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# Demystifying Multivariate Searches and the Hunt for the Higgs

Matthew Schwartz

Harvard University

Work down with Jason Gallicchio,

PRL, 105:022001, 2010

and with Gallicchio, Tweedie, Huth, Kagan and Black

*... in preparation*

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Johns Hopkins University  
September 20, 2010

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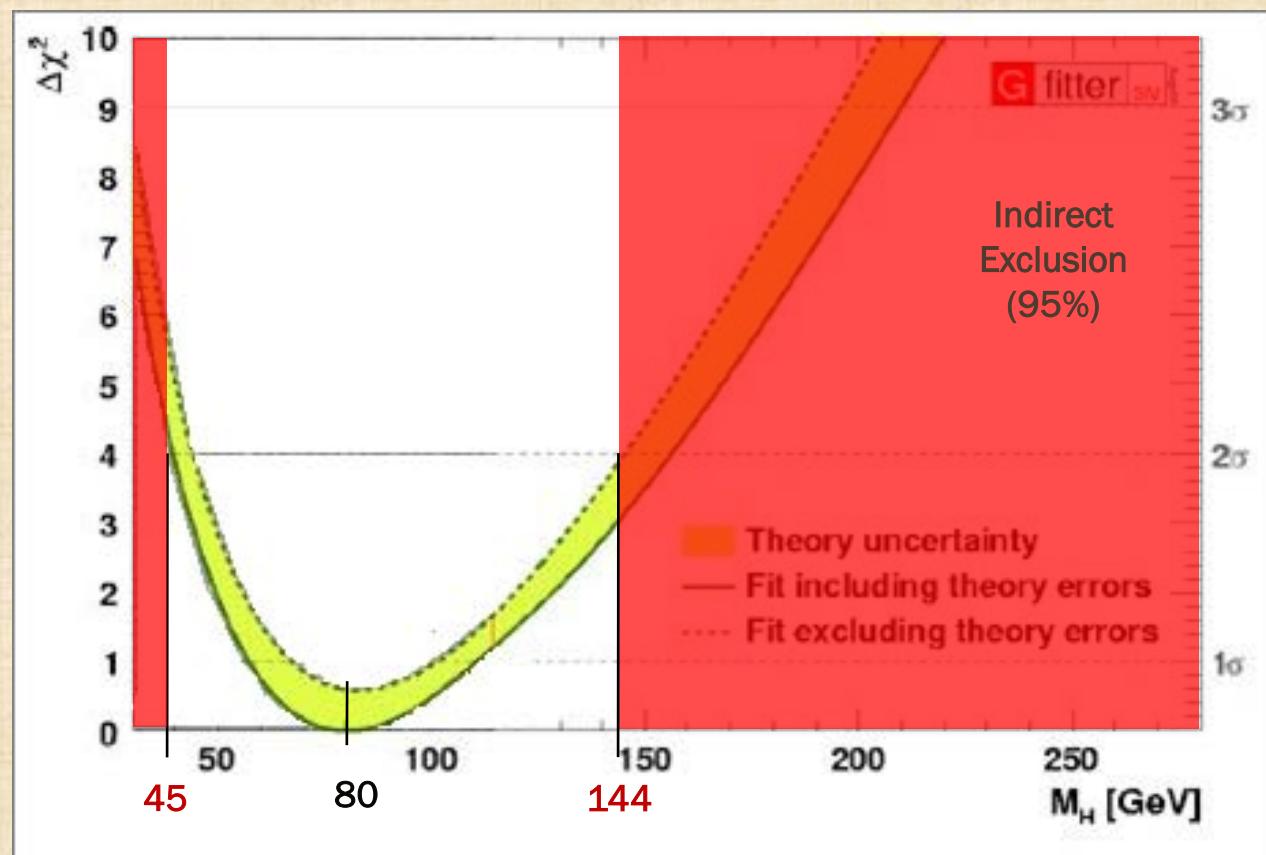
# Part 1:

# Motivation

# WHERE IS THE HIGGS?

Parameter	Input value
$M_Z$ [GeV]	$91.1875 \pm 0.0021$
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$
$\sigma_{\text{had}}^0$ [nb]	$41.540 \pm 0.037$
$R_\ell^0$	$20.767 \pm 0.025$
$A_{\text{FB}}^{0,\ell}$	$0.0171 \pm 0.0010$
$A_\ell^{(*)}$	$0.1499 \pm 0.0018$
$A_c$	$0.670 \pm 0.027$
$A_b$	$0.923 \pm 0.020$
$A_{\text{FB}}^{0,c}$	$0.0707 \pm 0.0035$
$A_{\text{FB}}^{0,b}$	$0.0992 \pm 0.0016$
$R_c^0$	$0.1721 \pm 0.0030$
$R_b^0$	$0.21629 \pm 0.00066$
$\sin^2\theta_{\text{eff}}^\ell(Q_{\text{FB}})$	$0.2324 \pm 0.0012$

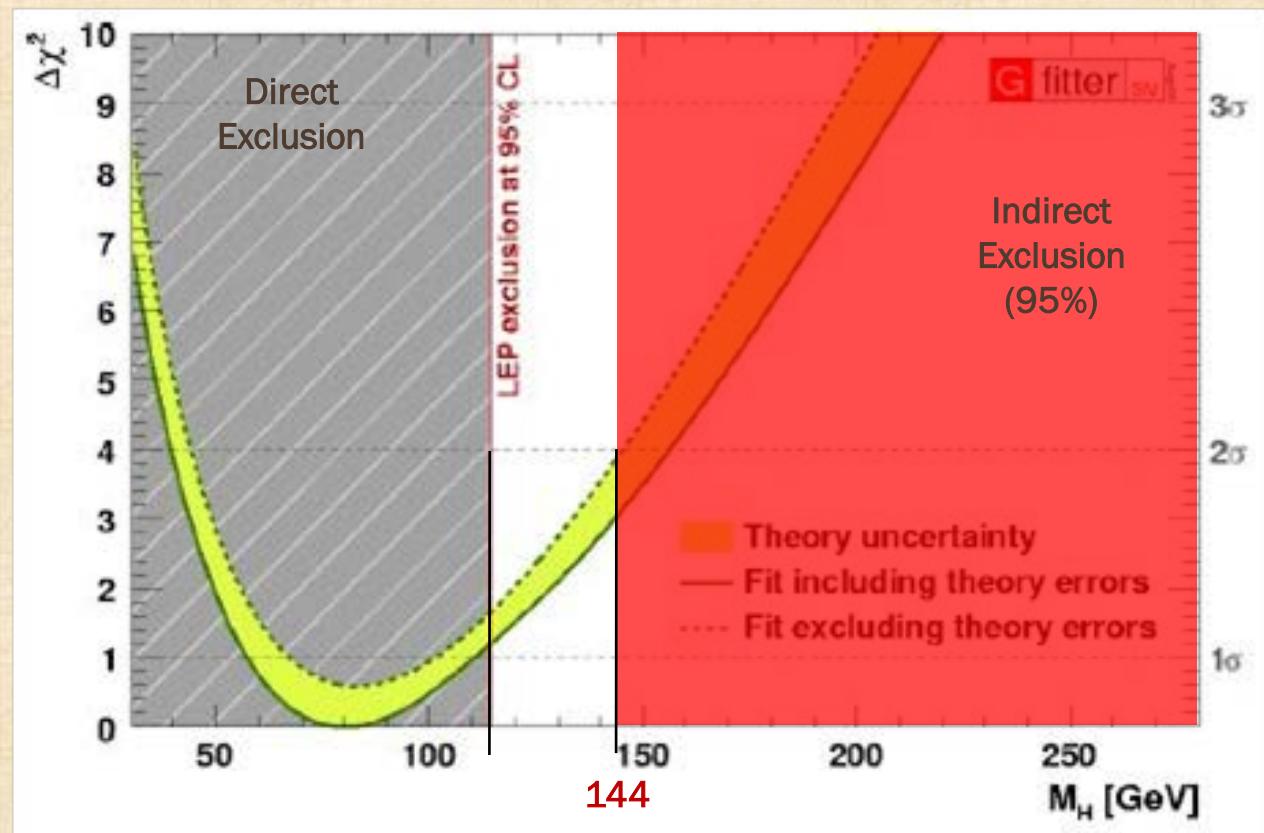
Combine many  
observables to constrain  
Higgs mass



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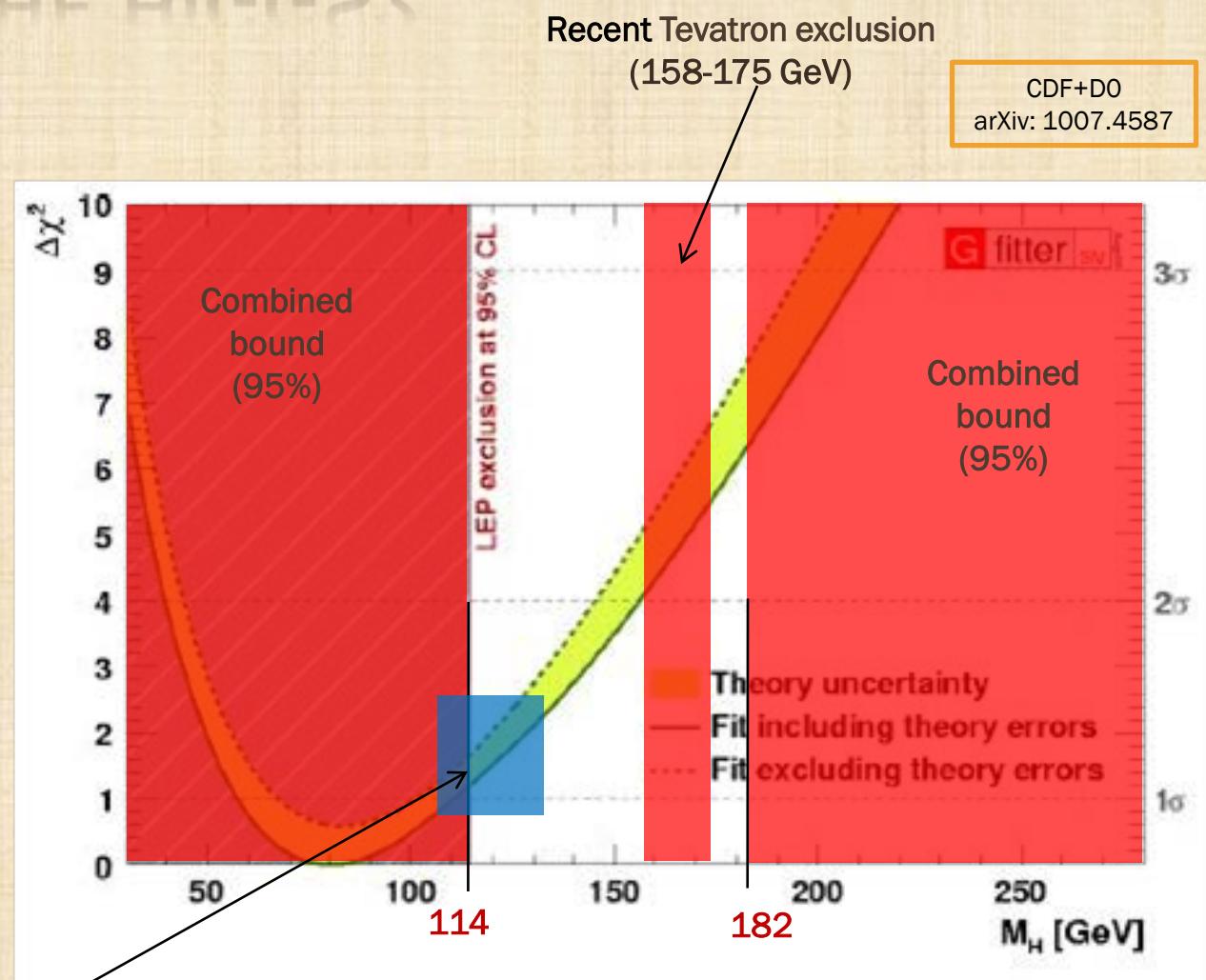
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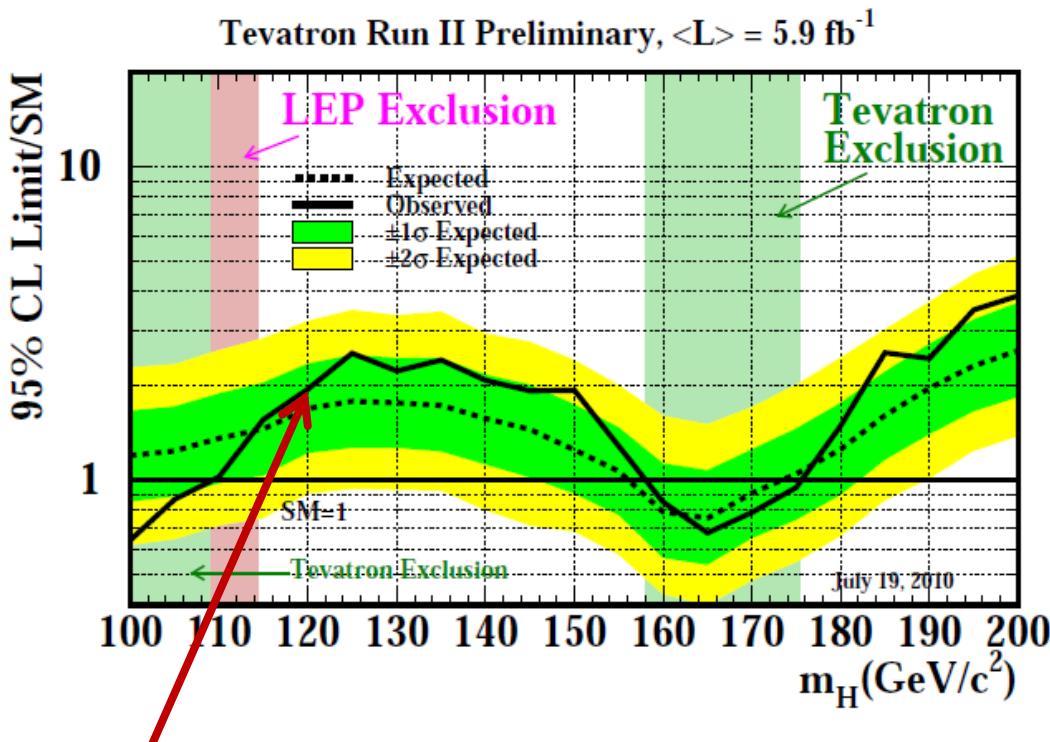
Combine many observables to constrain Higgs mass



If it exists, Higgs is most likely light

# HOW DO WE FIND A LIGHT HIGGS?

## Tevatron



- Need a factor of 2 improvement in significance for  $m_H = 120$
- Double statistics gives  $\sqrt{2}$ , where will the other  $\sqrt{2}$  come from?

