
Demystifying Multivariate Searches and the **Hunt for the Higgs**

Matthew Schwartz

Harvard University

Work down with Jason Gallicchio,

PRL, 105:022001,2010

and with Gallicchio, Tweedie, Huth, Kagan and Black

... in preparation

Johns Hopkins University

September 20, 2010

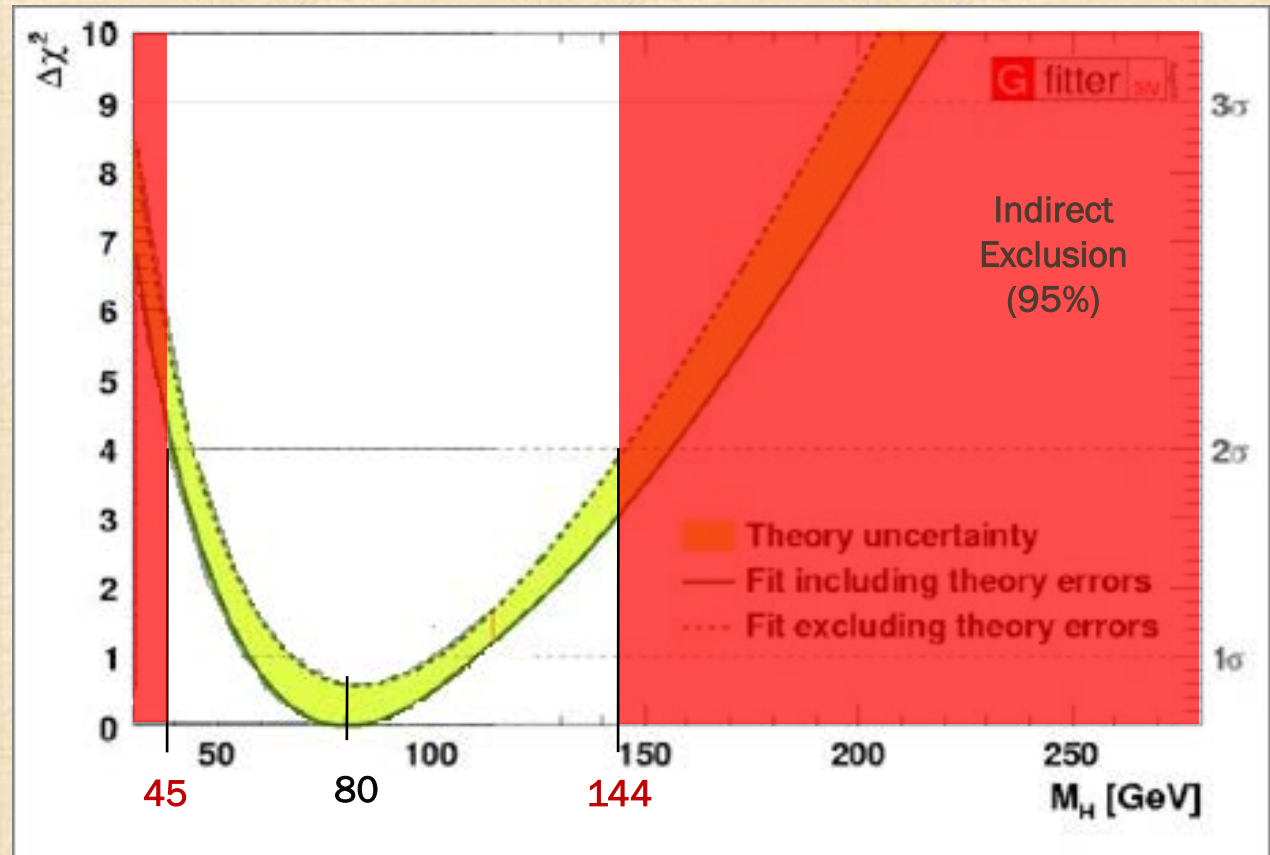
Part 1:

Motivation

WHERE IS THE HIGGS?

Parameter	Input value
M_Z [GeV]	91.1875 ± 0.0021
Γ_Z [GeV]	2.4952 ± 0.0023
σ_{had}^0 [nb]	41.540 ± 0.037
R_ℓ^0	20.767 ± 0.025
$A_{\text{FB}}^{0,\ell}$	0.0171 ± 0.0010
$A_\ell^{(*)}$	0.1499 ± 0.0018
A_c	0.670 ± 0.027
A_b	0.923 ± 0.020
$A_{\text{FB}}^{0,c}$	0.0707 ± 0.0035
$A_{\text{FB}}^{0,b}$	0.0992 ± 0.0016
R_c^0	0.1721 ± 0.0030
R_b^0	0.21629 ± 0.00066
$\sin^2\theta_{\text{eff}}^\ell(Q_{\text{FB}})$	0.2324 ± 0.0012

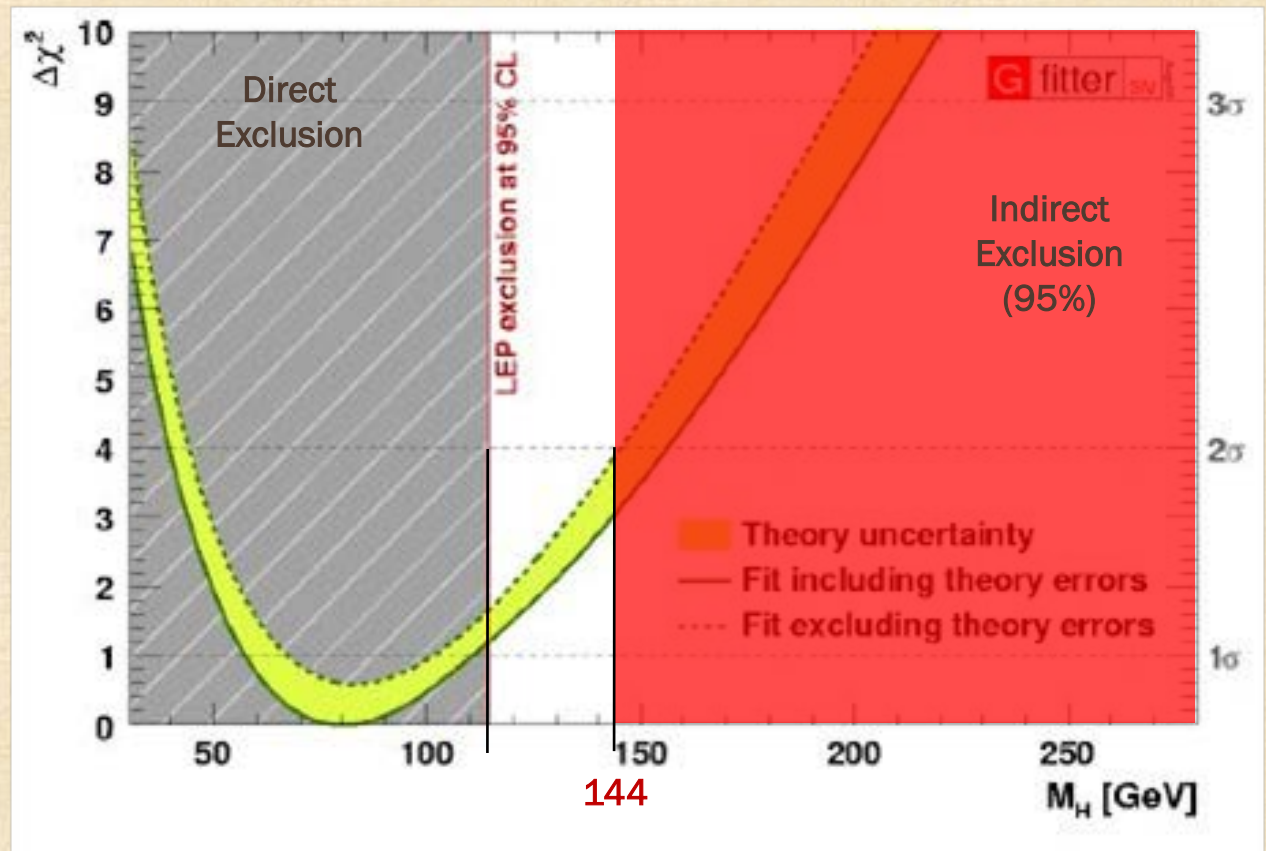
Combine many
observables to constrain
Higgs mass



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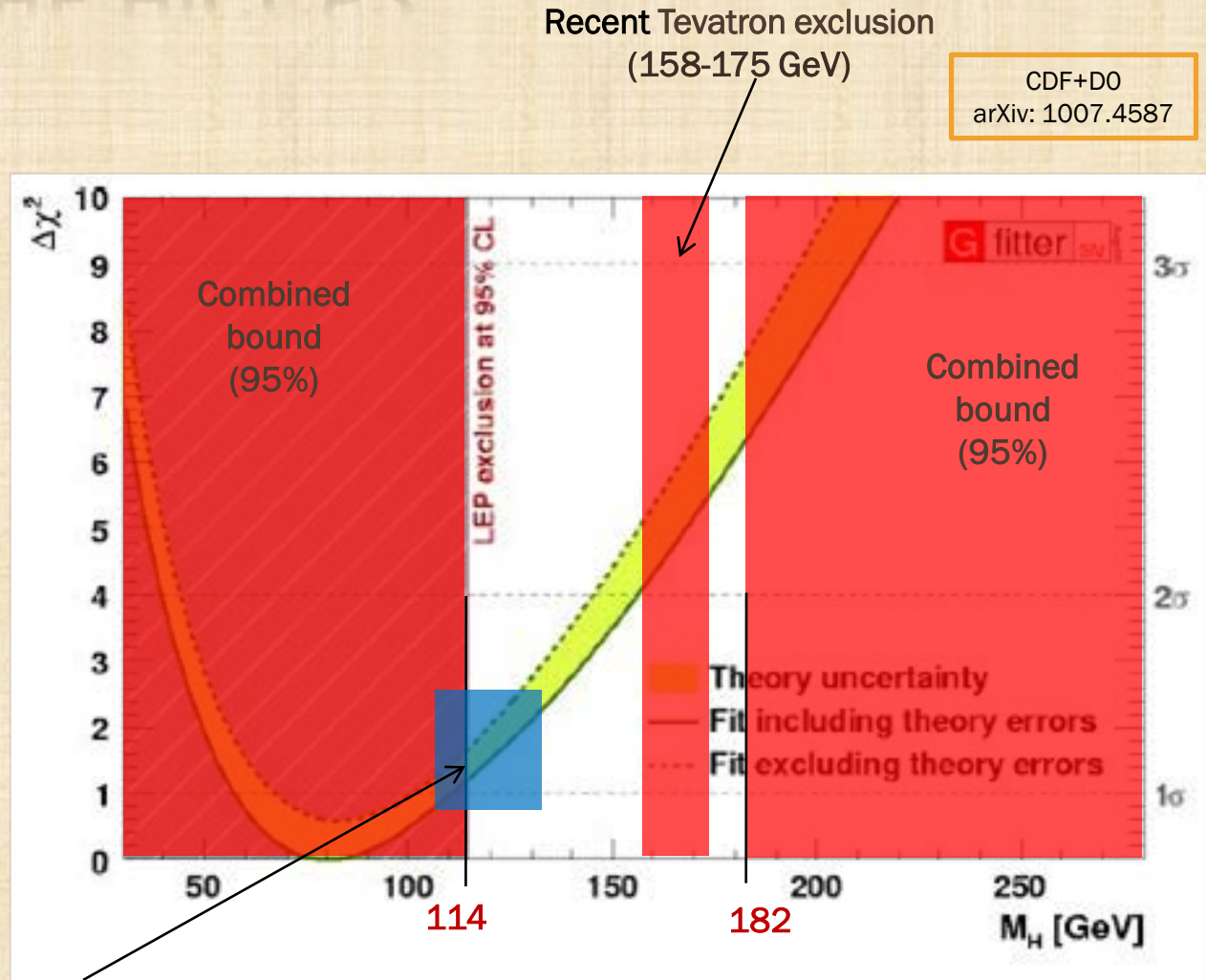
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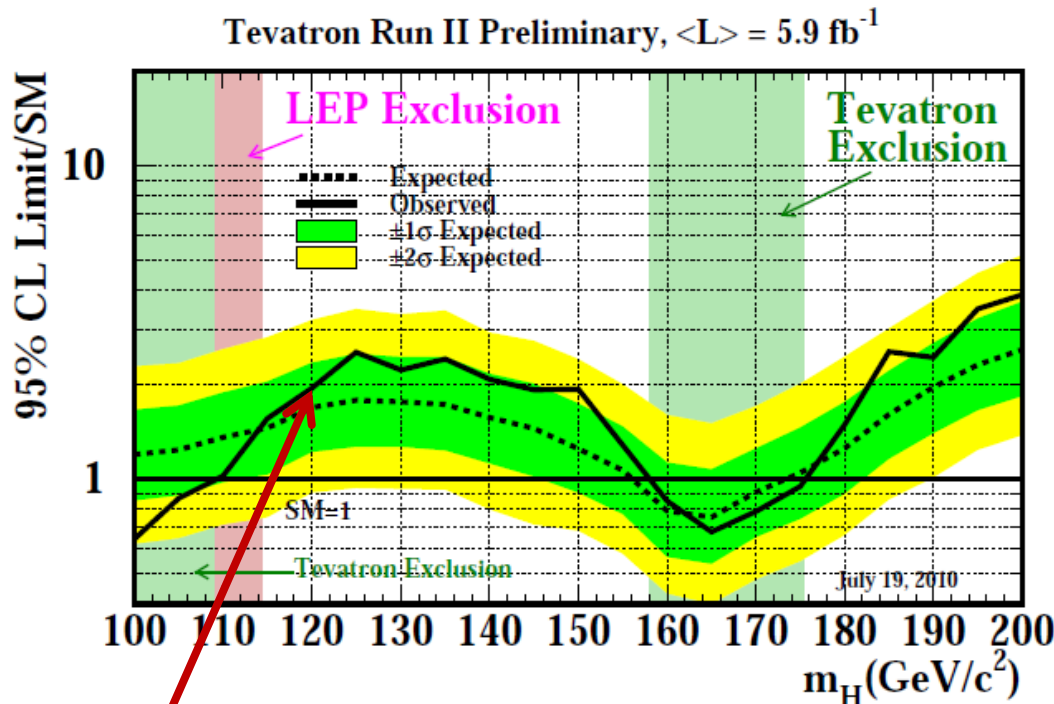
Combine many
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Higgs mass



If it exists, Higgs is
most likely light

HOW DO WE FIND A LIGHT HIGGS?

