
Multivariate Overview

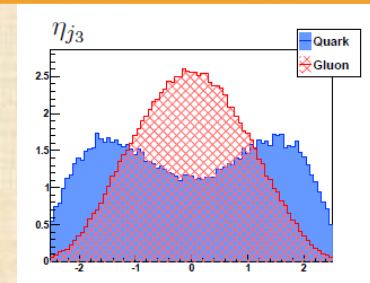
Matthew Schwartz

Harvard University

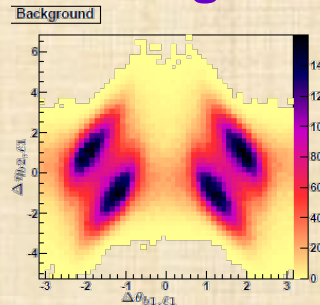
March 24, Boost 2011

WHY USE A MULTIVARIATE APPROACH?

- We can think about and visualize **single variables**



- Two variables are harder

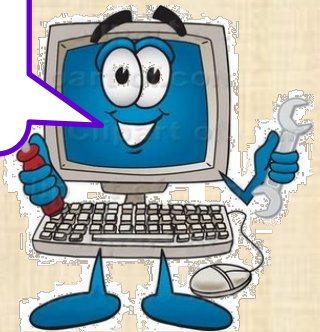


- Nobody who thinks in 11 dimensions is in this room!

- There things that **computers are just better** at.

- Multivariate approach lets you figure out how well you could **possibly do**

Let me do
the work
for you!



FRAMING

See if simple variables
can do as well (establishes the goal)

POWER

Sometimes they are really necessary (e.g. ZH)

EFFICIENCY

Save you the trouble or looking
for good variables (project killer)

MULTIVARIATE (MVA) BASICS

Lots of methods (all in the TMVA package for root)

- Boosted Decision Trees
- Artificial Neural Networks
- Fischer Discriminants
- Rectangular cut optimization
- Projective Likelihood Estimator
- H-matrix discriminant
- Predictive learning/Rule ensemble
- Support Vector Machines
- K-nearest neighbor
- ...

Useful in many
areas of science,
such as artificial intelligence

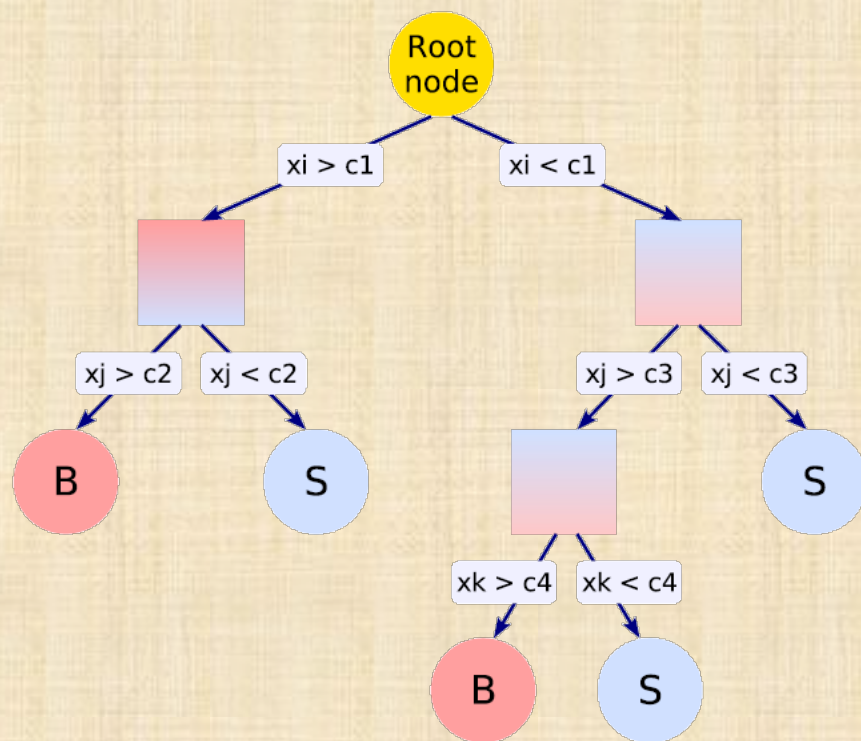
For particle physics, **Boosted Decision Trees** work best