## LHC

- Important search channel is  $\text{pp} \rightarrow W/Z + H$   
 $H \rightarrow bb$
- Abandoned by ATLAS and CMS  
too much background
- Recently high  $P_T$   $W/Z + H$  revived,
  - Requires  $P_T > 200$
  - Lose 95% of signal

How **good** can we do  
in  $W/Z + (H \rightarrow bb)$ ?

# FOCUS ON $pp \rightarrow HZ \rightarrow b\bar{b}l^+l^-$

CDF note 10235 (summer 2010)

$ZH$	0.7
$t\bar{t}$	9.9
$WW$	0.02
$WZ$	0.1
$ZZ$	3.6
$Z \rightarrow \ell\ell + b\bar{b}$	22.1
$Z \rightarrow \ell\ell + c\bar{c}$	2.4
$Z \rightarrow \ell\ell + l.f.$	1.2
fakes	0.9
Total Bkg	40.3

Dominant background  
is the irreducible one

CDF employs **multivariate** approach

Inputs to the neural net are

- Missing transverse energy
- Dijet mass
- $t\bar{t}$  **matrix element** output
- $ZH$  **matrix element** output
- Sum of leading jet Pt's
- number of jets

} Parton-level kinematics

Questions:

- Are there **smarter** more comprehensive inputs?
- Can we **trust** the multivariate approach?

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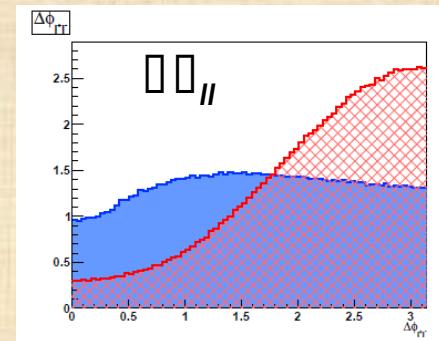
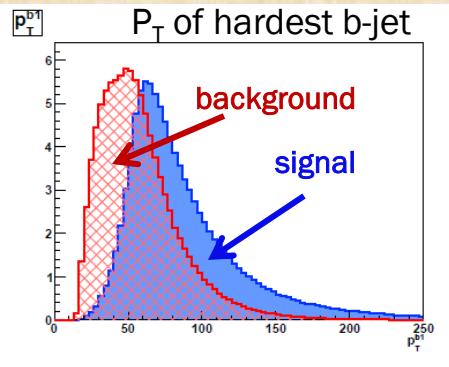
# Part 2:

# The Inputs

# KINEMATIC VARIABLES

## Standard Stuff

- $P_T$ 's of **b**'s and the **leptons**
- $\Delta\phi$  for the **b** jets and the **leptons**
- $\Delta R$  of the **b**'s and the **leptons**
- $P_T$  of the reconstructed **Z**
- $P_T$  of the reconstructed **Higgs**
- $m_{bb}$ : invariant **mass** of the **b**'s

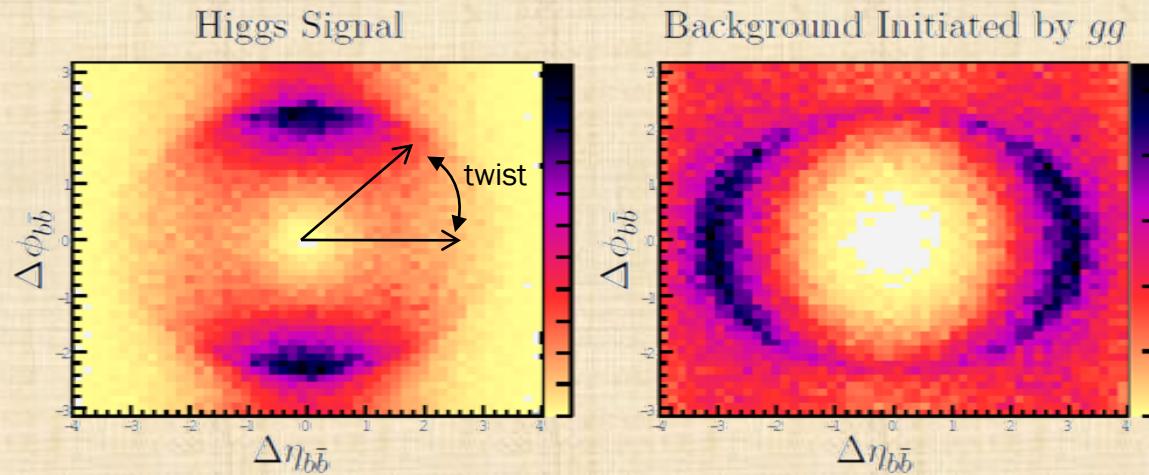


## Less Standard Stuff

- acoplanarity of the **b**'s:  $2\pi - \Delta\theta_{b\bar{b}} - \Sigma\theta_{b\bar{b}}$
- acoplanarity of the **leptons**
- transverse **mass** of the **bb** system  $m_T^{b\bar{b}} = \sqrt{m^2 + p_x^2 + p_y^2}$
- transverse **mass** of the **lepton** system
- invariant **mass** of 2 **leptons** and 1 **b** or 2 **b**'s and 1 **lepton**

# TWIST

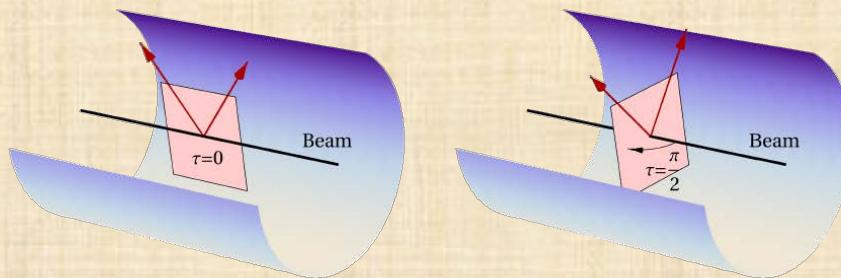
Look at 2D distribution in  $\Delta\eta_{b\bar{b}}$ - $\Delta\phi_{b\bar{b}}$  space:



It seems that neither **L** nor **R** nor **R** holds the right information

Introducing **twist** = polar angle in this plane

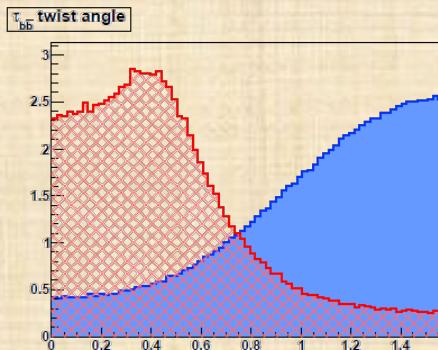
Background has pole for  
zero **twist**  
(t-channel singularity)



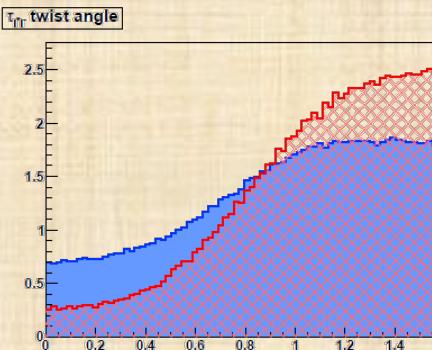
# TWIST

Parton level – no cuts

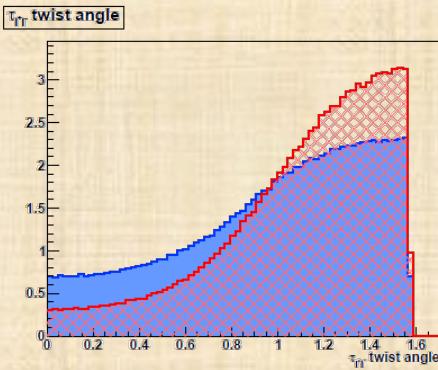
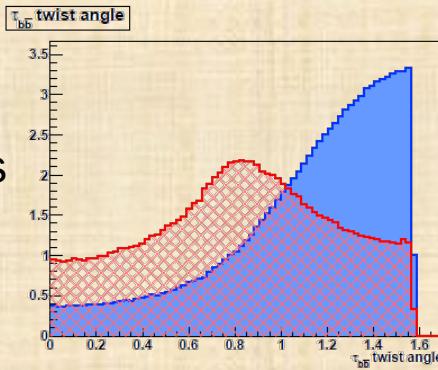
b-jet twist



lepton twist



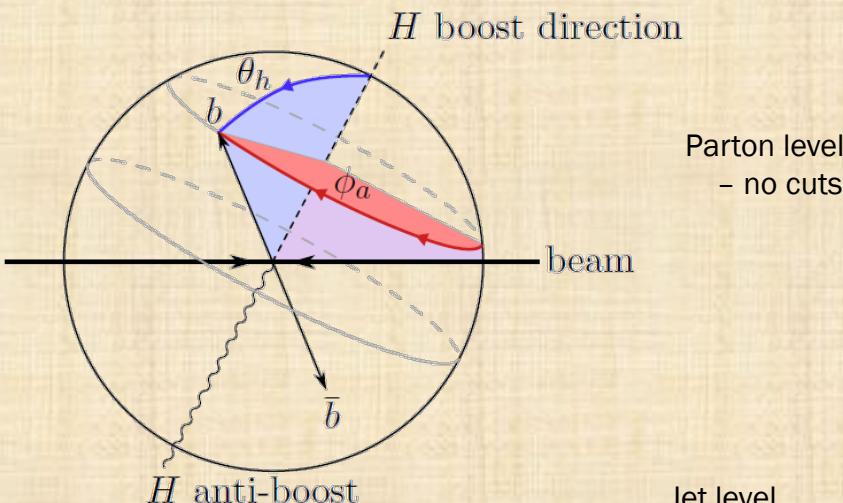
Jet level with detector cuts



Could be more generally useful....

# HELICITY AND AZILICITY ANGLES

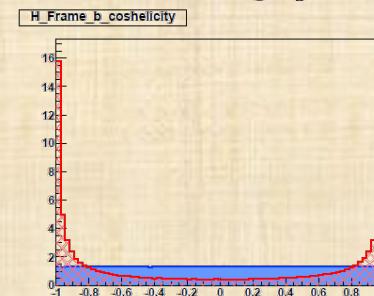
Angles in Higgs rest frame  
relative to H boost direction