Tevatron



- **Need** a factor of **2** improvement in significance for $m_H=120$
- Double statistics gives $\sqrt{2}$, **where** will **the other** $\sqrt{2}$ come from?

LHC

- Important search channel is
 $pp \rightarrow W/Z + H$
 $H \rightarrow bb$
- **Abandoned** by ATLAS and CMS
 too much background
- Recently high P_T $W/Z + H$ revived,
 - Requires $P_T > 200$
 - **Lose** 95% of signal

How **good** can we do
in $W/Z + (H \rightarrow bb)$?

FOCUS ON $pp \rightarrow HZ \rightarrow b\bar{b}l^+l^-$

CDF note 10235 (summer 2010)

ZH	0.7
$t\bar{t}$	9.9
WW	0.02
WZ	0.1
ZZ	3.6
$Z \rightarrow \ell\ell + b\bar{b}$	22.1
$Z \rightarrow \ell\ell + c\bar{c}$	2.4
$Z \rightarrow \ell\ell + l.f.$	1.2
fakes	0.9
Total Bkg	40.3

Dominant background
is the **irreducible** one

CDF employs **multivariate** approach

Inputs to the neural net are

- Missing transverse energy
 - Dijet mass
 - $t\bar{t}$ **matrix element** output
 - ZH **matrix element** output
 - Sum of leading jet Pt's
 - number of jets
- } **Parton-level** kinematics

Questions:

- Are there **smarter** more comprehensive inputs?
- Can we **trust** the multivariate approach?

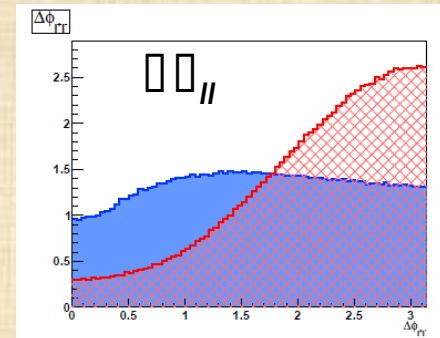
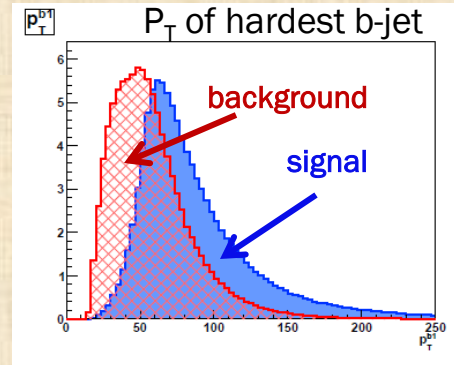
Part 2:

The Inputs

KINEMATIC VARIABLES

Standard Stuff

- P_T 's of b 's and the leptons
- $\Delta\phi_{ll}$ for the b jets and the leptons
- ΔR of the b 's and the leptons
- P_T of the reconstructed Z
- P_T of the reconstructed Higgs
- m_{bb} : invariant mass of the b 's

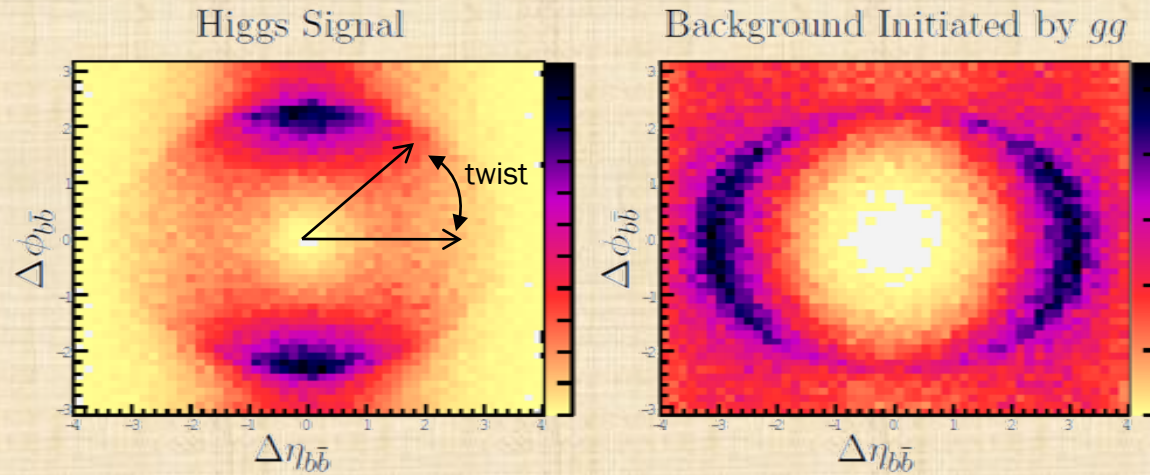


Less Standard Stuff

- acoplanarity of the b 's: $2\pi - \Delta\theta_{b\bar{b}} - \Sigma\theta_{b\bar{b}}$
- acoplanarity of the leptons
- transverse mass of the bb system $m_T^{b\bar{b}} = m^2 + p_x^2 + p_y^2$
- transverse mass of the lepton system
- invariant mass of 2 leptons and 1 b or 2 b 's and 1 lepton

TWIST

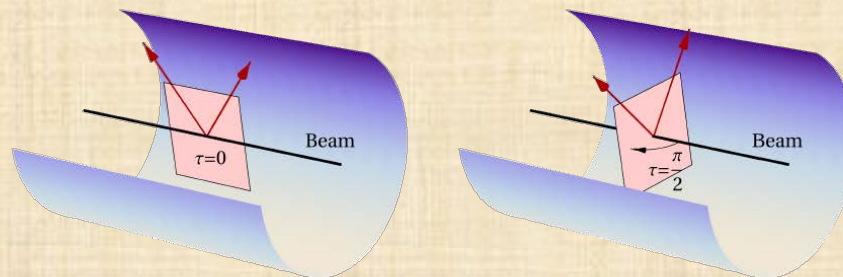
Look at **2D distribution** in $\Delta\eta_{b\bar{b}}$ space:



It seems that neither $\Delta\eta$ nor $\Delta\phi$ nor R holds the right information

Introducing **twist** = polar angle in this plane

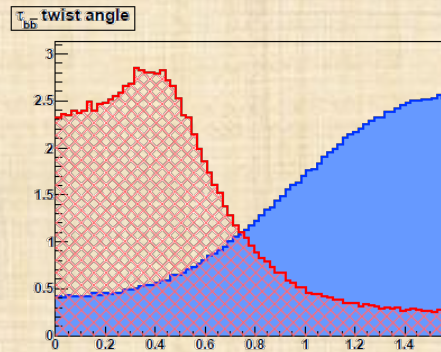
Background has pole for
zero **twist**
(t-channel singularity)



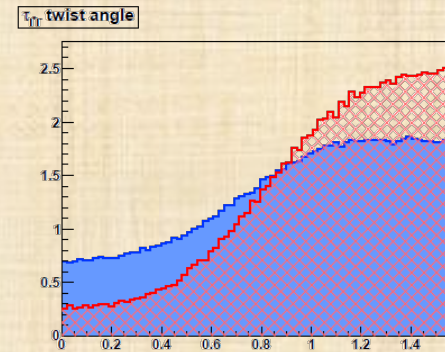
TWIST

Parton level – no cuts

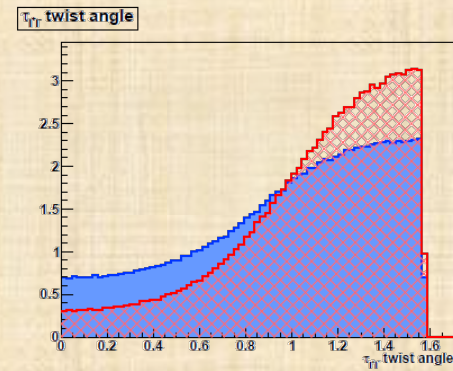
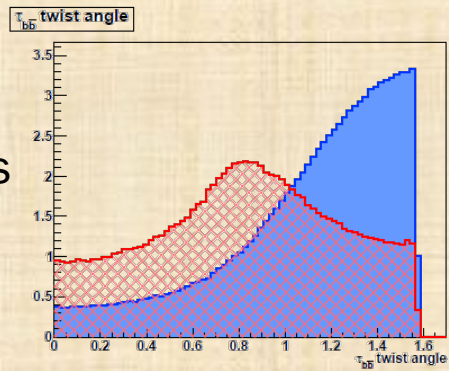
b-jet twist



lepton twist



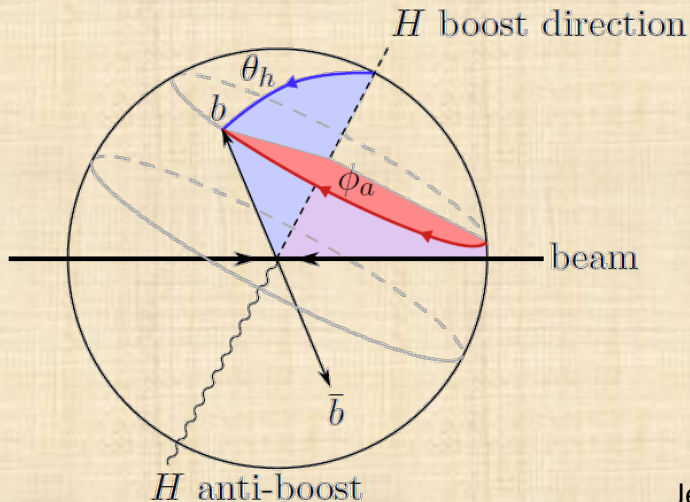
Jet level with detector cuts



Could be more generally useful....

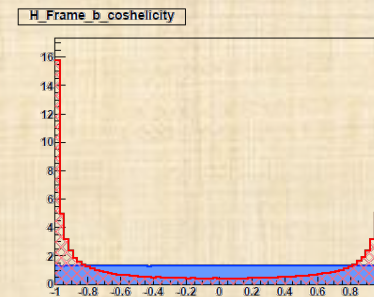
HELICITY AND AZILICITY ANGLES

Angles in **Higgs rest frame**
relative to H boost direction

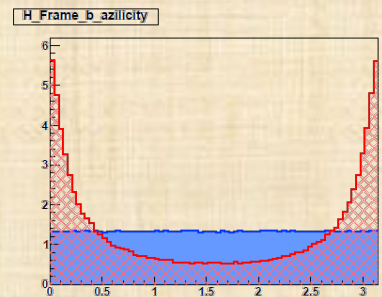


Parton level
– no cuts

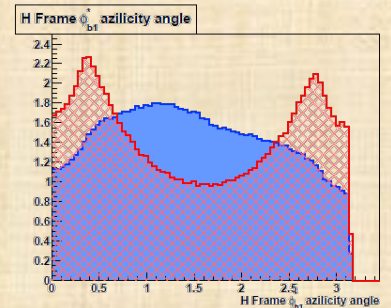
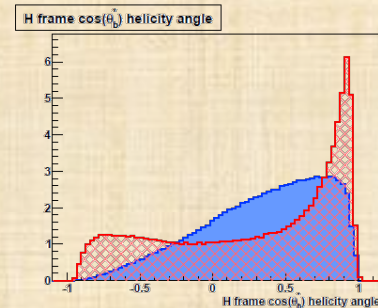
helicity angle



azilicity angle



Jet level
with detector cuts



Signal is on-shell

- angles **meaningful**

Background is not a resonance

- Angles **meaningless**
- Expect **peaked** due to collinear **singularities**

CHINESE MENU METHOD

- Pick a particle: high- p_T b -jet, low- p_T b -jet, high- p_T lepton, low- p_T lepton, Higgs, Z
- Optionally transform to a boosted frame: Higgs, Z , Center of Mass (CM)
- Optionally rotate the polar axis to point along the initial direction of the particle whose frame you're in (for Helicity and Azilicity Angles).
- Pick a kinematic property: p_T , η , ϕ , $\cos(\theta)$, etc.
- Optionally pick a second particle to form a sum or difference, sometimes with a coordinate transformation as in ΔR and twist τ , and sometimes with a more complicated combination as in invariant-mass.
- For vector quantities optionally take the magnitude of vector sums, $|\vec{p}_1 \pm \vec{p}_2|$ or scalar sums, $|\vec{p}_1| \pm |\vec{p}_2|$.



