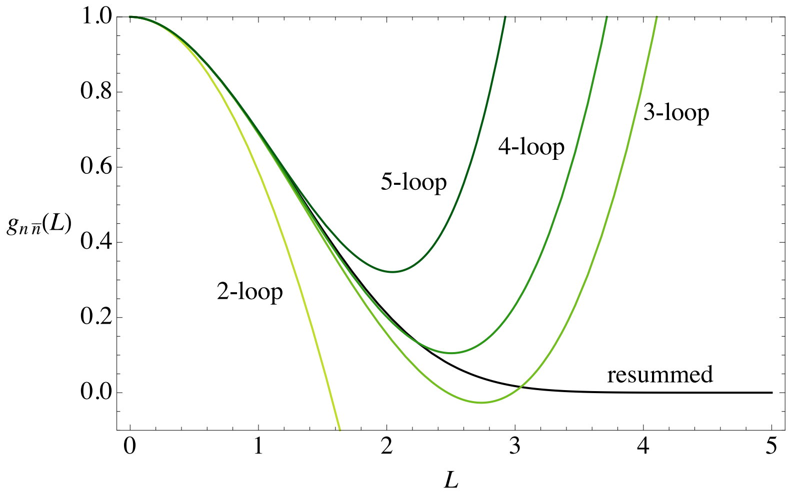
BMS (our solution)

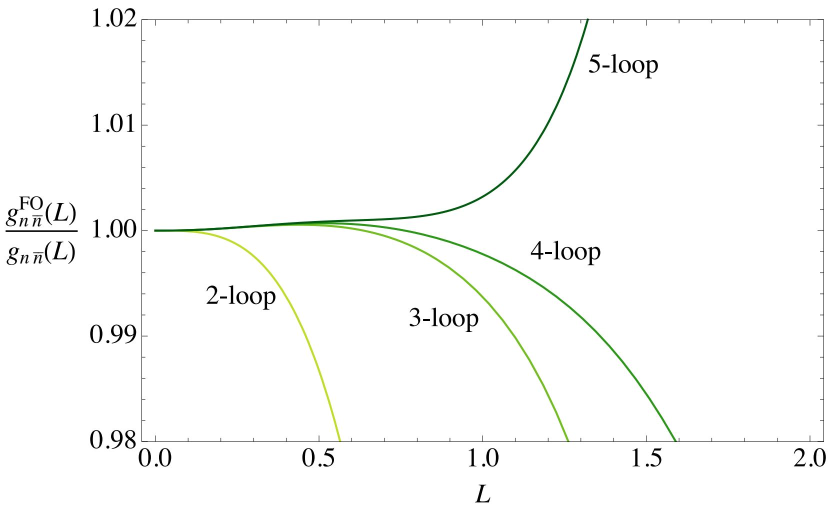
- Agreement to 0.1% for L < 2

Convergence

$$g_{n\bar{n}}(L) = 1 - \frac{\pi^2}{24}L^2 + \frac{\zeta(3)}{12}L^3 + \frac{\pi^4}{34560}L^4 + \left(-\frac{\pi^2\zeta(3)}{360} + \frac{17\zeta(5)}{480}\right)L^5 + \dots$$