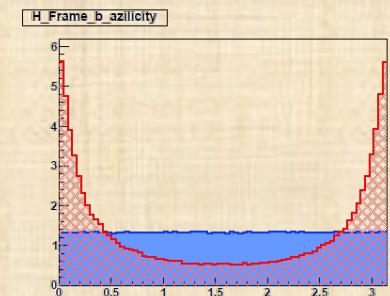
Parton level  
– no cuts

Jet level  
with detector cuts

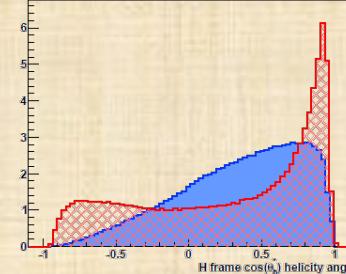
helicity angle



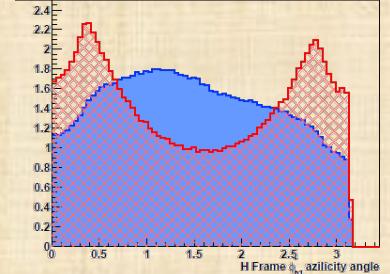
azilicity angle



$H$  frame  $\cos(\theta_b)$  helicity angle



$H$  Frame  $\phi_{b_1}$  azilicity angle



Signal is on-shell

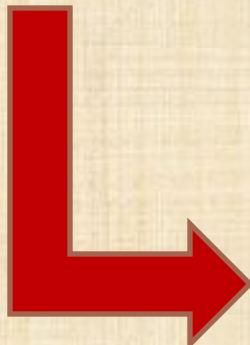
- angles **meaningful**

Background is not a resonance

- Angles **meaningless**
- Expect **peaked** due to collinear singularities

# CHINESE MENU METHOD

- Pick a particle: high- $p_T$   $b$ -jet, low- $p_T$   $b$ -jet, high- $p_T$  lepton, low- $p_T$  lepton, Higgs,  $Z$
- Optionally transform to a boosted frame: Higgs,  $Z$ , Center of Mass (CM)
- Optionally rotate the polar axis to point along the initial direction of the particle whose frame you're in (for Helicity and Azilicity Angles).
- Pick a kinematic property:  $p_T$ ,  $\eta$ ,  $\phi$ ,  $\cos(\theta)$ , etc.
- Optionally pick a second particle to form a sum or difference, sometimes with a coordinate transformation as in  $\Delta R$  and twist  $\tau$ , and sometimes with a more complicated combination as in invariant-mass.
- For vector quantities optionally take the magnitude of vector sums,  $|\vec{p}_1 \pm \vec{p}_2|$  or scalar sums,  $|\vec{p}_1| \pm |\vec{p}_2|$ .
  - $\Delta y_{H,b1}$  and  $\Delta y_{H,b2}$ : Difference in rapidity between Higgs and higher- $p_T$  or lower- $p_T$   $b$ -jet
  - $\cos(\theta_{b2}^*)$ : Center of Mass frame  $\cos(\theta)$  of the lower- $p_T$   $b$ -jet. Same for higher- $p_T$   $b$ -jet.
  - $\Delta p_T^{Z,l1}$ : Difference in  $p_T$  between the reconstructed  $Z$  and the higher- $p_T$  lepton
  - $\Delta p_T^{b1,l2}$ : Difference in  $p_T$  between the higher- $p_T$   $b$ -jet and the lower- $p_T$  lepton
  - $\Sigma p_T^{b1,l2}$ : Sum of  $p_T$ 's of the higher- $p_T$   $b$ -jet and the lower- $p_T$  lepton
  - $\Delta \eta_{b1,l2}$ : Difference in  $\eta$  between the higher- $p_T$   $b$ -jet and the lower- $p_T$  lepton



# EVENT SHAPE VARIABLES

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Nothing to do with the particular signal or background

- $H_T$  = Scalar sum of all  $E_T$
- $\Sigma p_T$  = Scalar sum of all  $p_T$  (which differs from  $H_T$  for massive jets)
- $H_z$  = Boost of the center-of-mass system along the beam
- $E_{vis}$  = Scalar sum of all visible energy
- $\hat{s}$  = CM energy for hard collision, or invariant mass of the reconstructed Higgs and Z.
- Centrality
- Aplanarity and Aplanority
- Sphericity and Spherocity
- DShape and Yvariable (related to the eigenvalues that go into defining the above)
- Fox-Wolfram Moments

# JET SUPERSTRUCTURE

What is not in the parton-level kinematics?

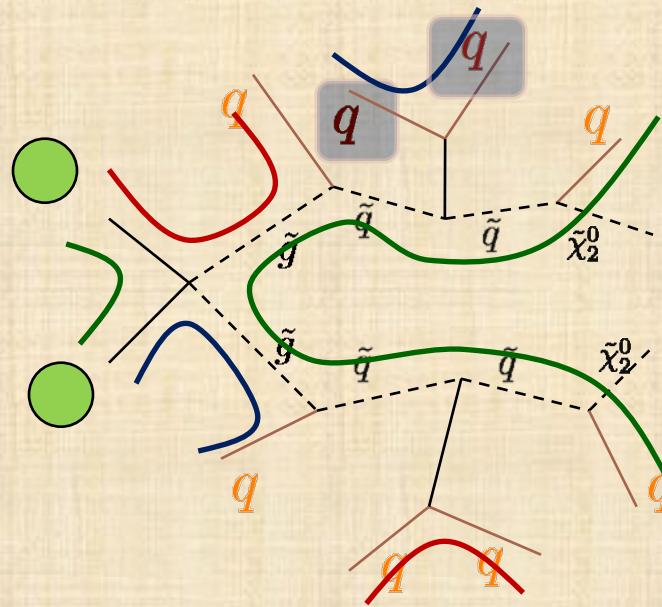
- **Global information**

- Event shapes

- **Color:**

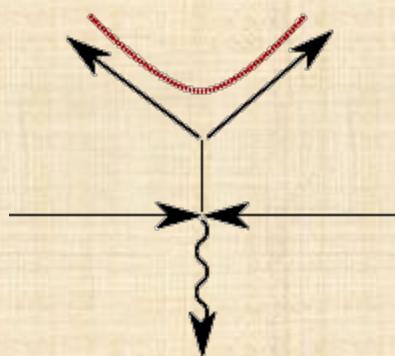
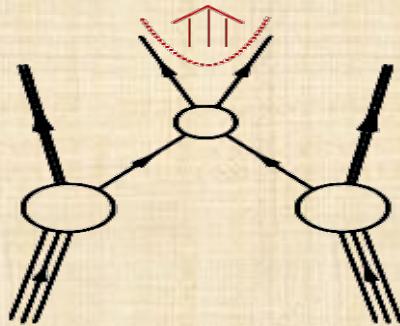
- Color **charge**: Quark vs. Gluon jets
- Color **connections**

$$\text{Tr}[T^A T^B] \propto \delta^{AB}$$

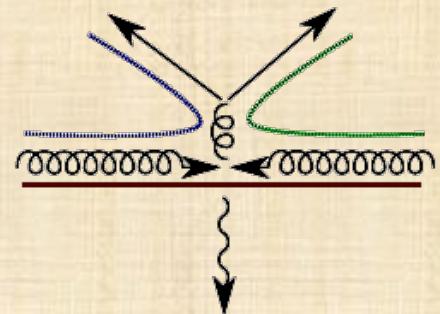
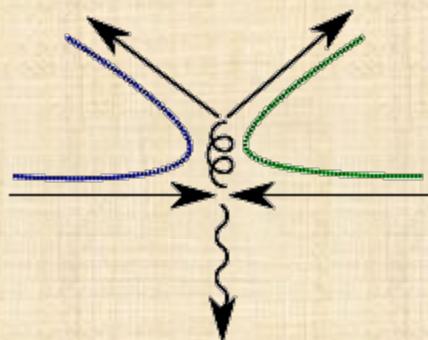
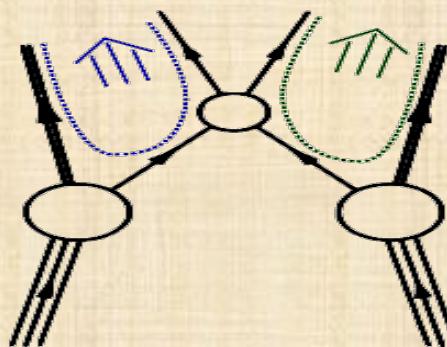


# COLOR CONNECTIONS

Signal



Background



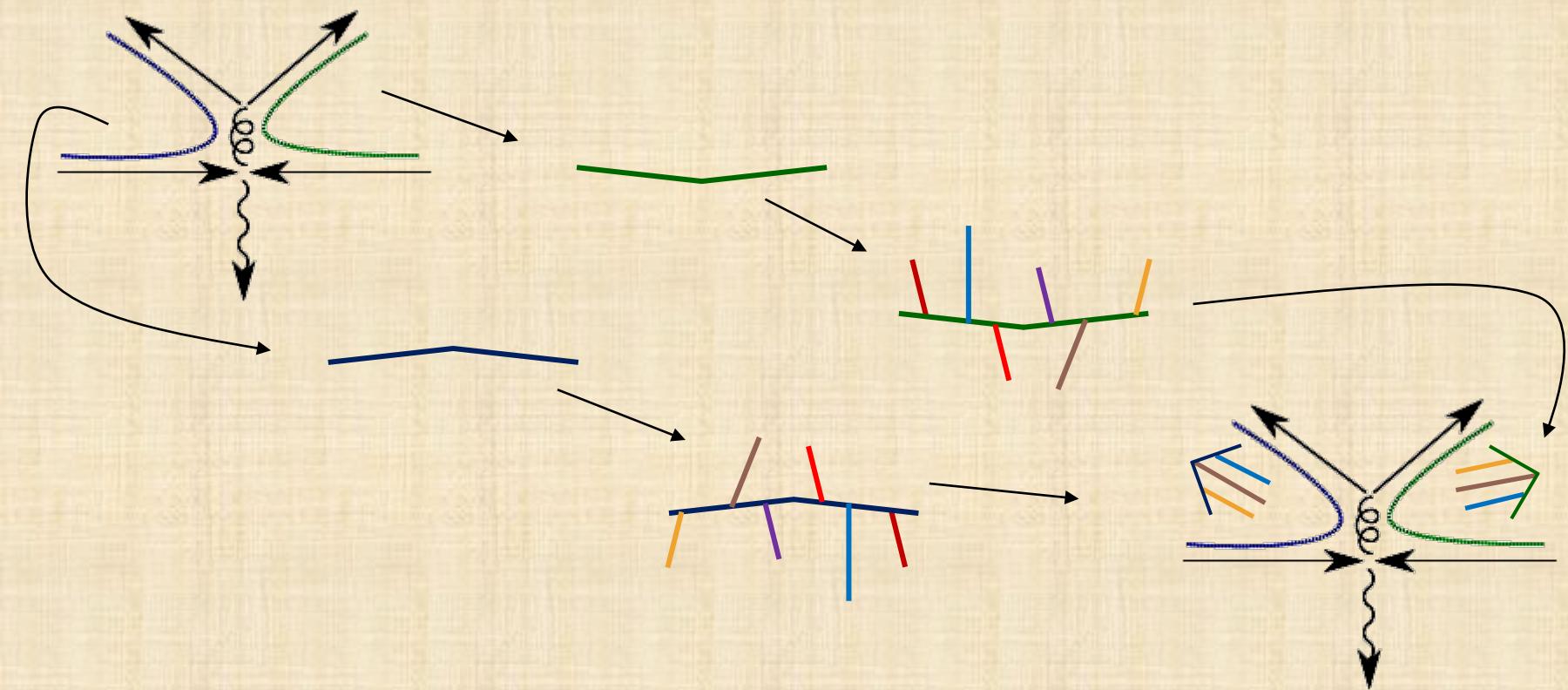
$q\bar{q} \rightarrow Zb\bar{b}$

$gg \rightarrow Zb\bar{b}$

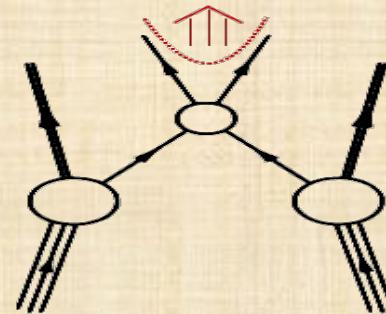
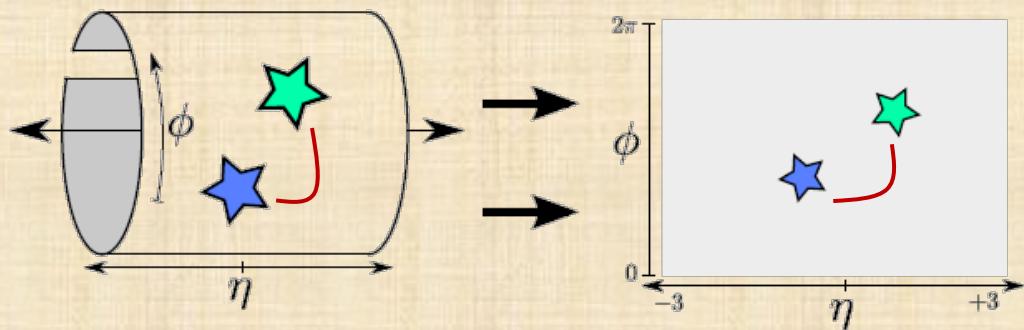
# HOW DO THEY SHOW UP?