- $\Delta y_{H,b1}$ and $\Delta y_{H,b2}$: Difference in rapidity between Higgs and higher- p_T or lower- p_T b -jet
- $\cos(\theta_{b2}^*)$: Center of Mass frame $\cos(\theta)$ of the lower- p_T b -jet. Same for higher- p_T b -jet.
- $\Delta p_T^{Z,l1}$: Difference in p_T between the reconstructed Z and the higher- p_T lepton
- $\Delta p_T^{b1,l2}$: Difference in p_T between the higher- p_T b -jet and the lower- p_T lepton
- $\Sigma p_T^{b1,l2}$: Sum of p_T 's of the higher- p_T b -jet and the lower- p_T lepton
- $\Delta \eta_{b1,l2}$: Difference in η between the higher- p_T b -jet and the lower- p_T lepton

EVENT SHAPE VARIABLES

Nothing to do with the particular signal or background

- H_T = Scalar sum of all E_T
- Σp_T = Scalar sum of all p_T (which differs from H_T for massive jets)
- H_z = Boost of the center-of-mass system along the beam
- E_{vis} = Scalar sum of all visible energy
- \hat{s} = CM energy for hard collision, or invariant mass of the reconstructed Higgs and Z.
- Centrality
- Aplanarity and Aplanority
- Sphericity and Spherocity
- DShape and Yvariable (related to the eigenvalues that go into defining the above)
- Fox-Wolfram Moments

JET SUPERSTRUCTURE

What is not in the parton-level kinematics?

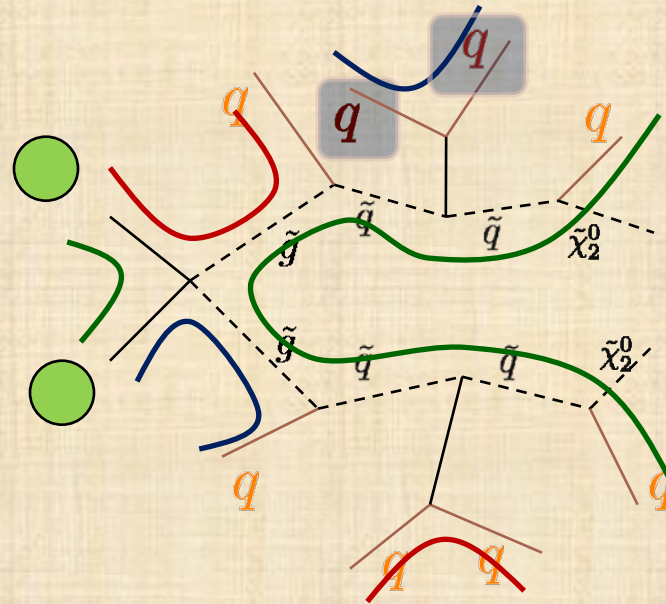
- **Global information**

- **Event shapes**

- **Color:**

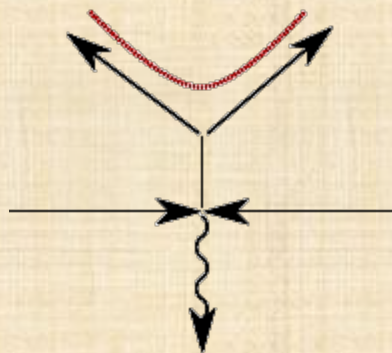
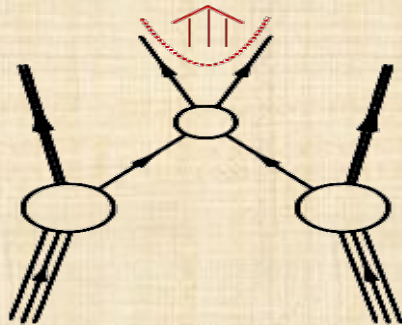
- Color **charge**: Quark vs. Gluon jets
- Color **connections**

$$\text{Tr}[T^A T^B] \propto \delta^{AB}$$

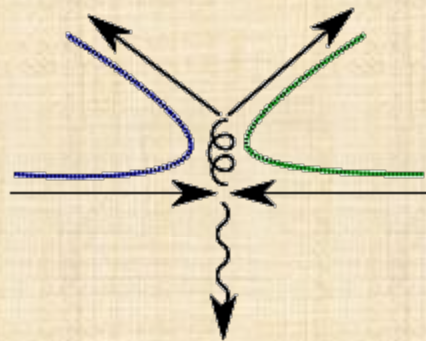
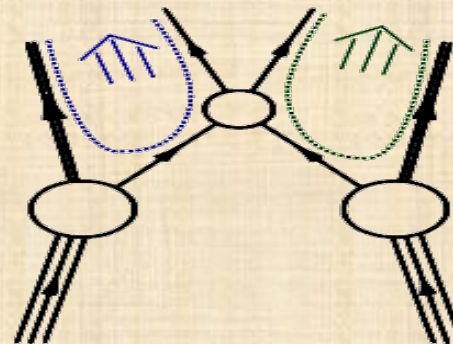


COLOR CONNECTIONS

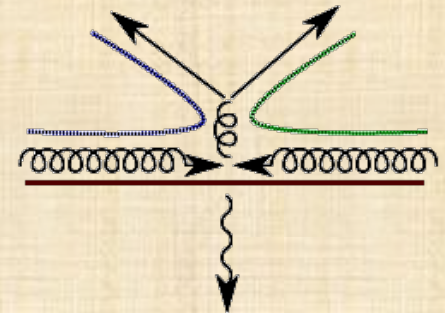
Signal



Background



$$q\bar{q} \rightarrow Zb\bar{b}$$

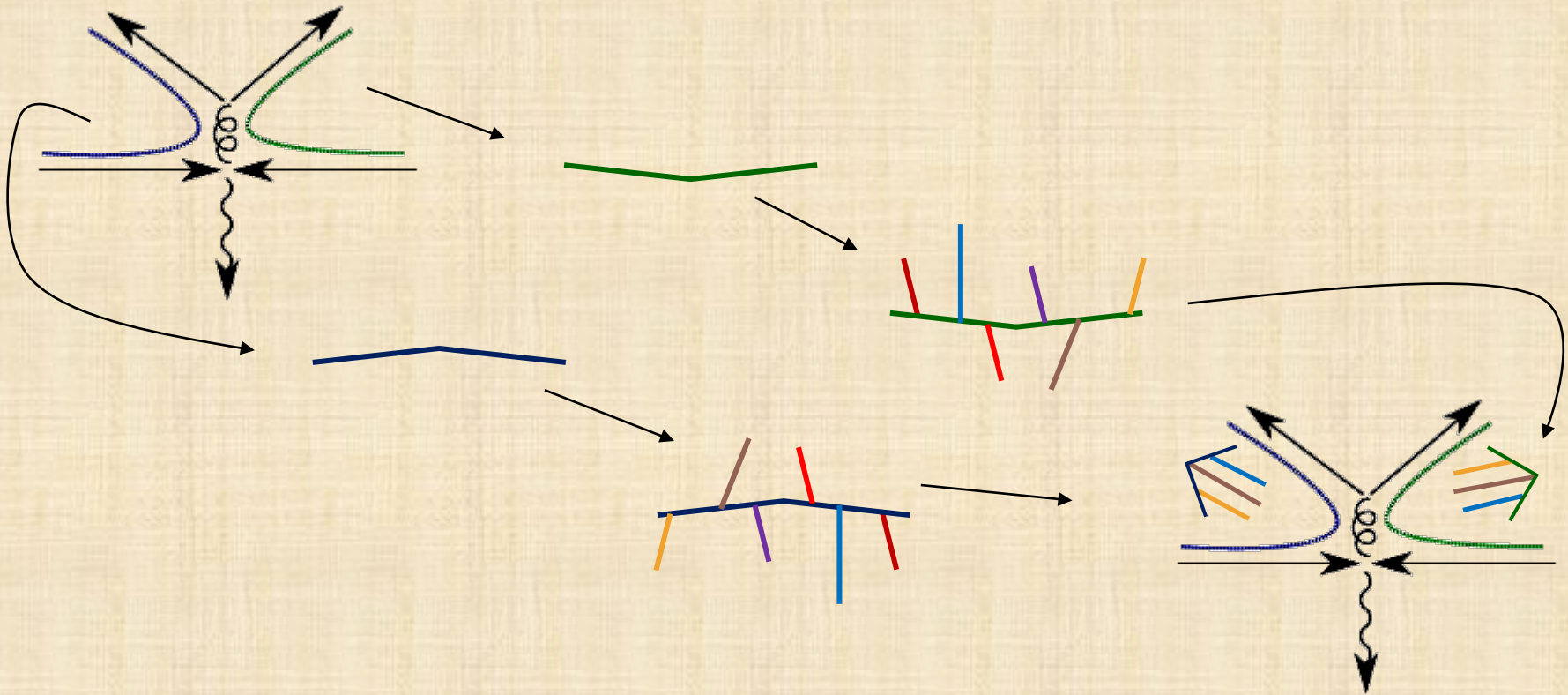


$$gg \rightarrow Zb\bar{b}$$

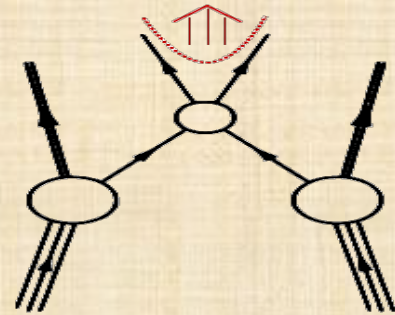
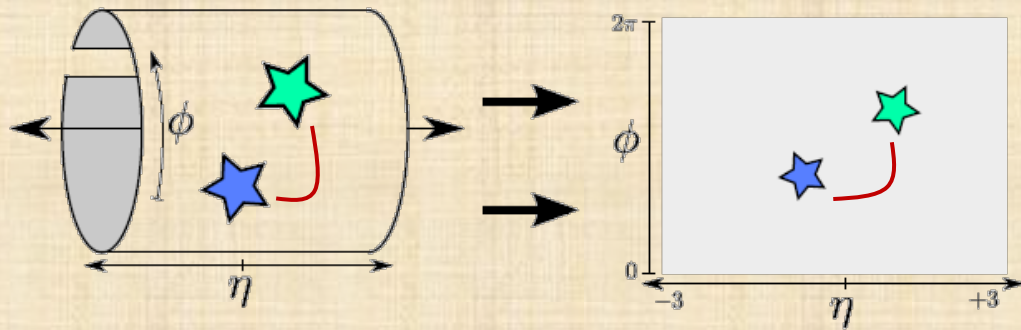
HOW DO THEY SHOW UP?

Monte Carlo simulation

- **Color coherence** (angular ordering, e.g. Herwig)
- Color string showers in its rest frame (pt ordering, e.g. Pythia)
 - Boost \rightarrow **string showers** in **string-momentum** direction



HOW DO THEY SHOW UP?