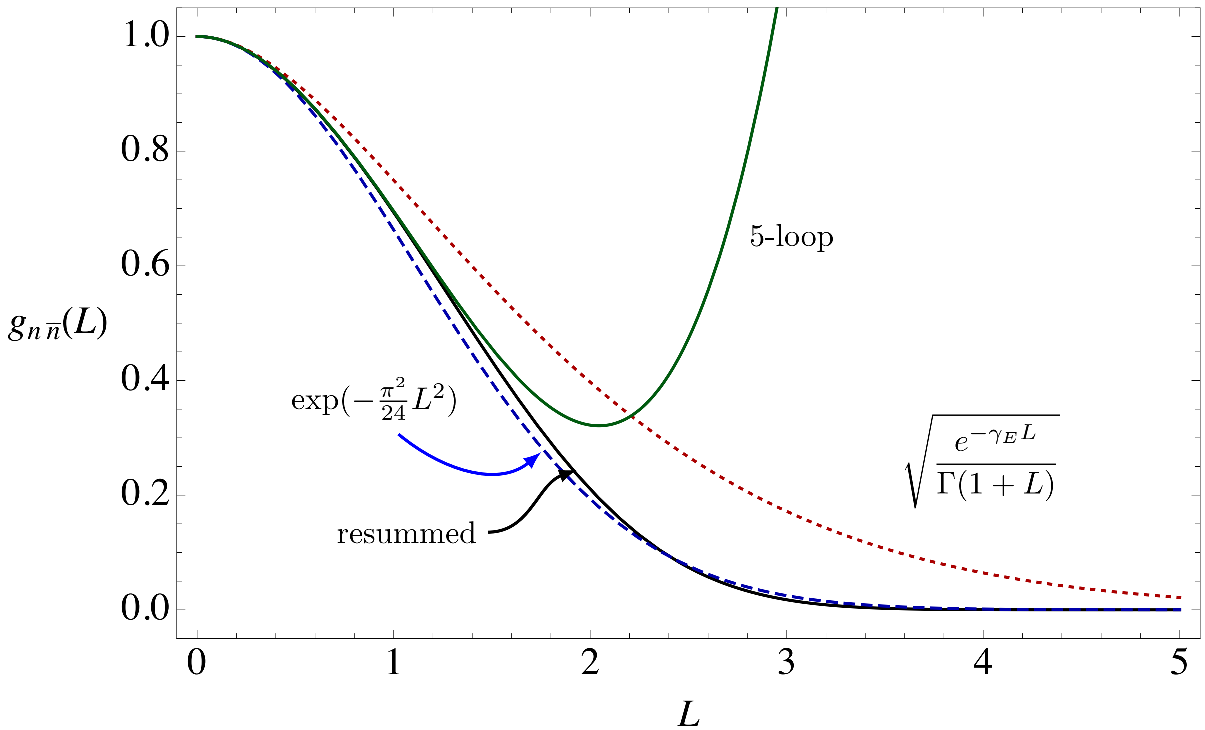
$$g_{n\bar{n}}(L) = 1 - 0.411233512L^2 + 0.10017141L^3 + 0.0028185501L^4 + 0.0037694522L^5 + \dots$$