## Monte Carlo simulation

- **Color coherence** (angular ordering, e.g. Herwig)
- Color string showers in its rest frame (pt ordering, e.g. Pythia)
  - Boost → **string showers** in **string-momentum** direction



# HOW DO THEY SHOW UP?

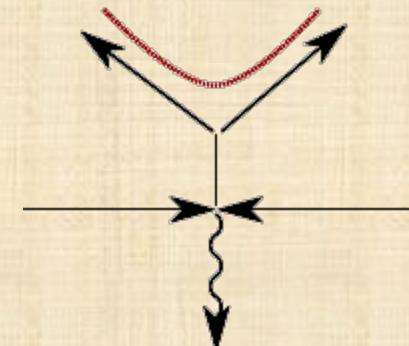
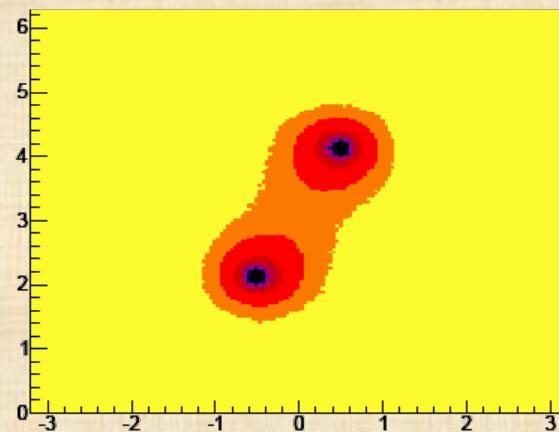


Shower same event  
*millions* of times

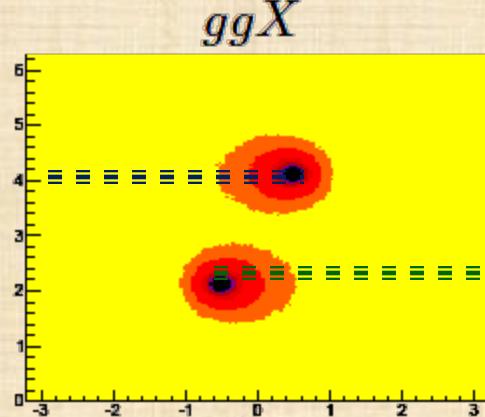
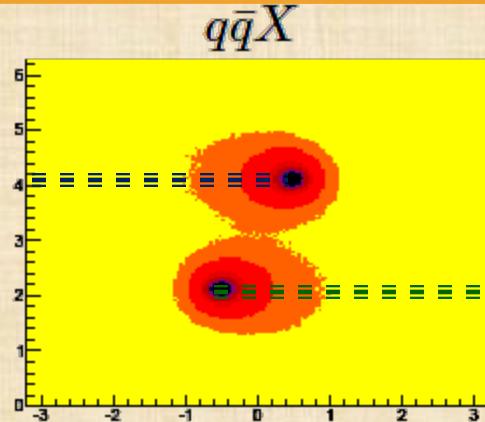
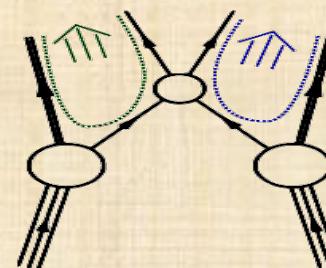
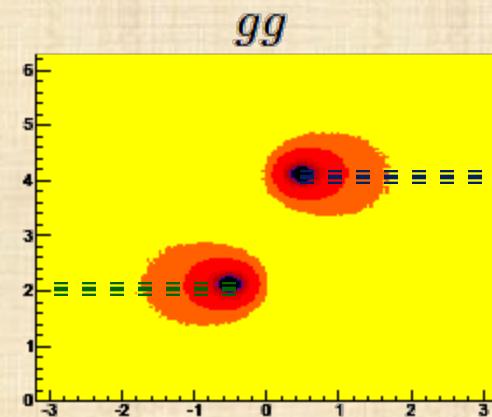
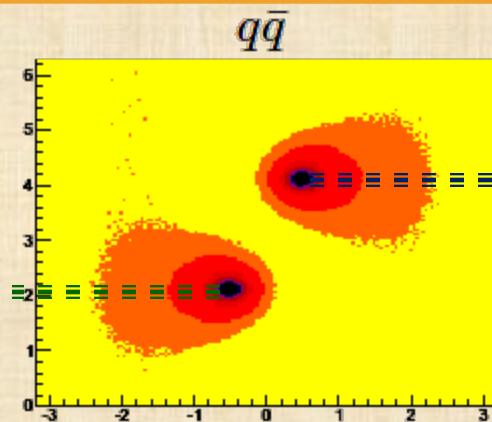
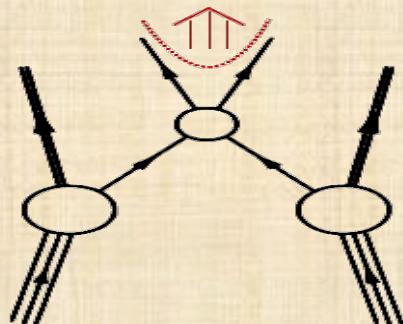
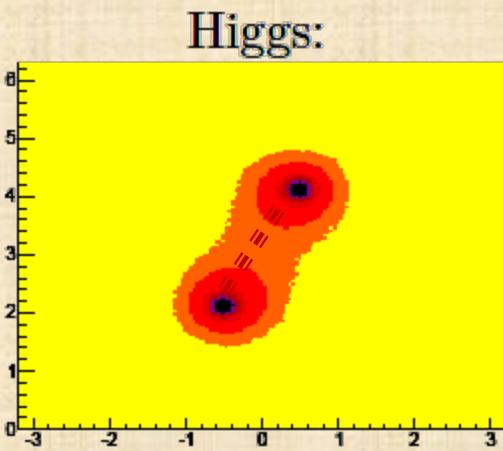
Higgs:

$$\Delta\eta_{b\bar{b}} = 1$$
$$\Delta\phi_{b\bar{b}} = 2$$

Add up  $E_T$  in  
each cell:

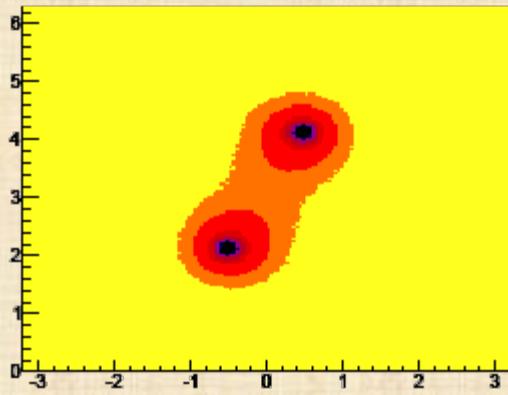


# SIGNAL VS BACKGROUND

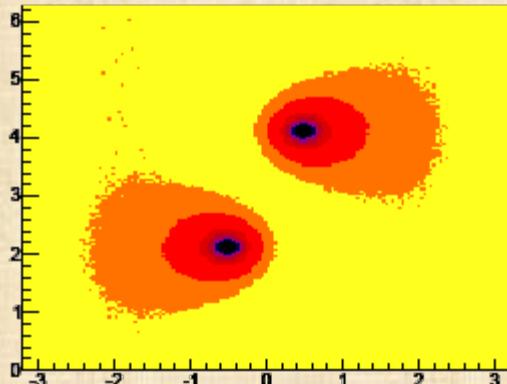


# HOW CAN WE USE IT?

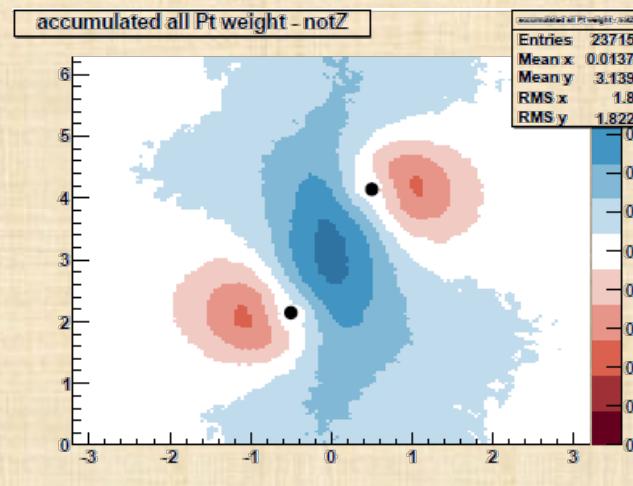
Higgs:



$q\bar{q}$

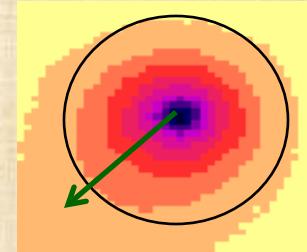
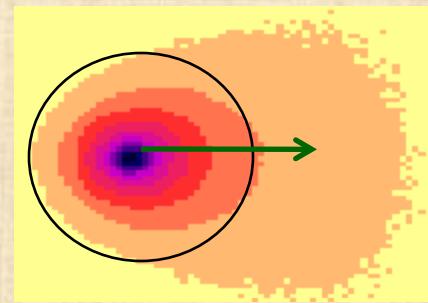
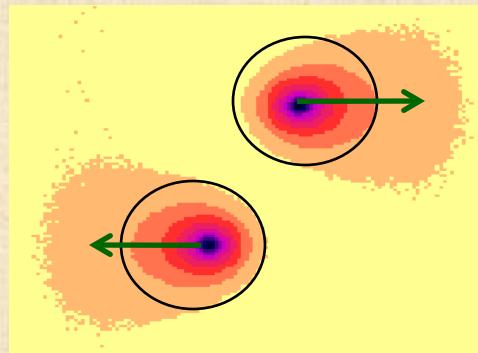
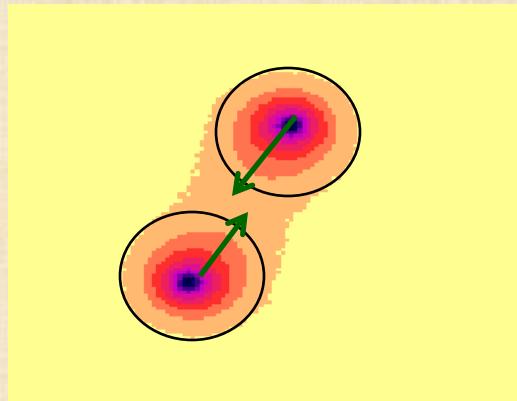


Baysean probability that  
each bit of radiation is **signal**



- Most useful radiation is  $R = 0.5 - 1.5$  away
- Pattern depends strongly on **kinematics**
- Can we find a *simpler* or more *universal* discriminant?

# PULL



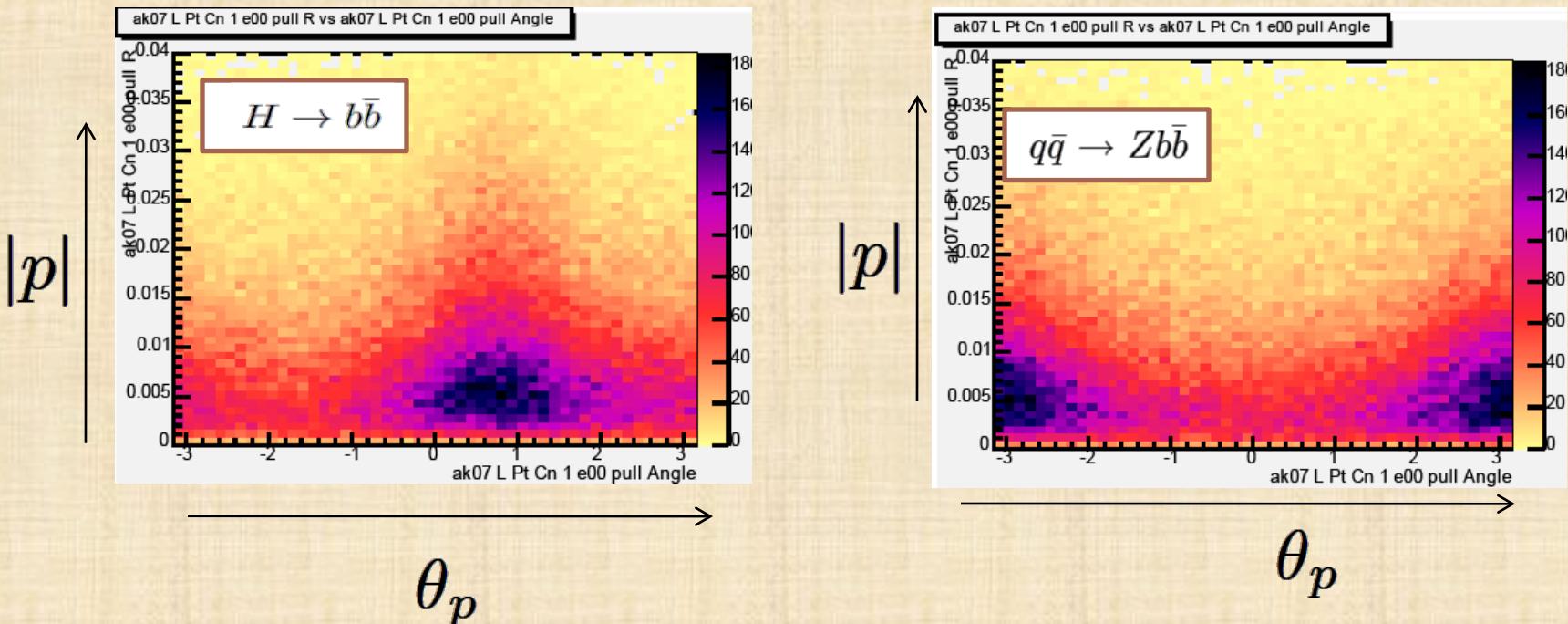
- Find jets (e.g. anti- $k_T$ )
- Construct **pull vector** ( $\sim$  dipole moment) on radiation in jet