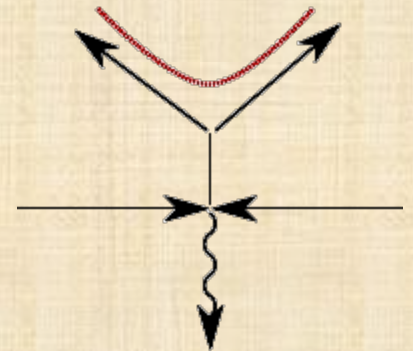
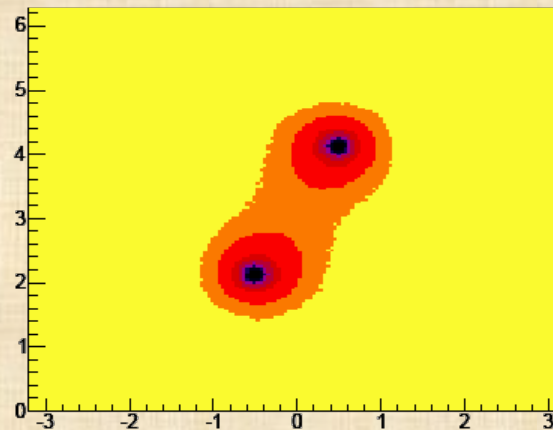
Shower same event
millions of times

Higgs:

$$\Delta\eta_{b\bar{b}} = 1$$

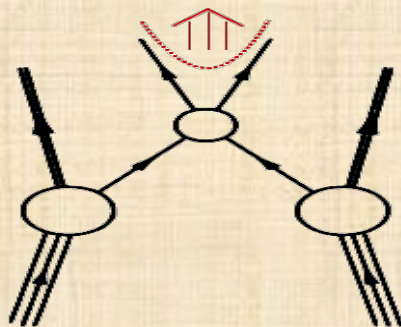
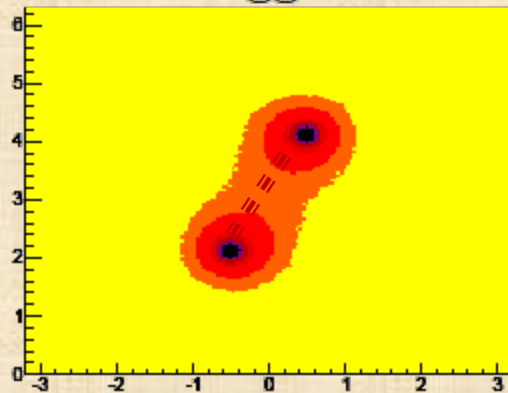
$$\Delta\phi_{b\bar{b}} = 2$$

Add up E_T in
each cell:

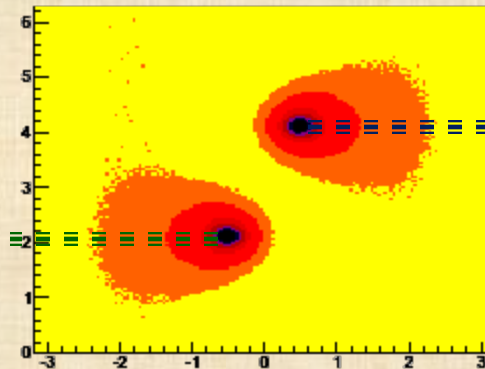


SIGNAL VS BACKGROUND

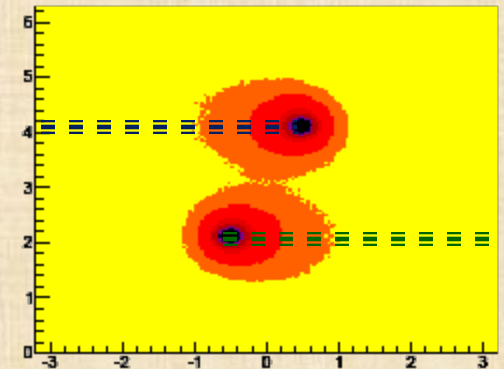
Higgs:



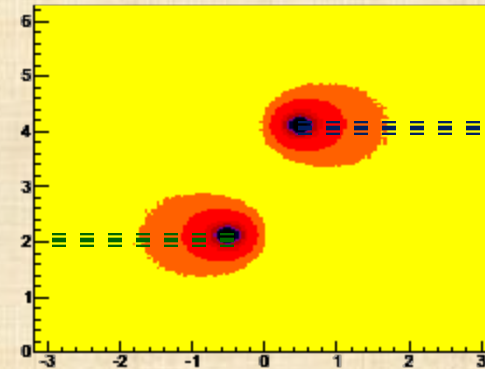
$q\bar{q}$



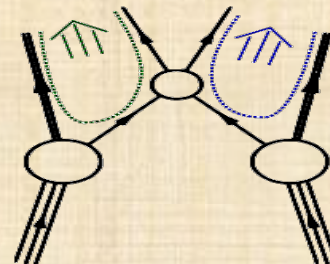
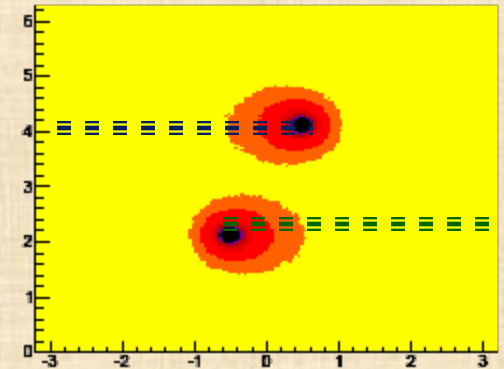
$q\bar{q}X$



gg

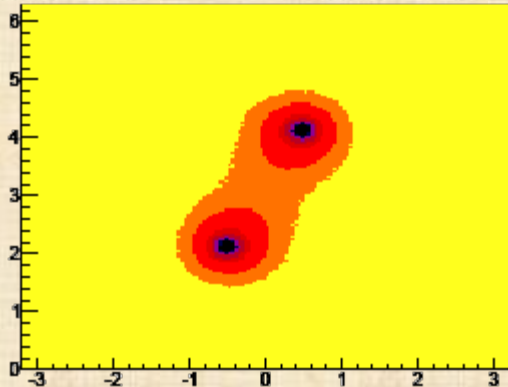


ggX

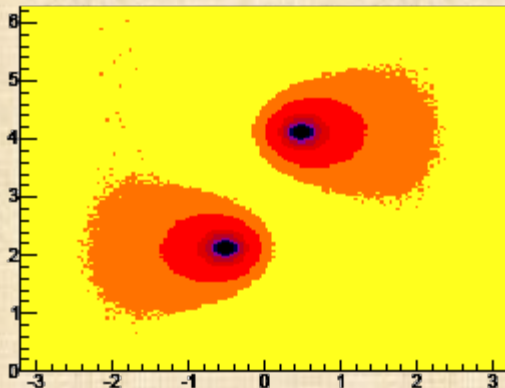


HOW CAN WE USE IT?

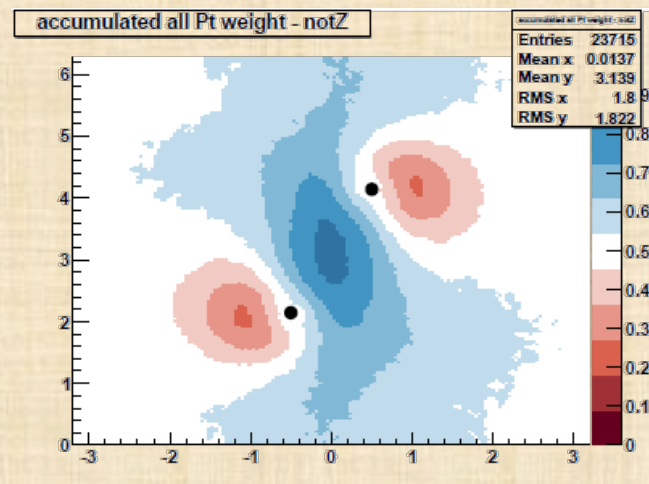
Higgs:



$q\bar{q}$

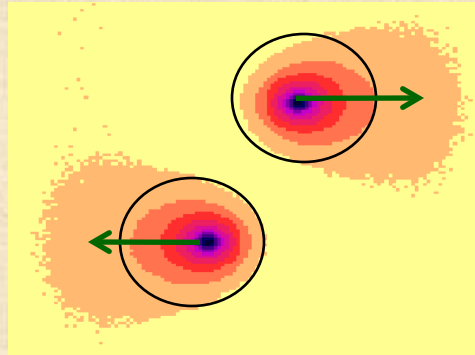
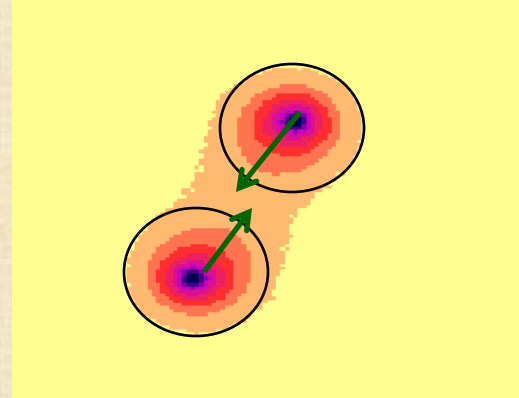


Baysean **probability** that
each bit of radiation is **signal**



- Most useful radiation is
 $R = 0.5 - 1.5$ away
- Pattern depends **strongly** on **kinematics**
- Can we find a **simpler** or more **universal** discriminant?

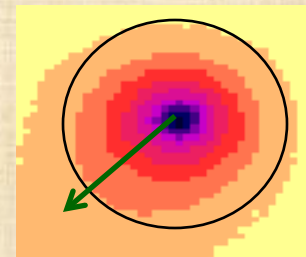
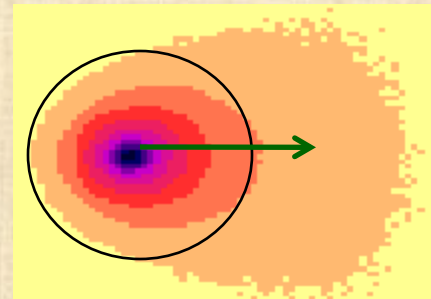
PULL



- Find **jets** (e.g. anti- k_T)
- Construct **pull vector** (\sim dipole moment) on radiation in **jet**

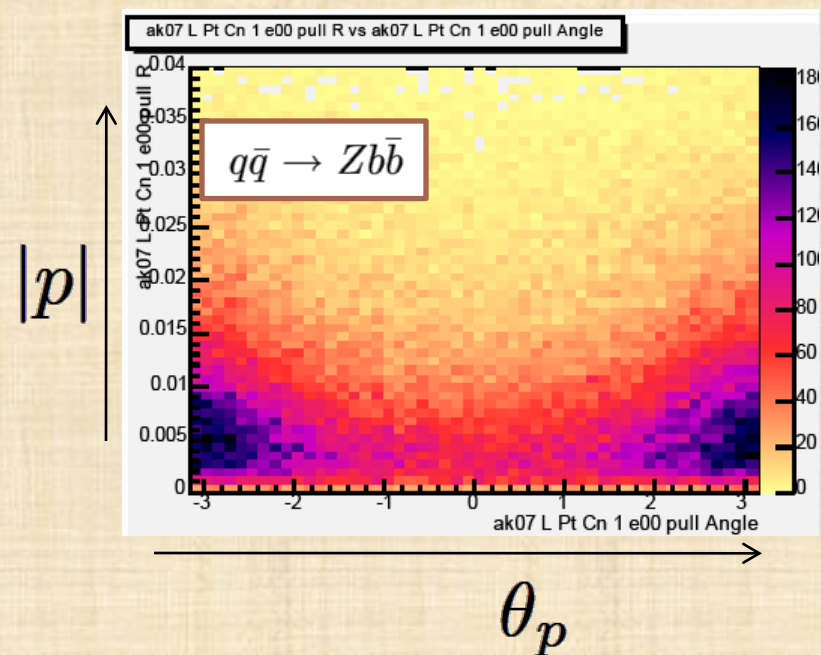
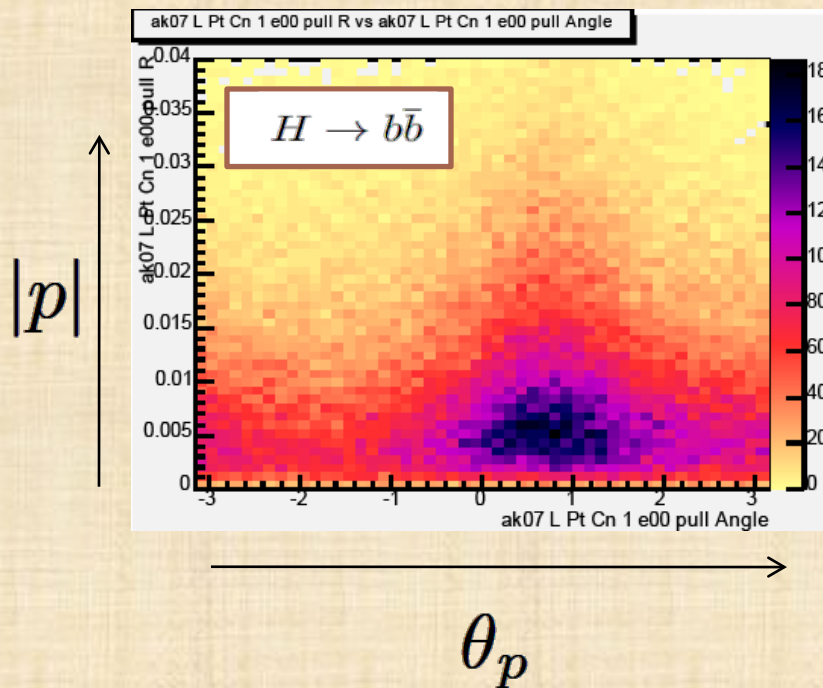
$$\vec{p} = \sum_i \frac{E_T^i |r_i|}{E_T^{jet}} \vec{r}_i$$