Seems to converge up to around L=1

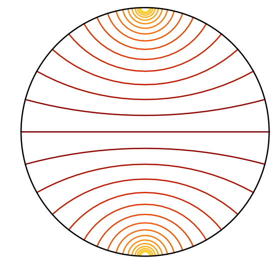
Exponentiation



Exponential of 2-loop result has excellent agreement with resummed result

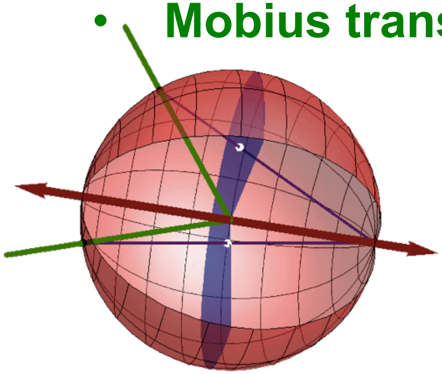
Very intriguing…

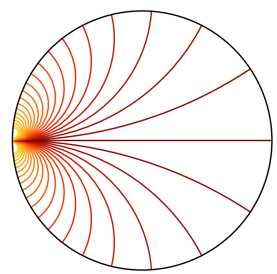
Summary

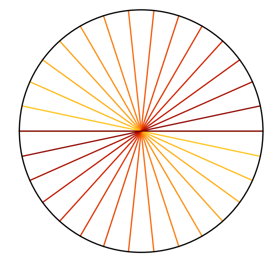


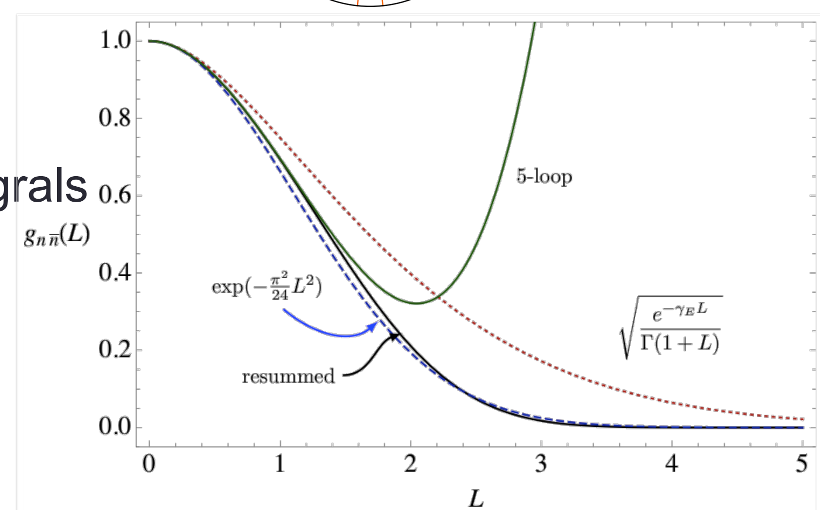
1. Strong-energy ordering lets us resum stuff SCET can't (so far)
   - Includes all real and virtual contributions
   - Easiest at large N
   - Leading non-global logs only (not systematically improvable)

2. Leading NGL has a hidden **PSL(2,R)** symmetry
   - **Mobius transformations** of Poincare disk









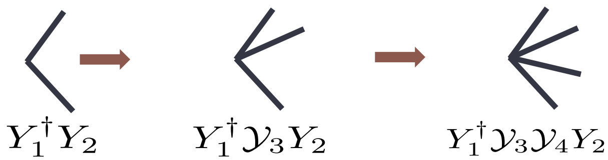
3. NGLs at each order involve iterated integrals

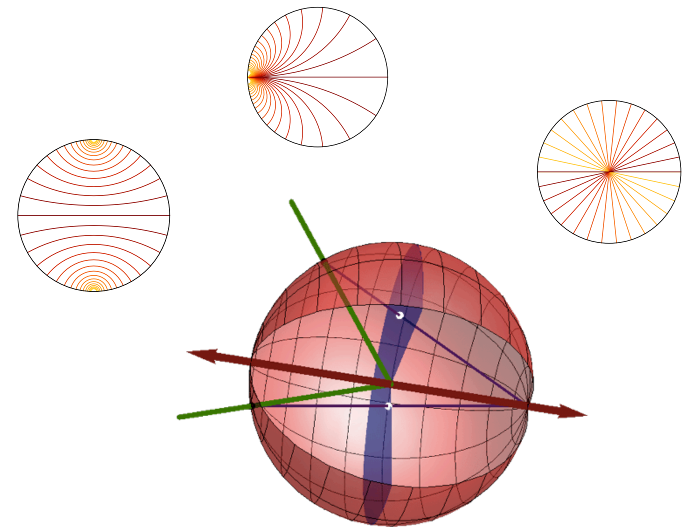
4. **Symbols** and Goncharov polylogarihms
   - Simplify expressions
   - Make polar integrals trivial

5. **Resummation** of leading NGL is possible

Questions

1. Can leading NGLs be resummed with Effective Field Theory?





2. **Physics** of PSL(2,R)?

3. Can the BMS equation be solved **anlaytically**?

4. Can **subleading NGLs** be resummed?
   - Probably not with strong-energy-ordering

5. Do these symmetries/methods work for **non-hemisphere NGLs**?
   - Yes, for out-of-jet energy

6. Does Exp[2-loop] **always work** well?
   - Important for phenomenology