$$\vec{p} = \sum_i \frac{E_T^i |r_i|}{E_T^{jet}} \vec{r}_i$$

- Can use bigger jets for pull, but  $R = 0.7$  seems optimal

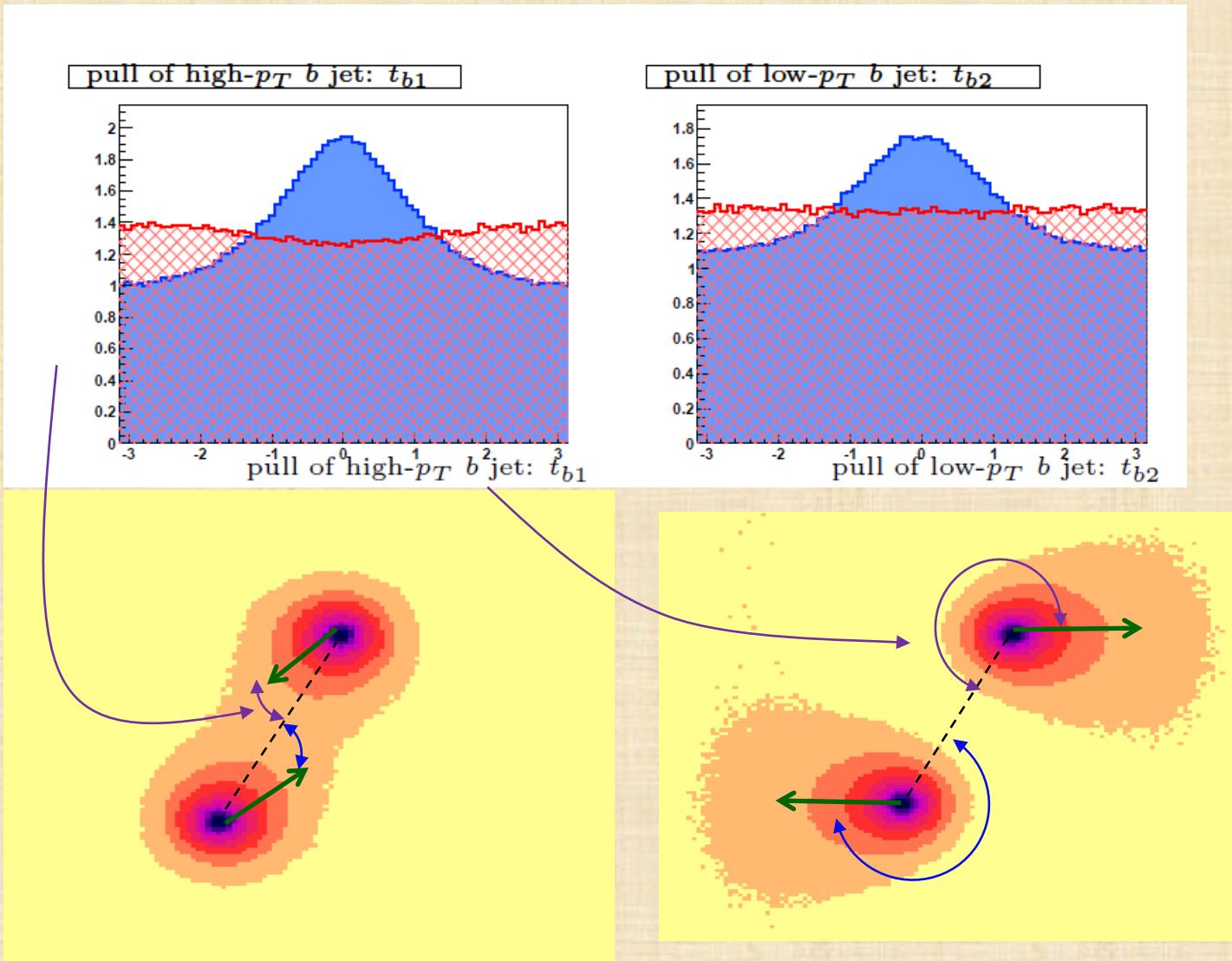
# PULL VECTOR IN RADIAL COORDS

$$\vec{p} = \sum_i \frac{E_T^i |r_i|}{E_T^{jet}} \vec{r}_i$$



- Angle much more important than length
- Look at radial pull angle (like for twist)

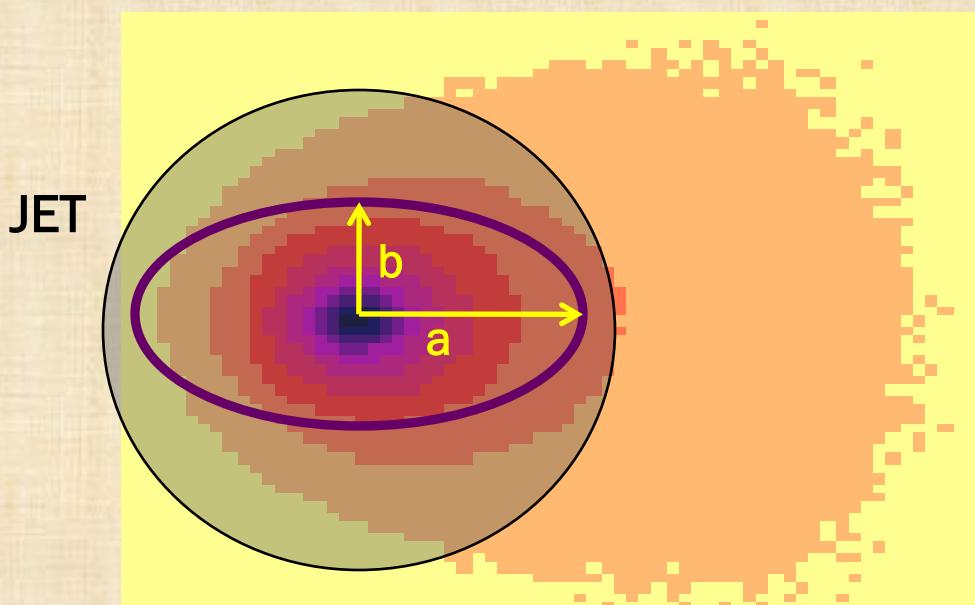
# SIGNAL VS BACKGROUND



# SECOND MOMENTS

What about **higher moments**?

$$\mathbf{I} = \sum_i \frac{E_T^i |r_i|}{E_T^{jet}} \begin{pmatrix} \Delta\phi_i^2 & -\Delta\phi_i \Delta\eta_i \\ -\Delta\eta_i \Delta\phi_i & \Delta\eta_i^2 \end{pmatrix} \longrightarrow \text{Eigenvalues } \mathbf{a} \text{ and } \mathbf{b}$$



Eccentricity

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Girth

$$g = \sqrt{a^2 + b^2}$$

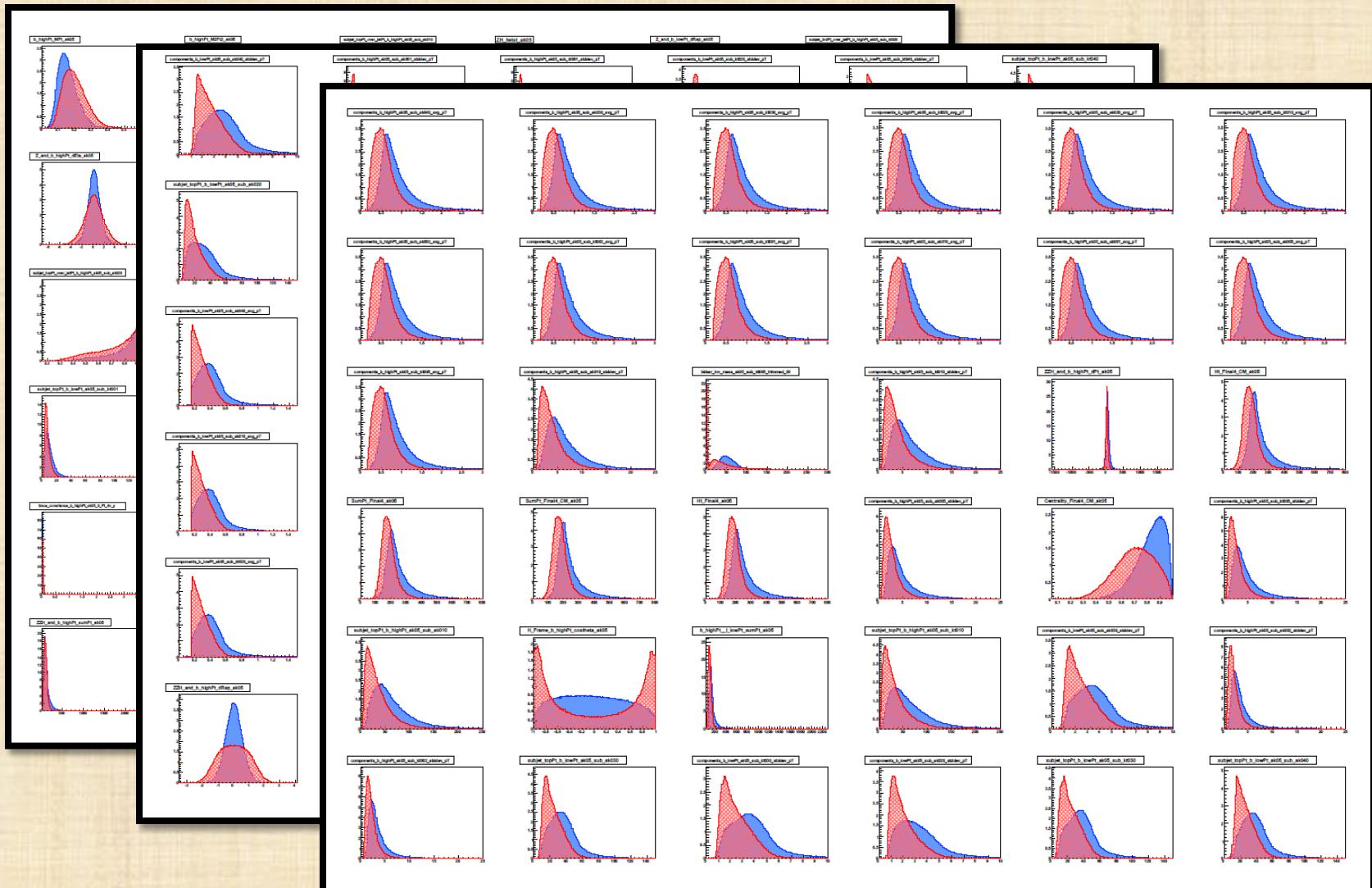
# OTHER “SHOWERED” VARIABLES

Many variables **vanish** at the **parton level**

- Do **not** enter the **matrix element** method
- **Complimentary** and **uncorrelated** with kinematic variables
- Mass of each b-jet and the jet mass to  $p_T$  ratio
- Rapidity  $y$  in addition to pseudorapidity  $\eta$  of each massive b-jet
- Subjet multiplicity for each b-jet
- Average  $p_T$  of the small subjets within each b-jet
- $p_T$  of hardest, 2nd hardest, and 3rd hardest subjets within each b-jet
- Radial moments (“girth”) of each b-jet:  $g = \sum_i \frac{p_T^i |r_i|}{p_T^{jet}}$
- Angularity:  $\tau_a = \frac{1}{m_{jet}} \sum_i E_i \sin^a \left( \frac{\pi \theta_i}{2R} \right) \left[ 1 - \cos \left( \frac{\pi \theta_i}{2R} \right) \right]^{1-a}$  for  $-1 < a < 1$

# SUMMARY

- We looked at ~ 900 discriminants!



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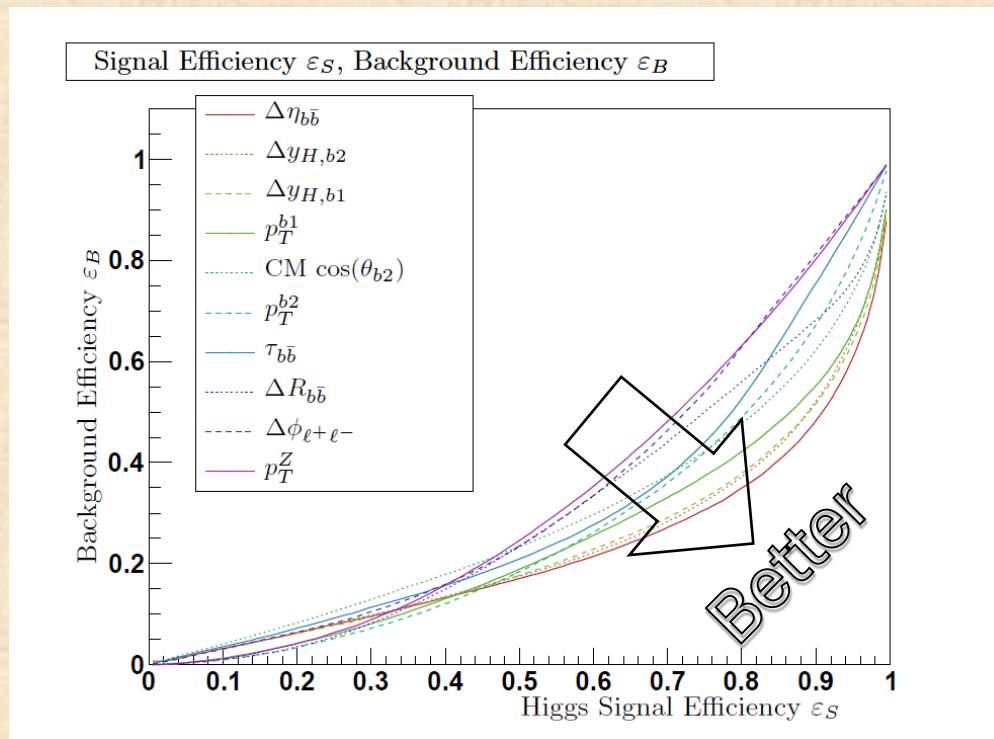
# Part 3: The Output

# EFFICIENCIES

ROC  
curve:

Receiver  
Operator  
Characteristic

Background efficiency as a function of signal efficiency



Which variable is **best**?