- Can use bigger jets for pull, but $R = 0.7$ seems optimal



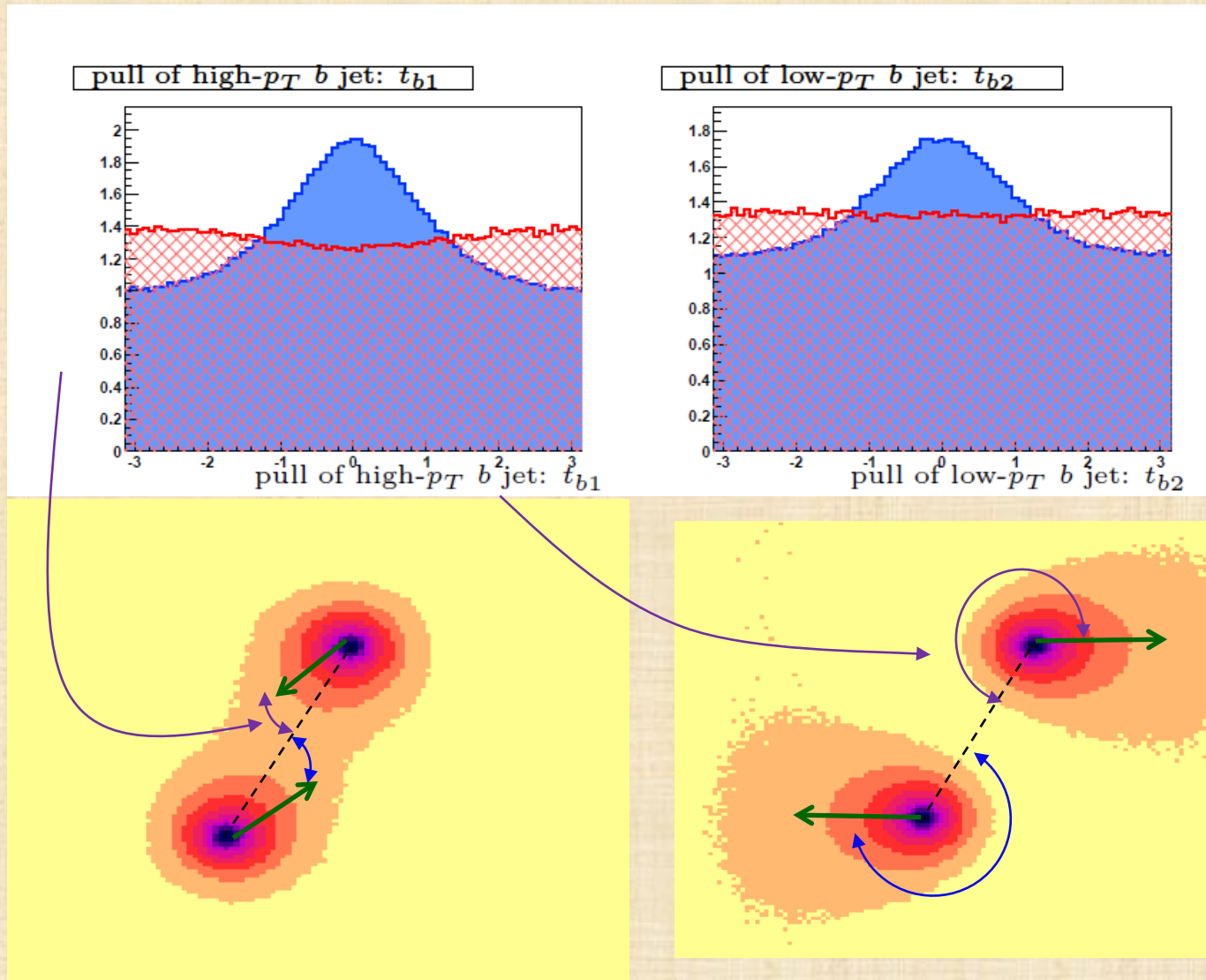
PULL VECTOR IN RADIAL COORDS

$$\vec{p} = \sum_i \frac{E_T^i |r_i|}{E_T^{jet}} \vec{r}_i$$



- Angle much more important than length
- Look at radial pull angle (like for twist)

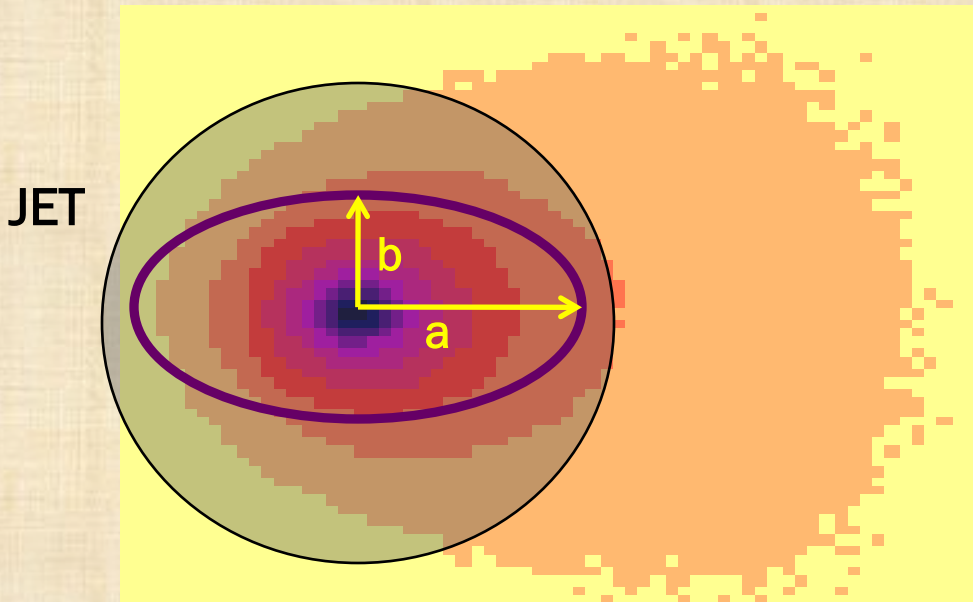
SIGNAL VS BACKGROUND



SECOND MOMENTS

What about **higher moments**?

$$\mathbf{I} = \sum_i \frac{E_T^i |r_i|}{E_T^{jet}} \begin{pmatrix} \Delta\phi_i^2 & -\Delta\phi_i \Delta\eta_i \\ -\Delta\eta_i \Delta\phi_i & \Delta\eta_i^2 \end{pmatrix} \longrightarrow \text{Eigenvalues } \mathbf{a} \text{ and } \mathbf{b}$$



Eccentricity

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Girth

$$g = \sqrt{a^2 + b^2}$$

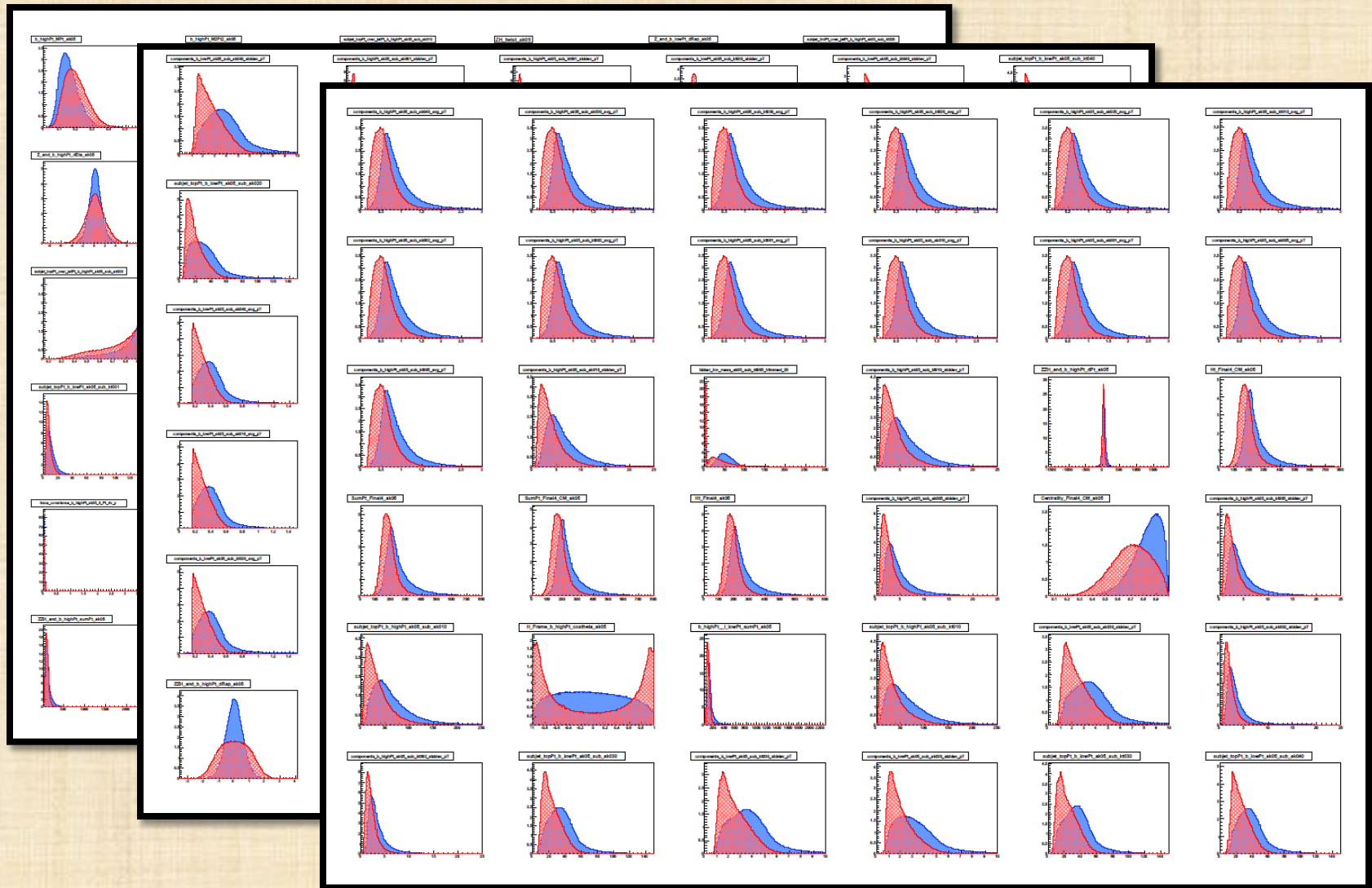
OTHER “SHOWERED” VARIABLES

Many variables **vanish** at the **parton level**

- Do **not** enter the **matrix element** method
- **Complimentary** and **uncorrelated** with kinematic variables
- Mass of each b-jet and the jet mass to p_T ratio
- Rapidity y in addition to pseudorapidity η of each massive b-jet
- Subjet multiplicity for each b-jet
- Average p_T of the small subjets within each b-jet
- p_T of hardest, 2nd hardest, and 3rd hardest subjets within each b-jet
- Radial moments (“girth”) of each b-jet: $g = \sum_i \frac{p_T^i |r_i|}{p_T^{jet}}$
- Angularity: $\tau_a = \frac{1}{m_{jet}} \sum_i E_i \sin^a \left(\frac{\pi \theta_i}{2R} \right) \left[1 - \cos \left(\frac{\pi \theta_i}{2R} \right) \right]^{1-a}$ for $-1 < a < 1$

SUMMARY

- We looked at ~ 900 discriminants!



Part 3:

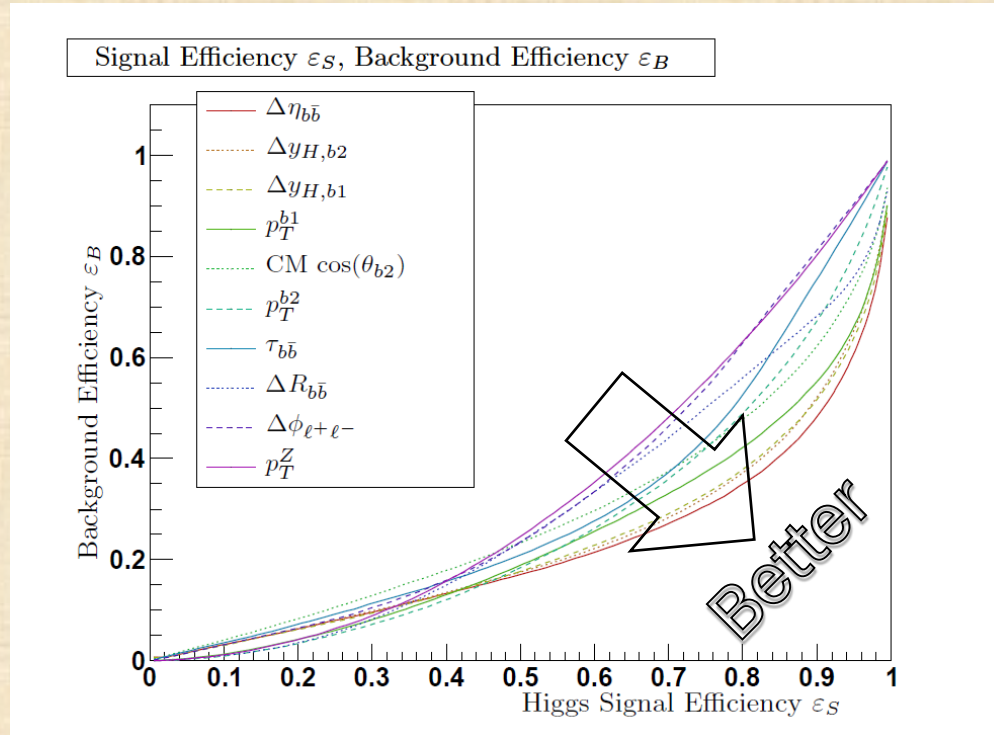
The Output

EFFICIENCIES

ROC
curve:

Background efficiency as a function of signal efficiency

Receiver
Operator
Characteristic



Which variable is best?

OTHER VISUALIZATIONS

$$\frac{S}{B} \xrightarrow{\text{cut}} \frac{\varepsilon_S S}{\varepsilon_S B} = \left(\frac{\varepsilon_S}{\varepsilon_B} \right) \frac{S}{B}$$

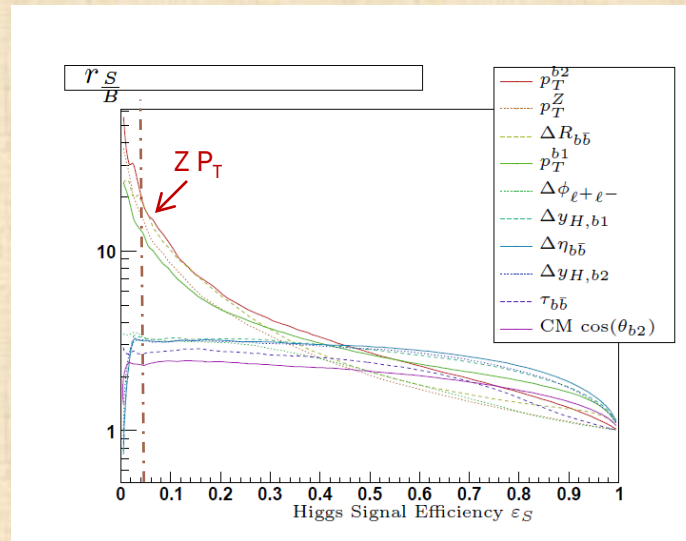
$$r_{\frac{S}{B}} \equiv \frac{\varepsilon_S}{\varepsilon_B}$$

Significance
Improvement
Characteristic

$$\sigma \equiv \frac{S}{\sqrt{B}} \xrightarrow{\text{cut}} \frac{\varepsilon_S S}{\sqrt{\varepsilon_B B}} = \left(\frac{\varepsilon_S}{\sqrt{\varepsilon_B}} \right) \sigma$$

$$r_{\sigma} \equiv \frac{\varepsilon_S}{\sqrt{\varepsilon_B}}$$

- Has **maximum**
- Maximum r_{\square} can **rank** variables
- Effective **visualization**
 - Contains **lots of information**



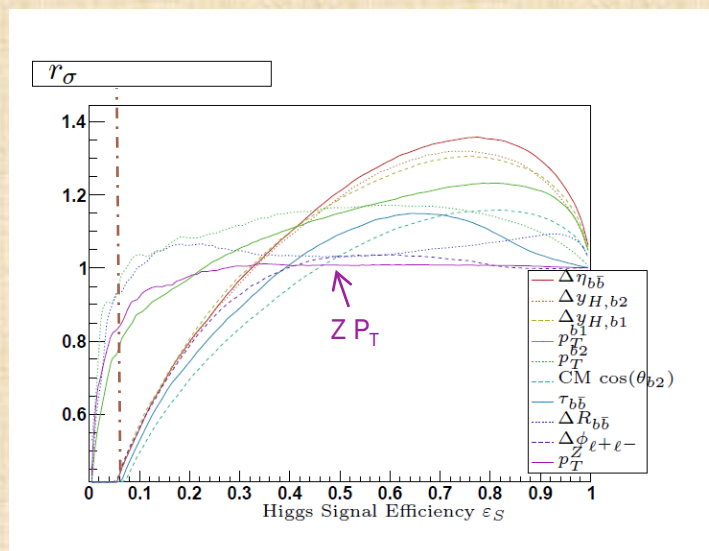
Butterworth et al.
arXiv:0802.2470

$$p_T^Z > 200 \text{ GeV}$$

$$\square_S = 1/20$$

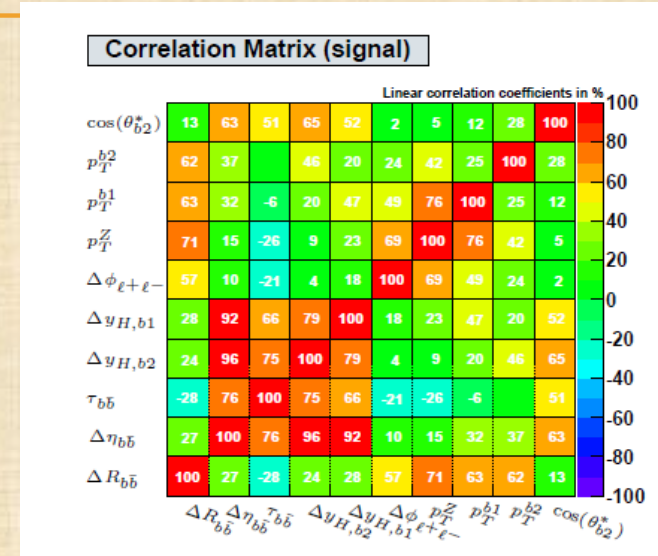
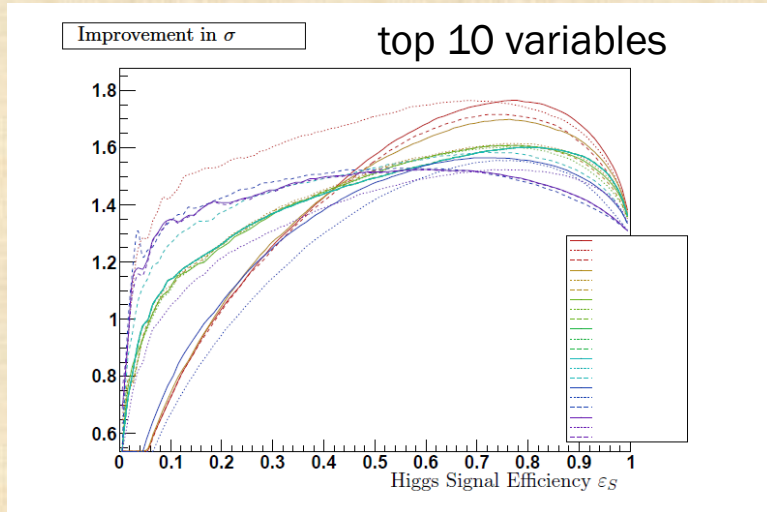
$$\square_B = 1/360$$

$$\frac{\square_S}{\square_B} = 18$$

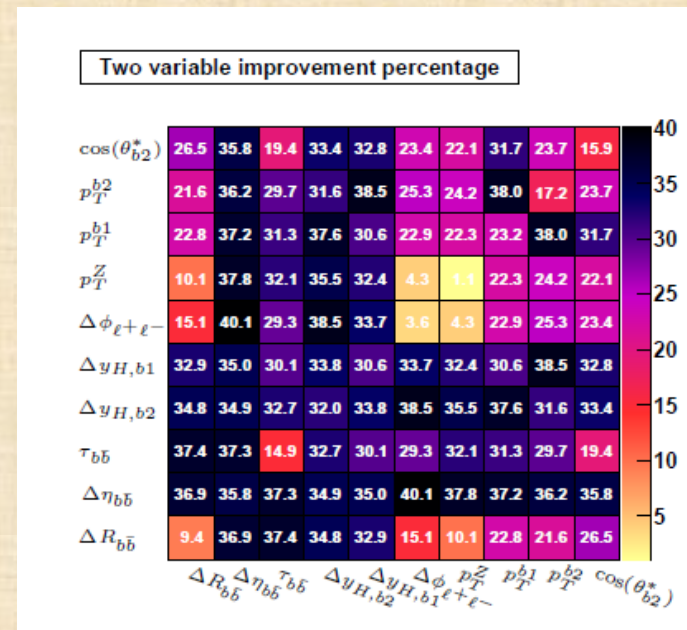
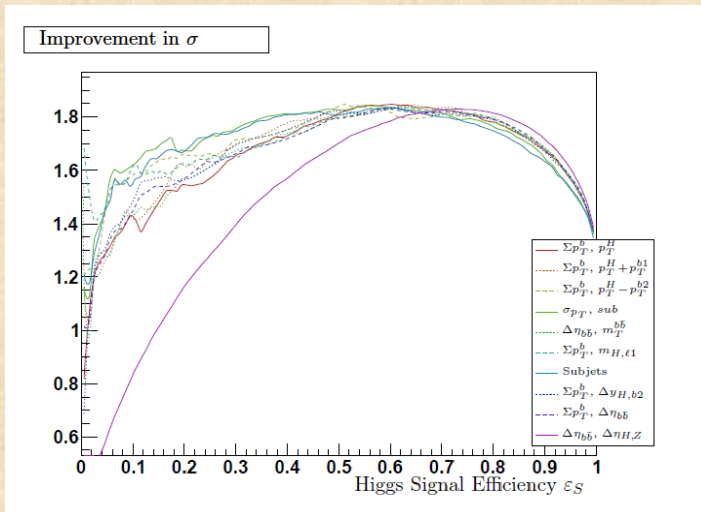


$$\frac{\varepsilon_S}{\sqrt{\varepsilon_B}} = 0.94$$

TOP VARIABLES



top 10 pairs, with boosted decision trees

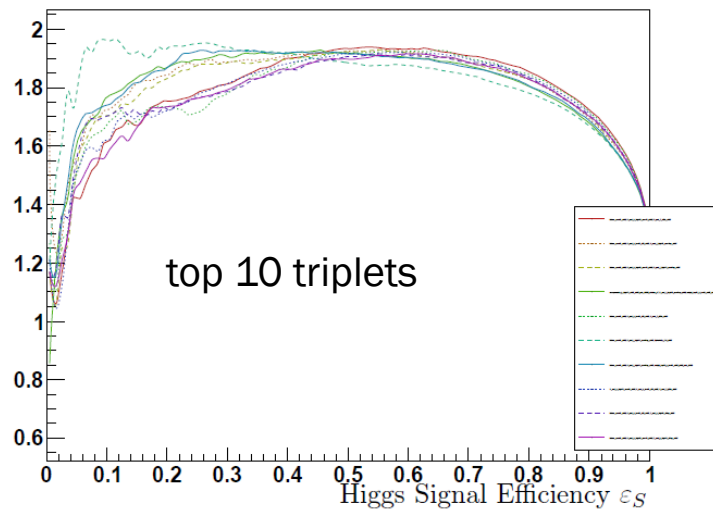


ADDING MORE

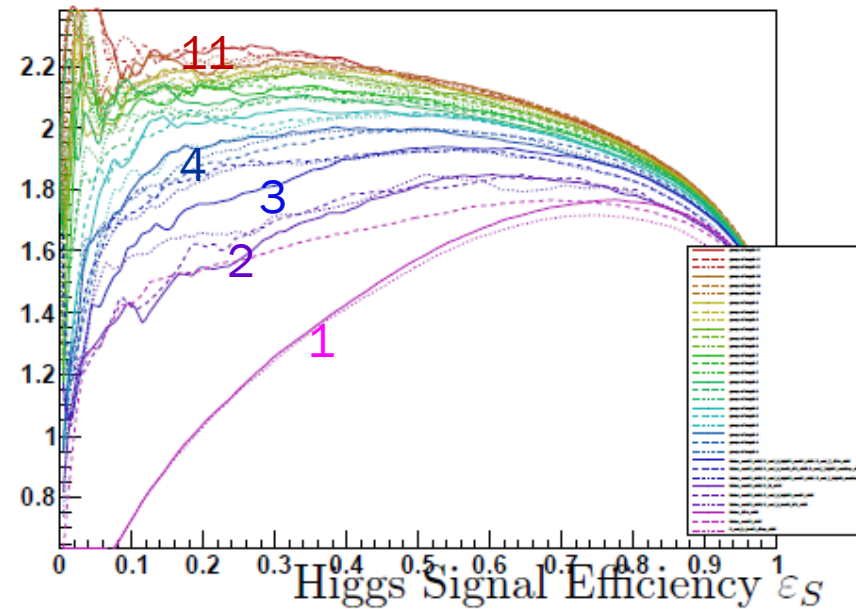
Sequential variable addition

- Take **top 3** sets of n variables
- **Add** any of original **900**
- Take **top 3** sets of $n+1$ variables