# OTHER VISUALIZATIONS

$$\frac{S}{B} \xrightarrow{\text{cut}} \frac{\varepsilon_S S}{\varepsilon_S B} = \left( \frac{\varepsilon_S}{\varepsilon_B} \right) \frac{S}{B}$$

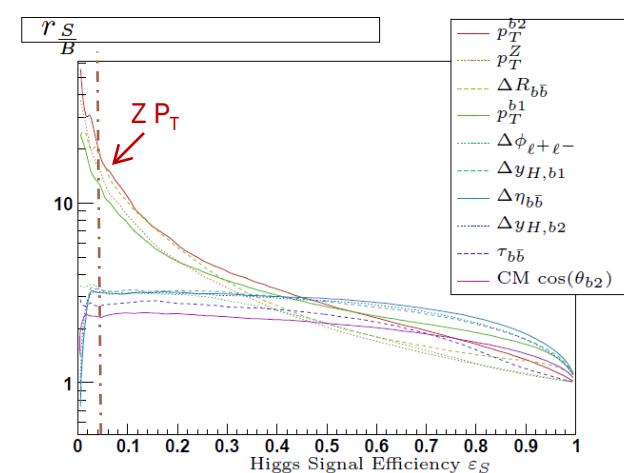
$$r_{\frac{S}{B}} \equiv \frac{\varepsilon_S}{\varepsilon_B}$$

Significance  
Improvement  
Characteristic

$$\sigma \equiv \frac{S}{\sqrt{B}} \xrightarrow{\text{cut}} \frac{\varepsilon_S S}{\sqrt{\varepsilon_B B}} = \left( \frac{\varepsilon_S}{\sqrt{\varepsilon_B}} \right) \sigma$$

$$r_\sigma \equiv \frac{\varepsilon_S}{\sqrt{\varepsilon_B}}$$

- Has maximum
- Maximum  $r_{\mathbb{I}}$  can rank variables
- Effective visualization
  - Contains lots of information



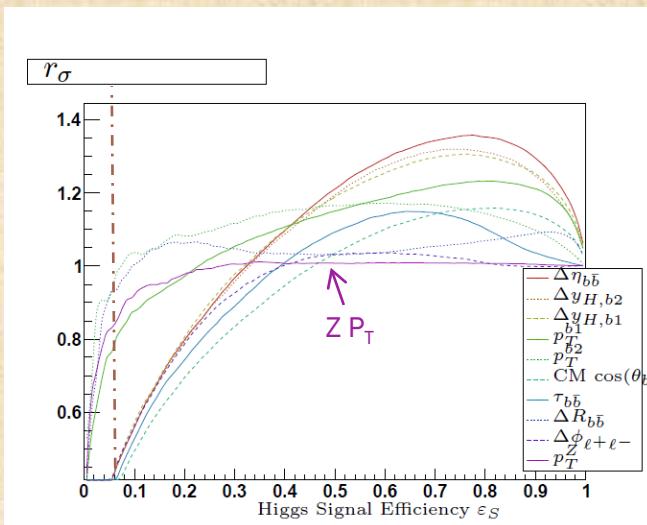
Butterworth et al.  
arXiv:0802.2470

$$p_T^Z > 200 \text{ GeV}$$

$$\mathbb{I}_S = 1/20$$

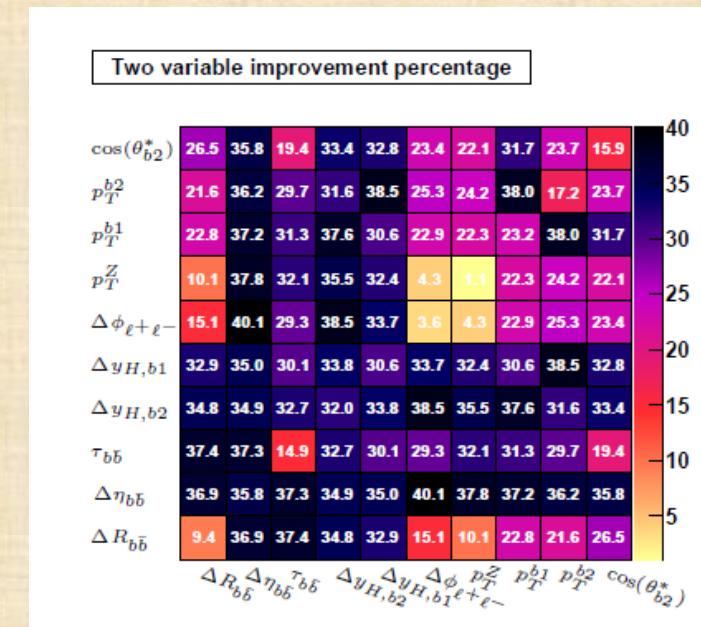
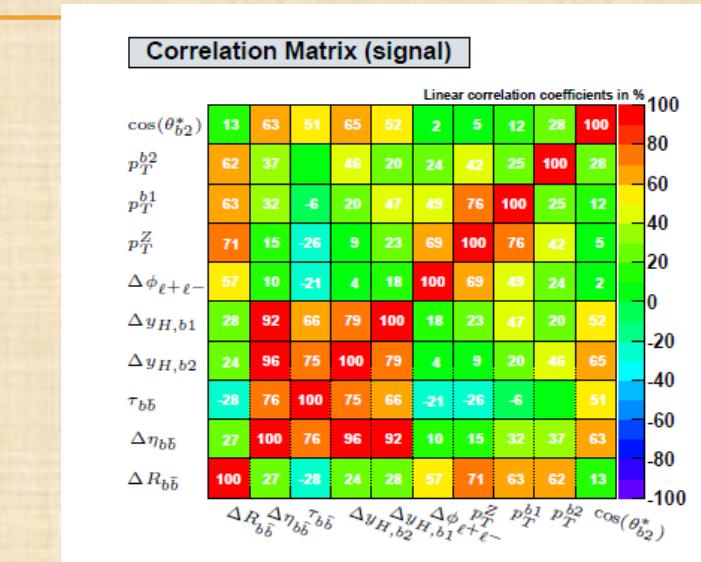
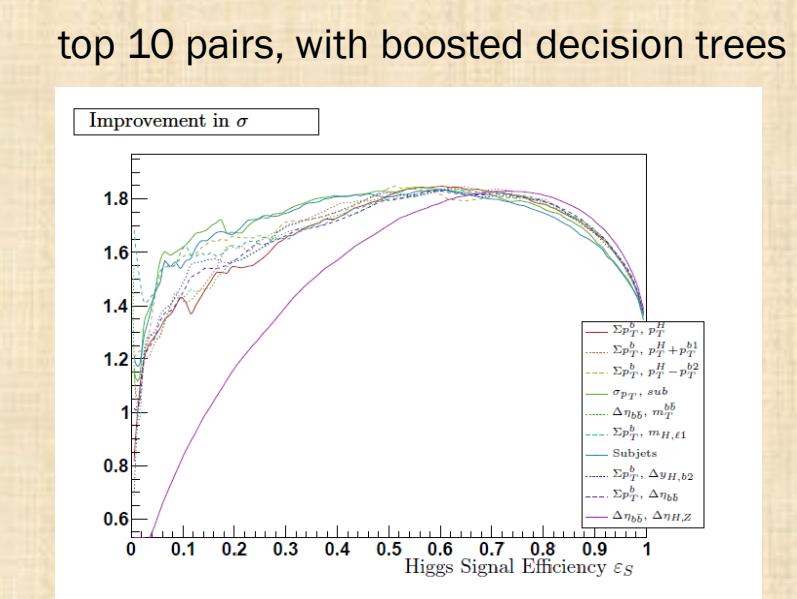
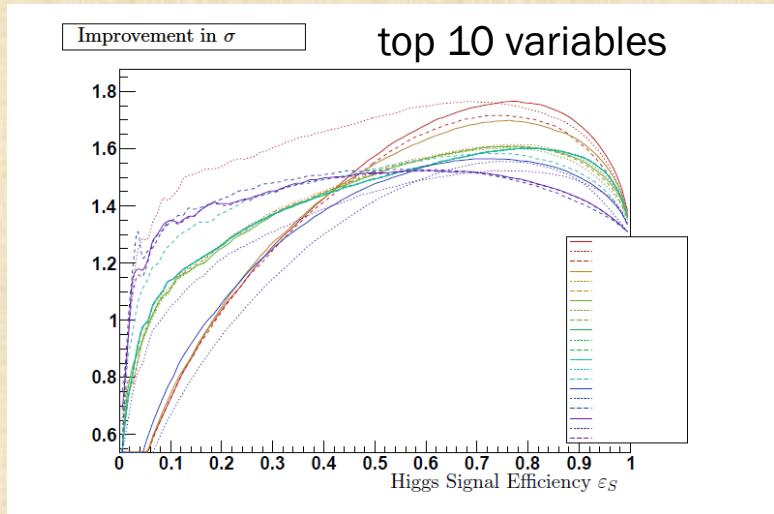
$$\mathbb{I}_B = 1/360$$

$$\frac{\mathbb{I}_S}{\mathbb{I}_B} = 18$$



$$\frac{\varepsilon_S}{\sqrt{\varepsilon_B}} = 0.94$$

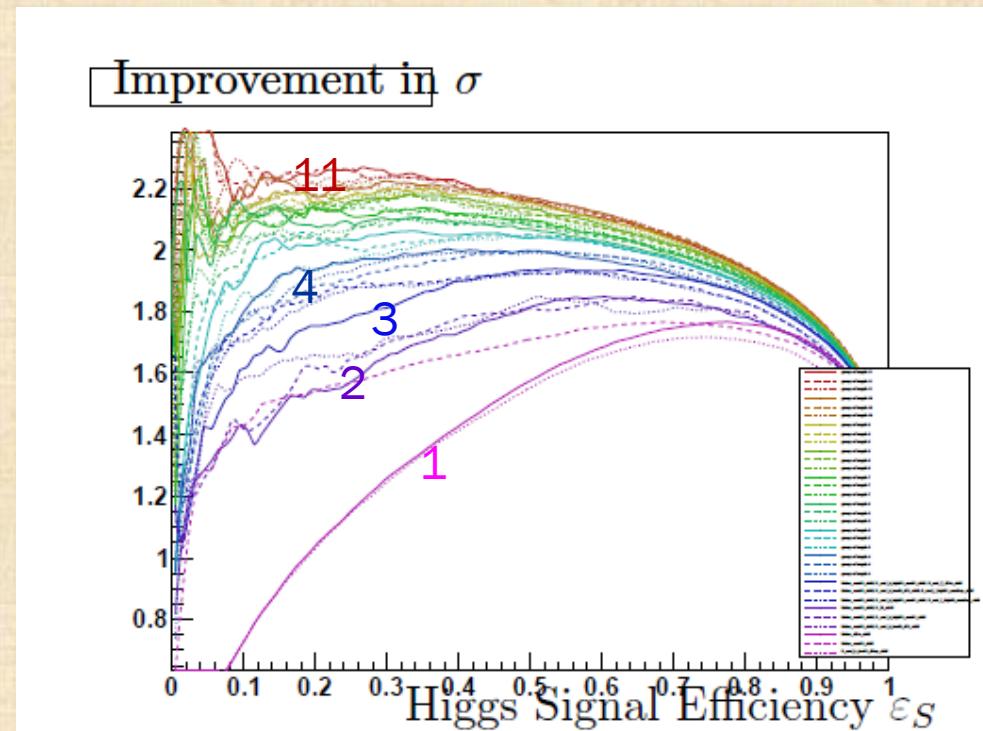
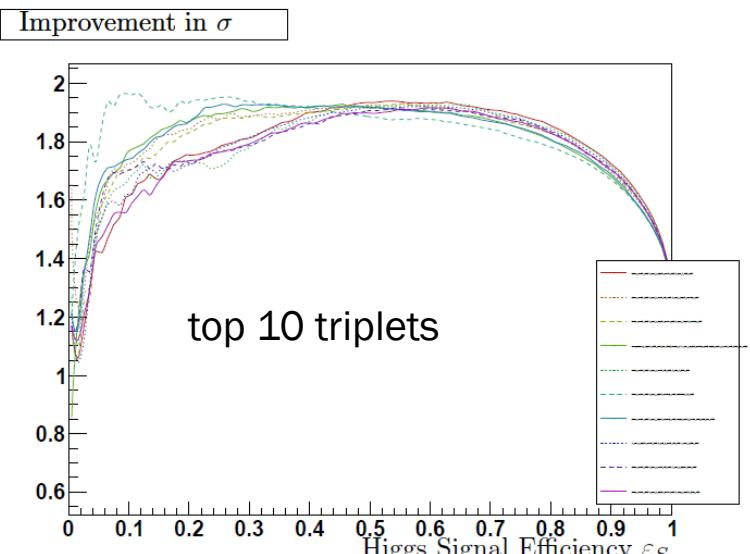
# TOP VARIABLES