Improvement in σ

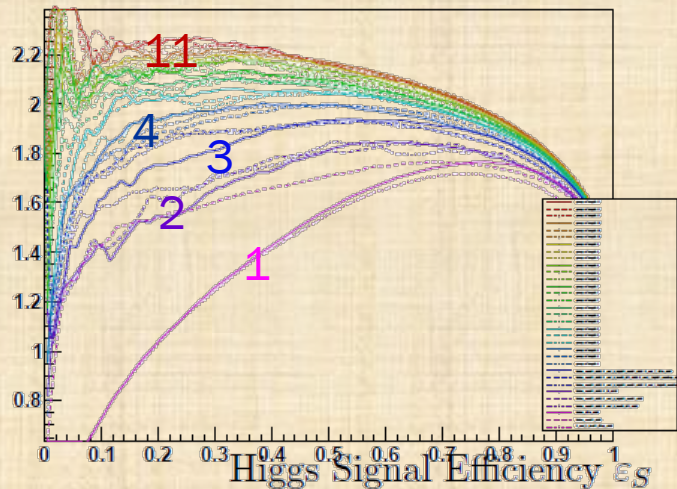


Improvement in σ



OBSERVATIONS

Improvement in σ

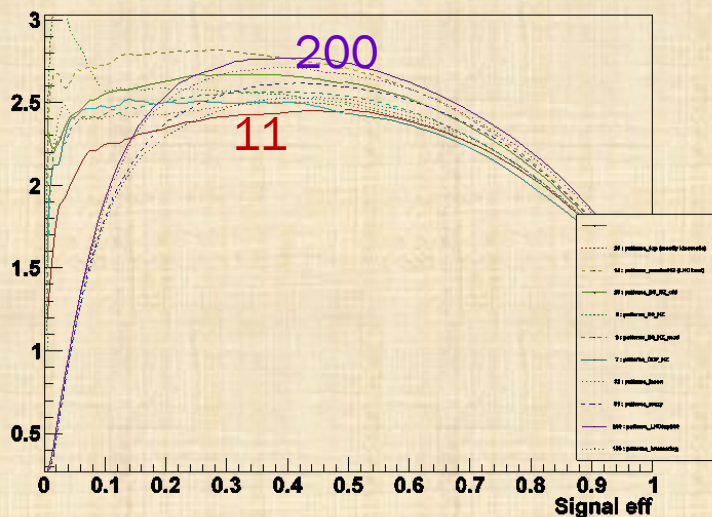


- Converges **slowly**
- **Sensitivity to statistics** apparent
 - $r_b = 2$ $\epsilon_s = 0.05$ gives $\epsilon_b = 1/1600$
 - 1 million events down to 600
- Some variables very **poor** by **themselves**, but show up as **5th** or **6th** variable

Top 10 include

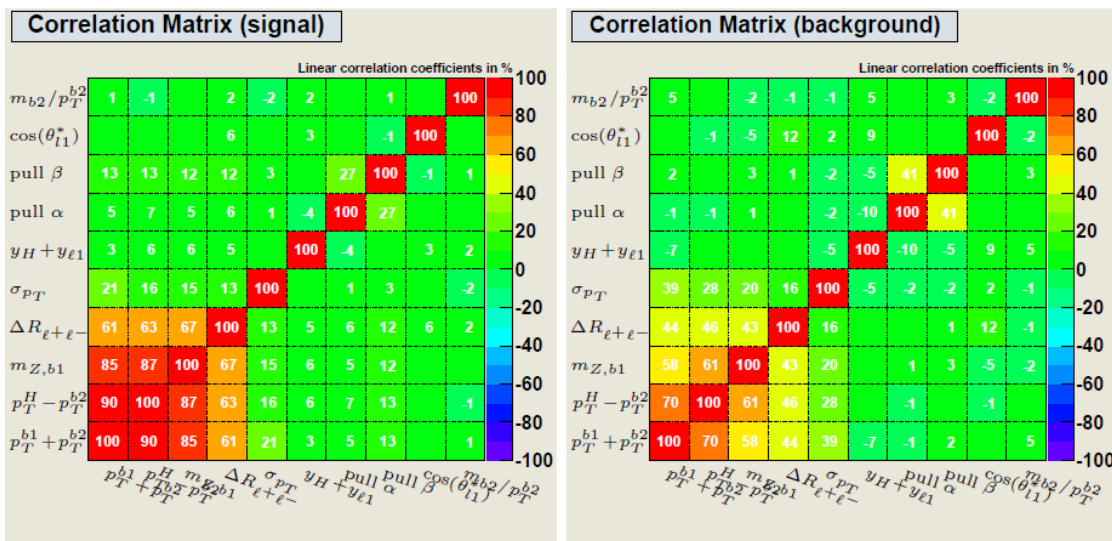
- Higgs p_T
- R_{ZH}
- **Pull**
- **Twist** y (twist with y not ϵ)
- **Event shape D**
- Determinant of covariance matrix for **radiation** in low p_T b jet
- **Scalar sum** of the b jet p_{Ts}

LHC HZ : Significance

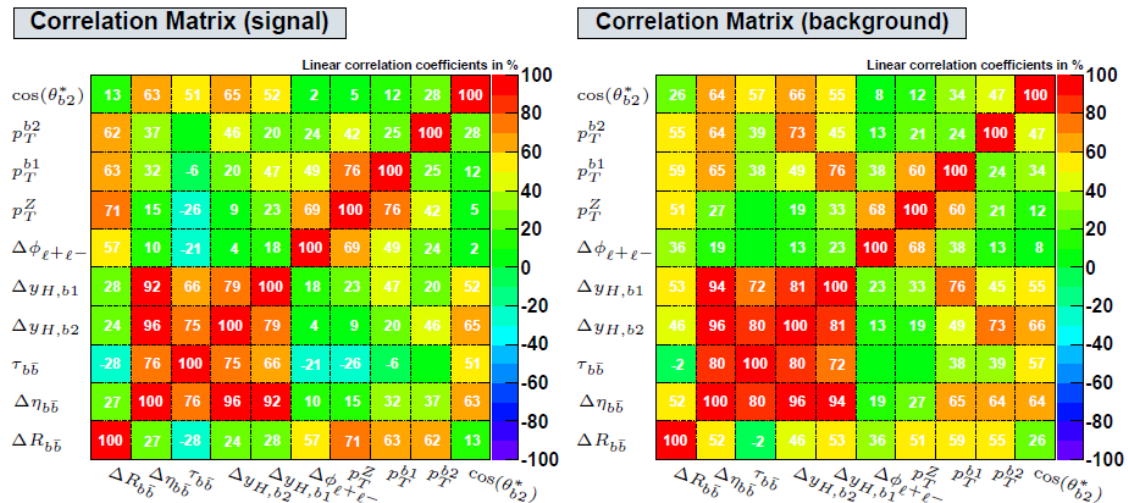


CORRELATIONS OF GOOD 10 COMBO

Best 10
combo

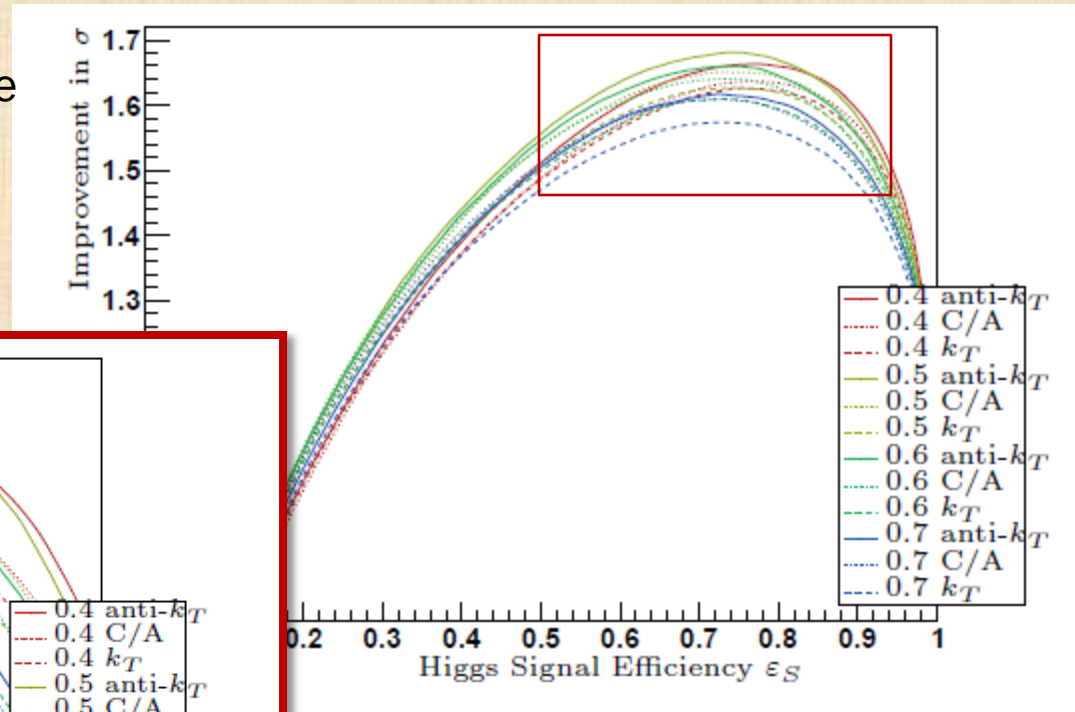
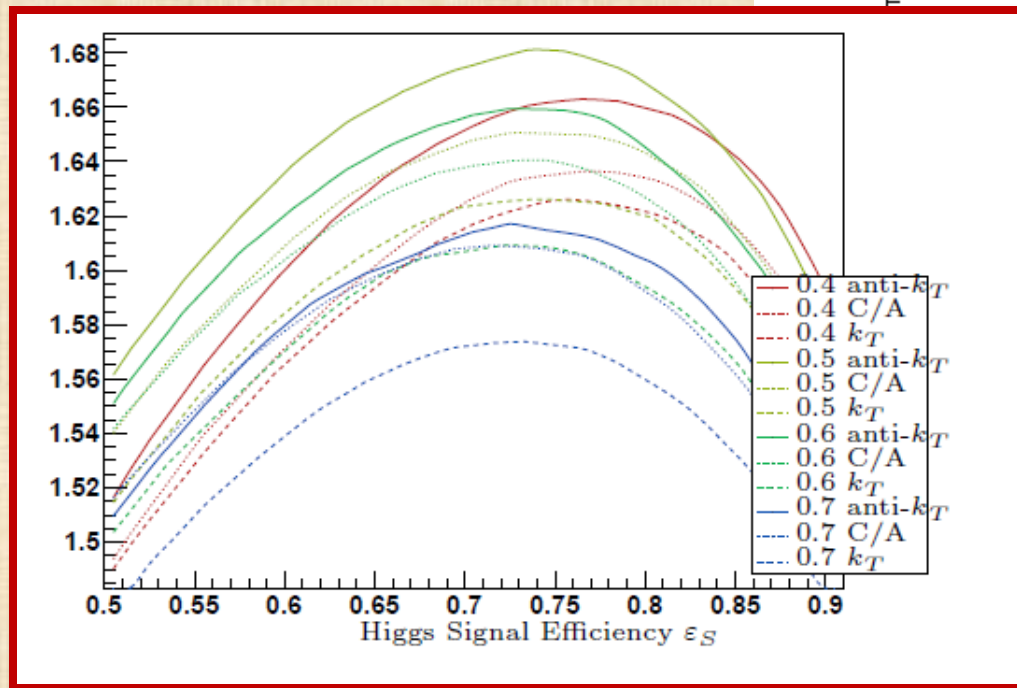


Best 10
individuals



JET ALGORITHMS

- Main observable is $m_{b\bar{b}}$
- Look at jet algorithm dependence



- The **winner** is ...
anti- k_T with $R = 0.5$
- Optimal **mass window**
 $90 \text{ GeV} < m_{b\bar{b}} < 124 \text{ GeV}$