# ADDING MORE

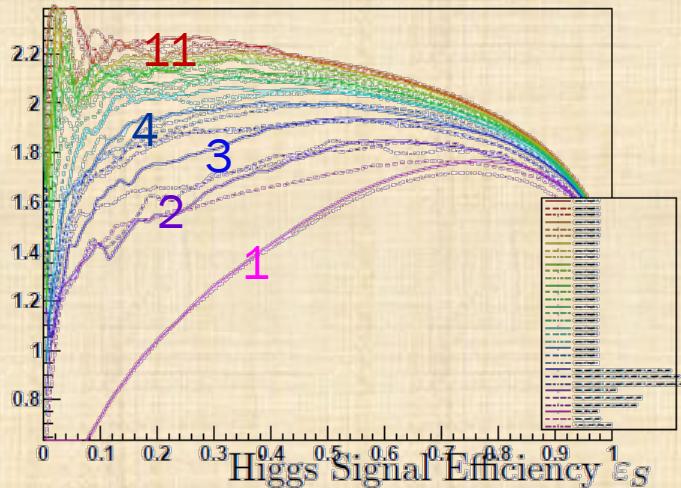
## Sequential variable addition

- Take **top 3** sets of  $n$  variables
- **Add** any of original **900**
- Take **top 3** sets of  $n+1$  variables

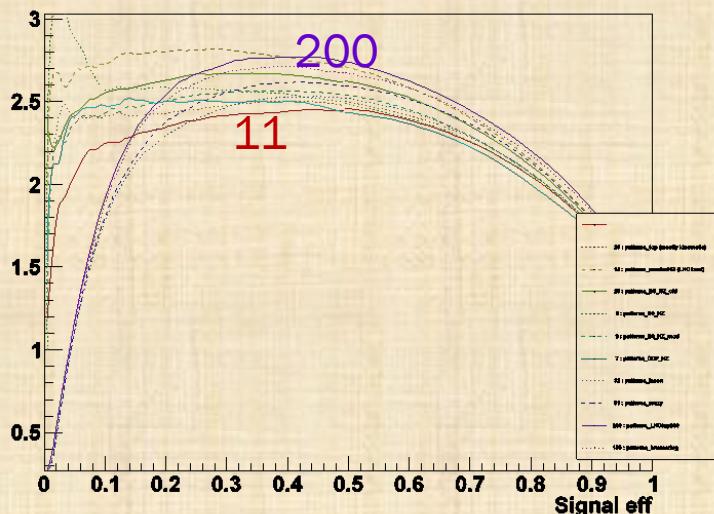


# OBSERVATIONS

### Improvement in $\sigma$



## LHC HZ : Significance



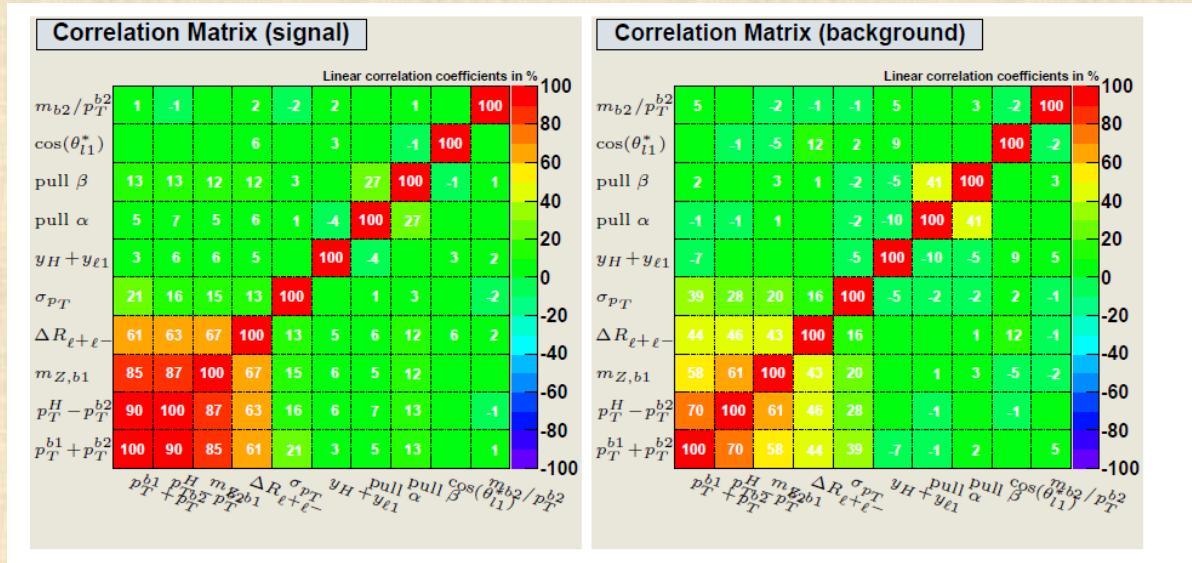
- Converges slowly
- Sensitivity to statistics apparent
  - $r_0 = 2 \cdot \bar{L}_S = 0.05$  gives  $\bar{L}_B = 1/1600$
  - 1 million events down to 600
- Some variables very poor by themselves, but show up as 5<sup>th</sup> or 6<sup>th</sup> variable

## Top 10 include

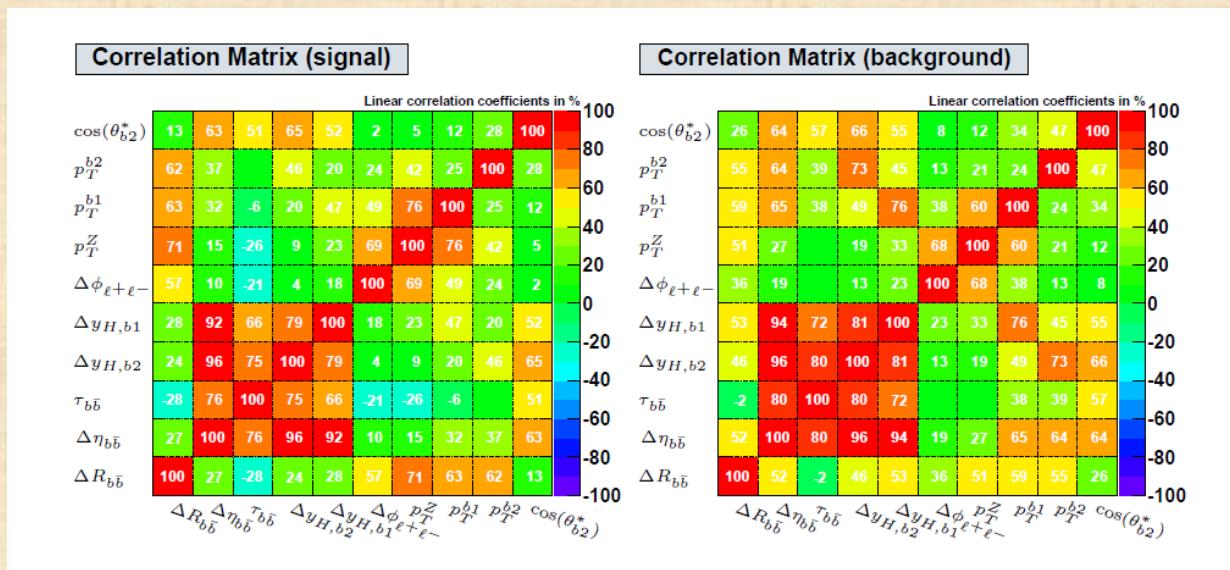
- Higgs  $p_T$
- $\Box R_{ZH}$
- Pull
- Twist  $y$  (twist with  $y$  not  $\Box$ )
- Event shape  $D$
- Determinant of covariance matrix for radiation in low  $p_T$  b jet
- Scalar sum of the b jet  $p_T s$

# CORRELATIONS OF GOOD 10 COMBO

Best 10  
combo

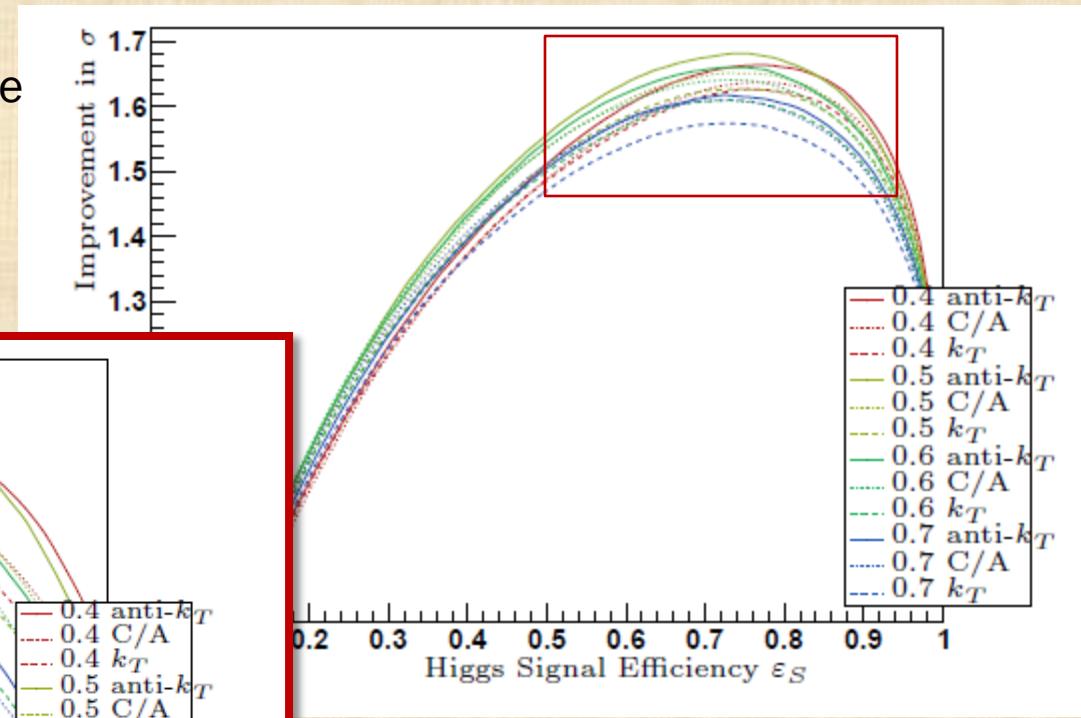
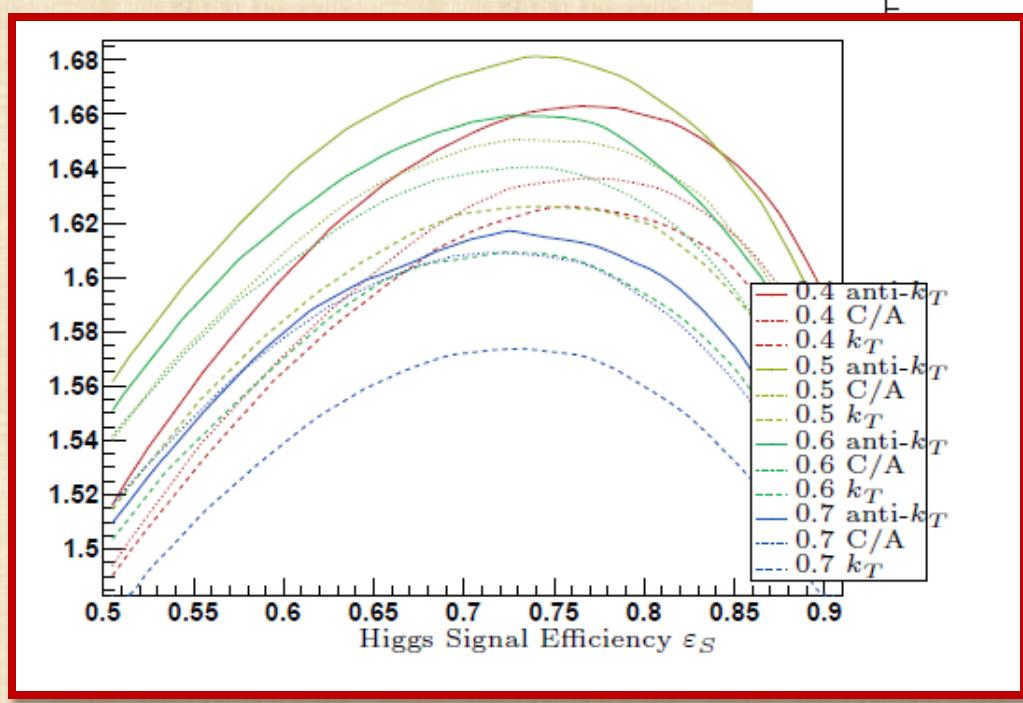


Best 10  
individuals



# JET ALGORITHMS

- Main observable is  $m_{b\bar{b}}$
- Look at jet algorithm dependence



- The **winner** is ...  
anti- $k_T$  with  $R = 0.5$
- Optimal **mass window**  
 $90 \text{ GeV} < m_{b\bar{b}} < 124 \text{ GeV}$

# TRIMMING

Krohn, Thaler, Wang

1. **Recluster** jet constituents into *very thin* jets



# TRIMMING

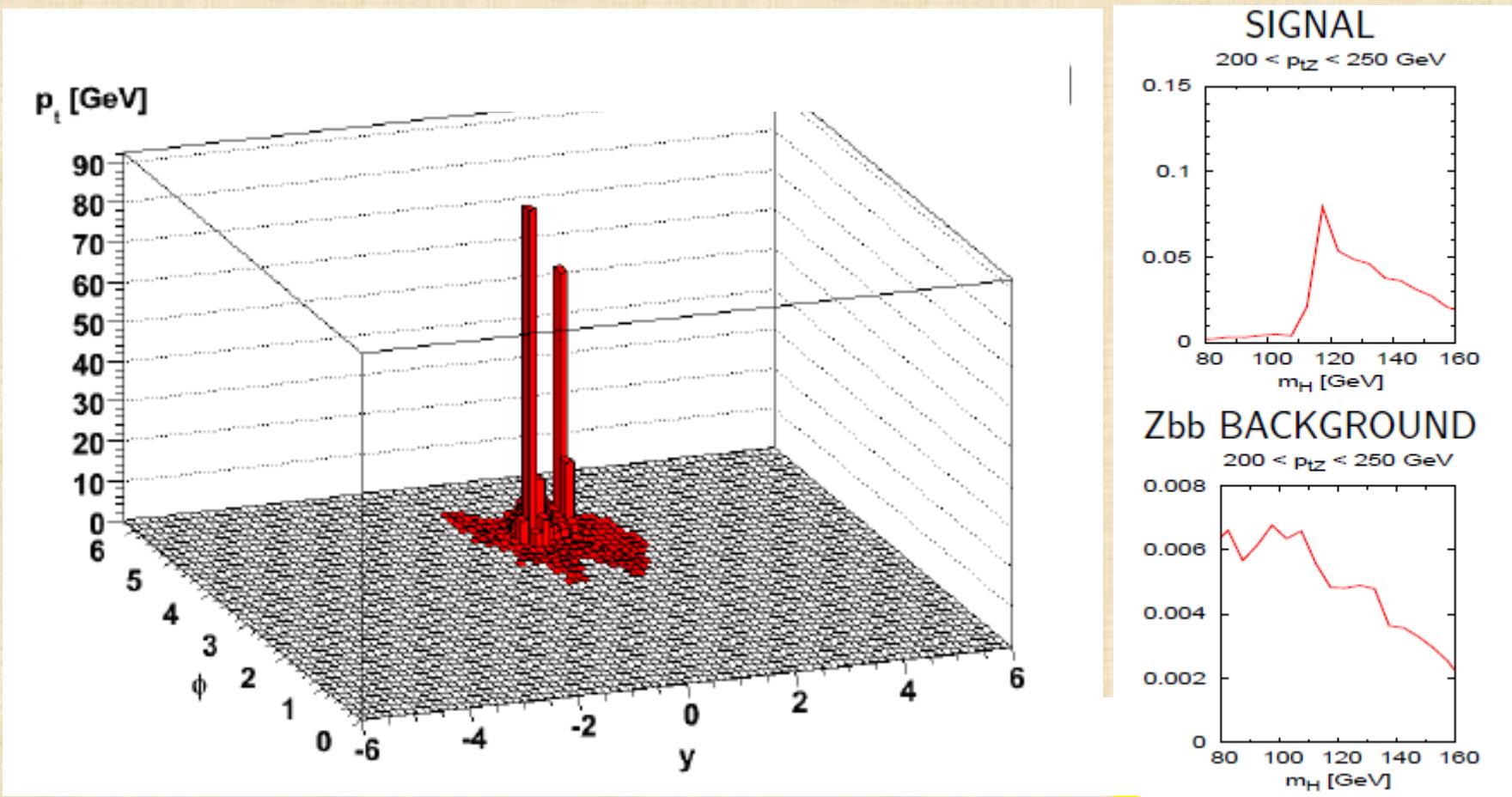
Krohn, Thaler, Wang

1. **Recluster** jet constituents into **very thin** jets

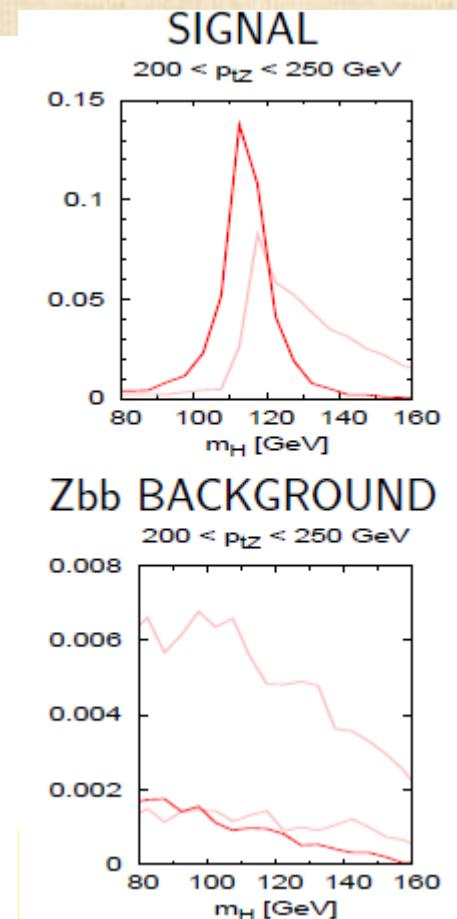
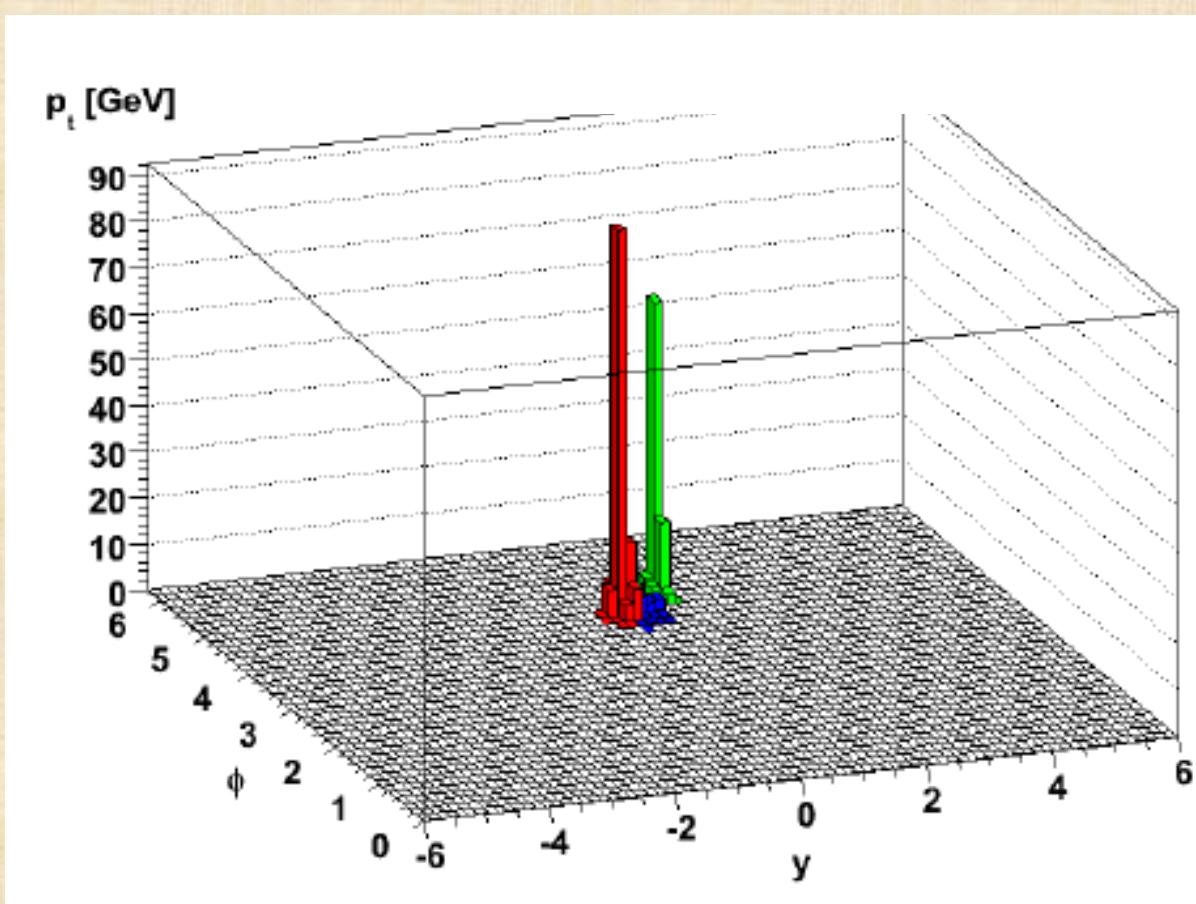
2. **Throw away** thin jets that are **too soft**



## TRIMMING

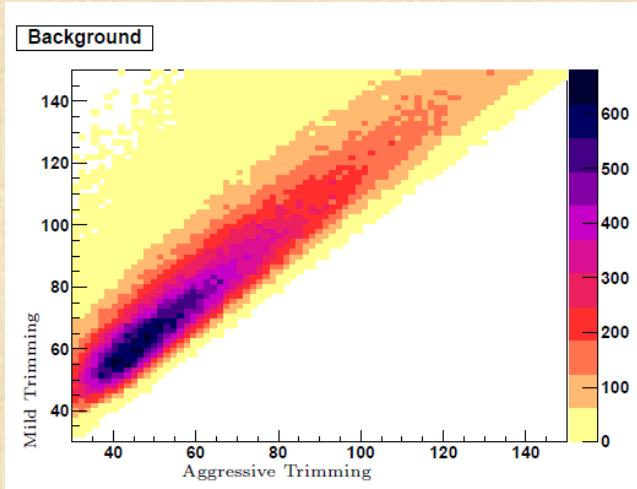
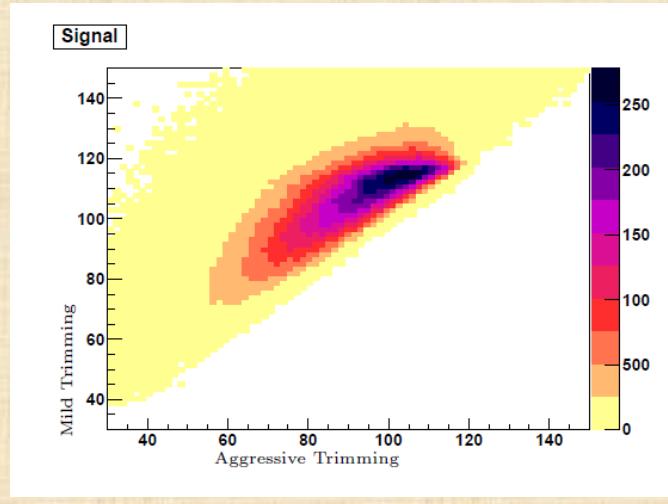
Boosted  $H \rightarrow bb$ 

## TRIMMING

Boosted  $H \rightarrow bb$ 

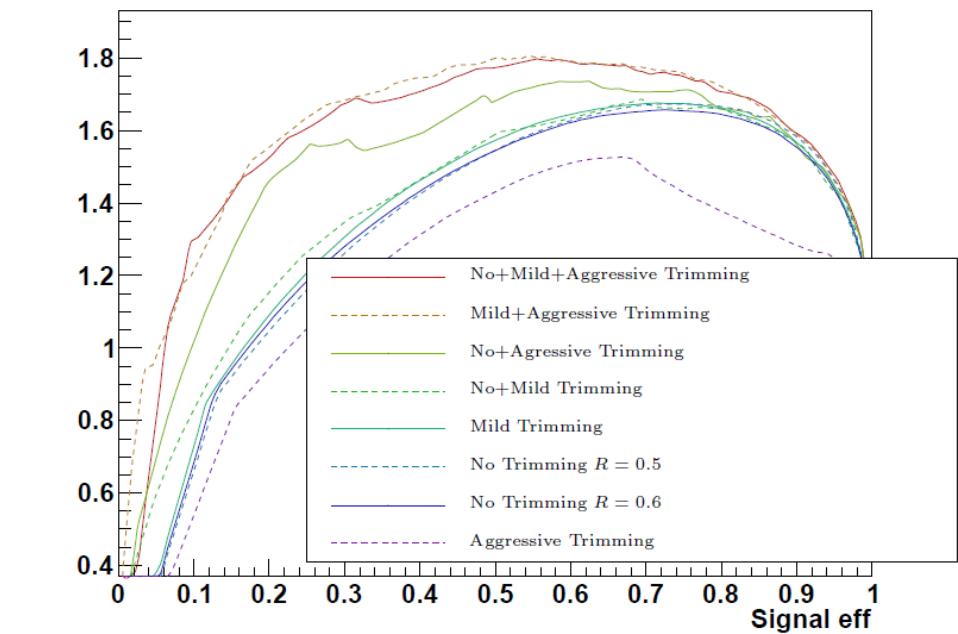
# MULTIPLE TRIMMINGS

Trimming does **not** seem to **help** much in our case...



**Higgs : Significance**

Multiple trimmings do **help!**



(inspired by Soper and Spannowsky)

# CONCLUSIONS



- Final efficiencies still **under construction**
- Looks like we can **help** the Tevatron searches
  - around **10%** with **variables** (relative to the ones they use)
  - around **10%** with **masses** (assuming they can trim)
- **W/Z + H** is totally **feasible** at the **LHC**
  - Do **not** need **large  $p_T$**
  - Discovery potential with  $30 \text{ fb}^{-1}$

## General Observations

- **SIC** curves provide a useful visualization
  - demonstrate **instabilities**
  - show **covergence**
  - **visually compare** variables' performance
- **Uncorrelated variables** helpful after kinematics exhausted
- **Multiple mass measures** useful

## Future

- Compare boosted decision trees, random forest, neural networks, etc.
- Compare different generators (**Herwig/Pythia**)
- Study **reducible** backgrounds