TRIMMING

Krohn, Thaler, Wang

1. **Recluster** jet constituents into **very thin** jets



TRIMMING

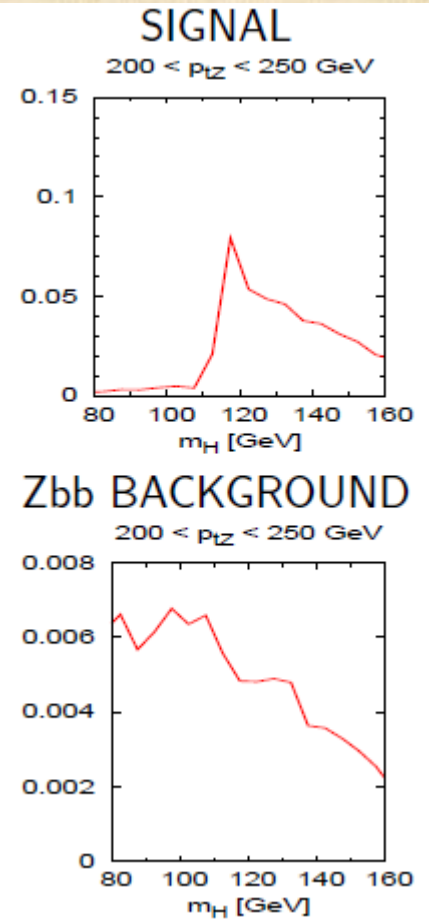
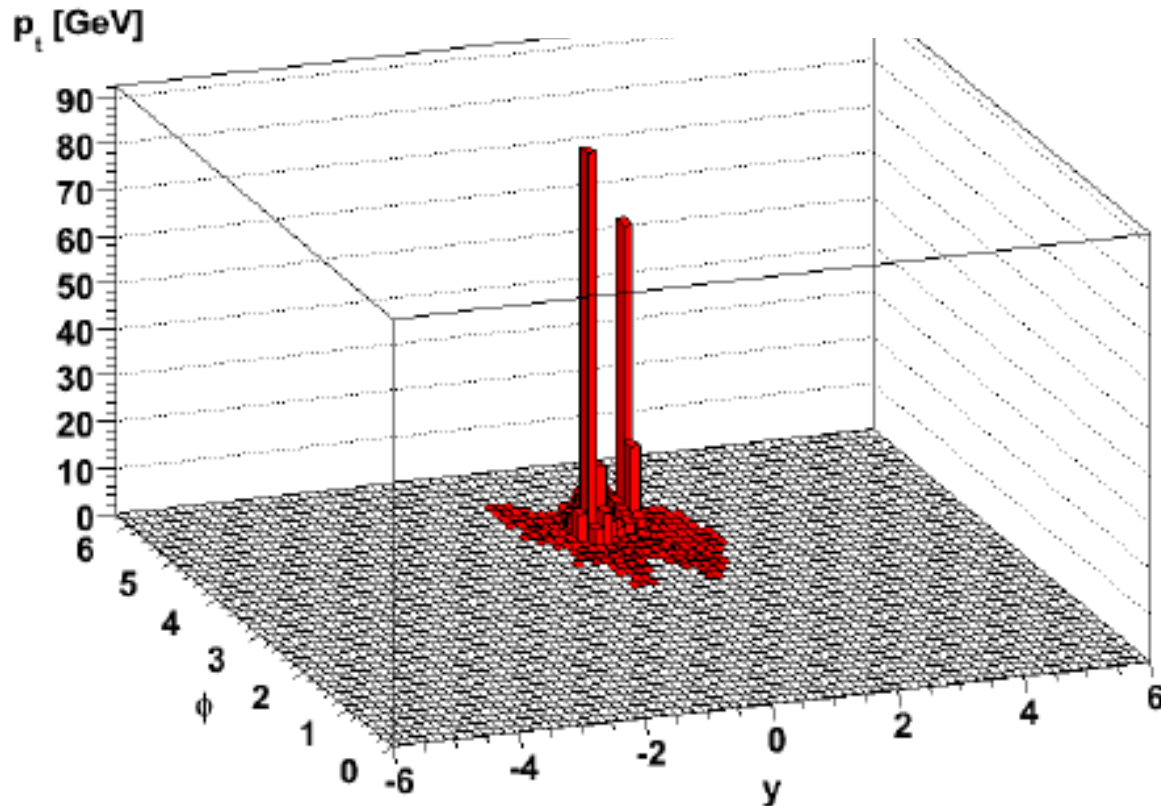
Krohn, Thaler, Wang

1. **Recluster** jet constituents into **very thin** jets
2. **Throw away** thin jets that are **too soft**



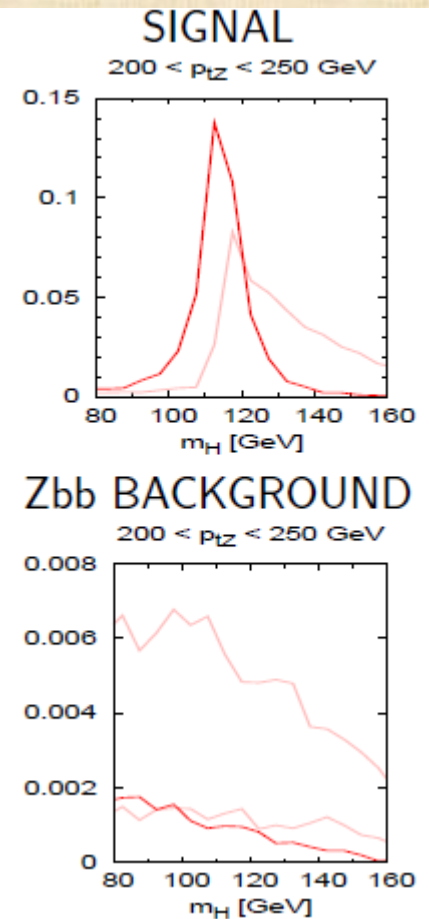
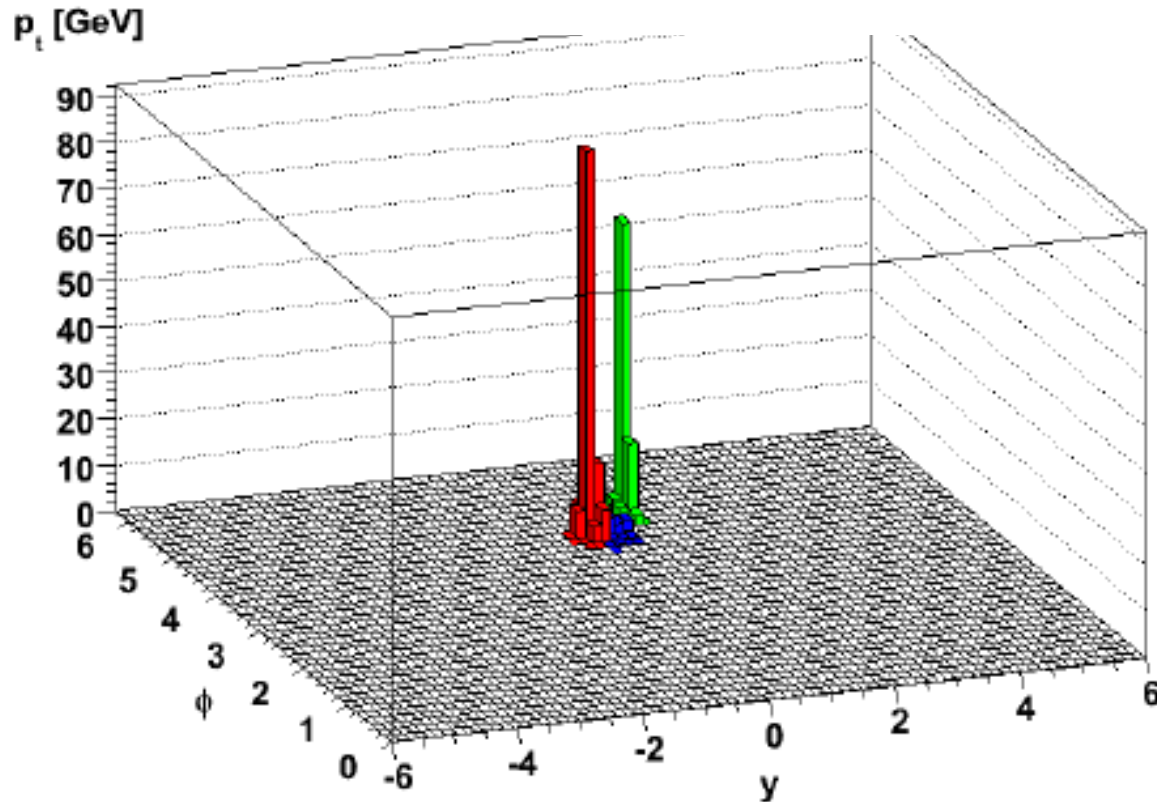
TRIMMING

Boosted $H \rightarrow b\bar{b}$



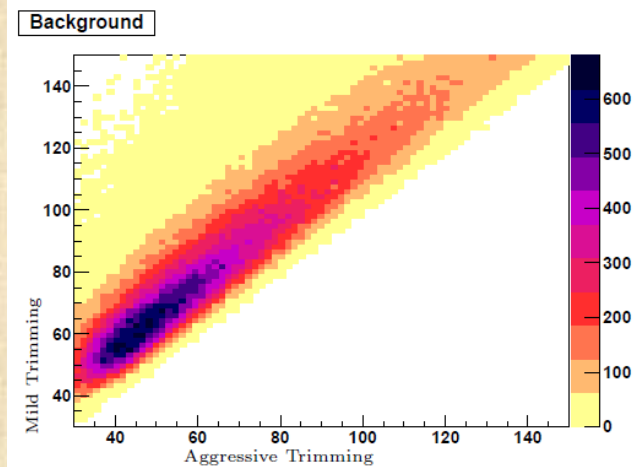
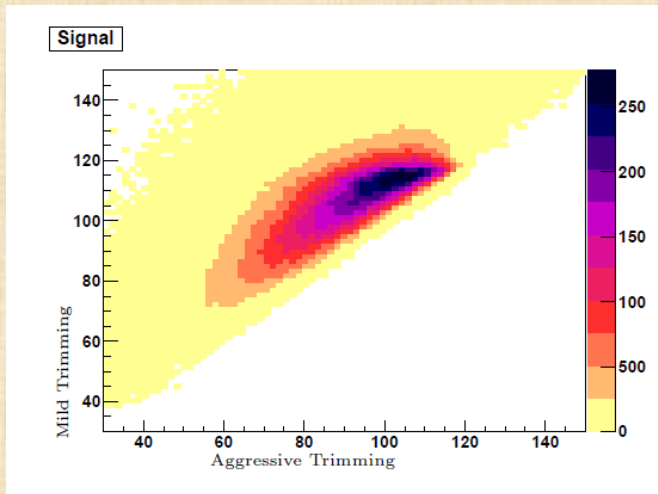
TRIMMING

Boosted $H \rightarrow b\bar{b}$



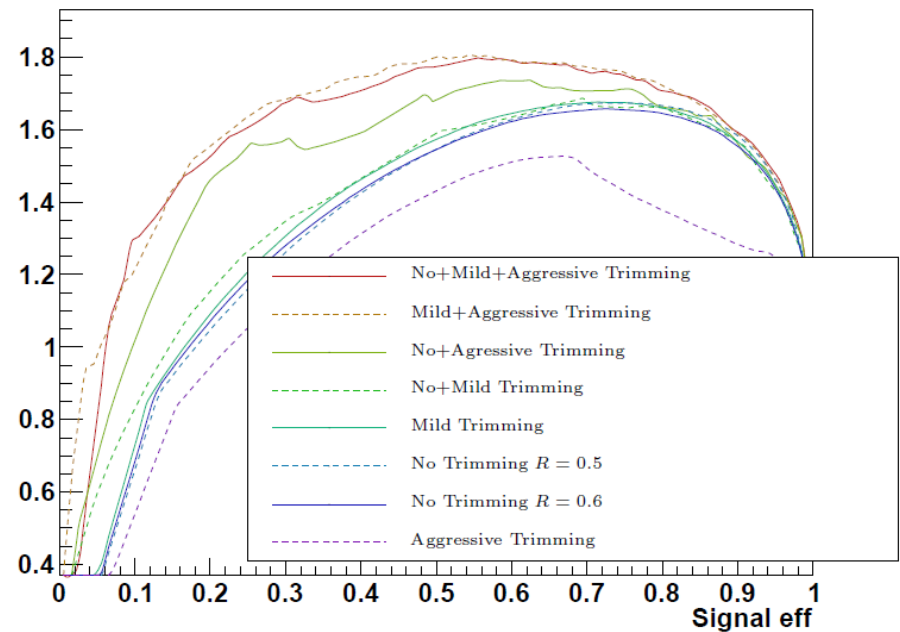
MULTIPLE TRIMMINGS

Trimming does **not** seem to **help** much in our case...



Higgs : Significance

Multiple trimmings do **help**!



(inspired by Soper and Spannowsky)

CONCLUSIONS



- **Final** efficiencies still **under construction**
- Looks like we can **help** the Tevatron searches
 - around **10%** with **variables** (relative to the ones they use)
 - around **10%** with **masses** (assuming they can trim)
- **W/Z + H** is totally **feasible** at the **LHC**
 - Do **not** need **large** p_T
 - Discovery potential with 30 fb^{-1}

General Observations

- **SIC** curves provide a useful visualization
 - demonstrate **instabilities**
 - show **coverage**
 - **visually compare** variables' performance
- **Uncorrelated variables** helpful after kinematics exhausted
- **Multiple mass measures** useful

Future

- Compare **boosted decision trees**, **random forest**, **neural networks**, etc.
- Compare different generators (**Herwig/Pythia**)
- Study **reducible** backgrounds