Easy to
understand

Train fast

Excellent efficiencies

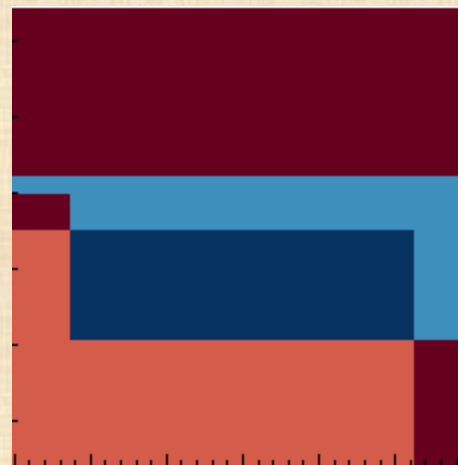
BOOSTED DECISION TREES



One decision tree



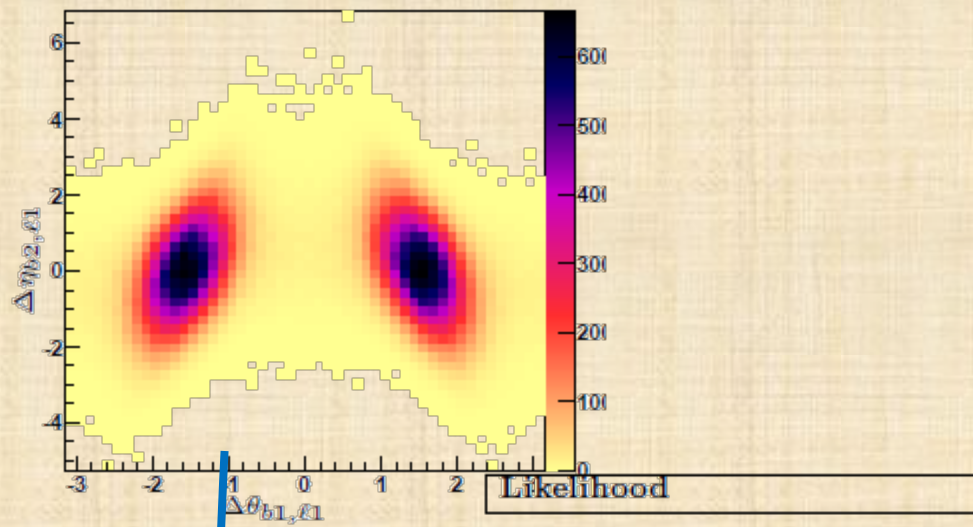
Two decision trees



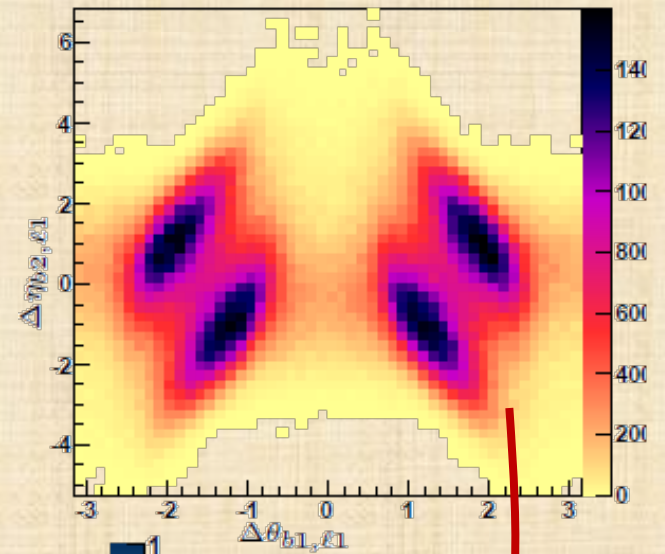
- **Boosting**: train successive trees on misclassified events by enhancing their importance

EXACT SOLUTION: LIKELIHOOD

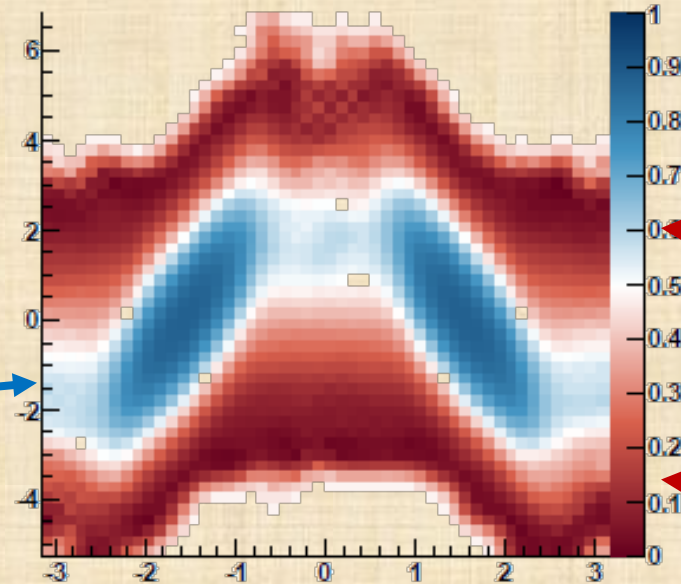
Signal



Background

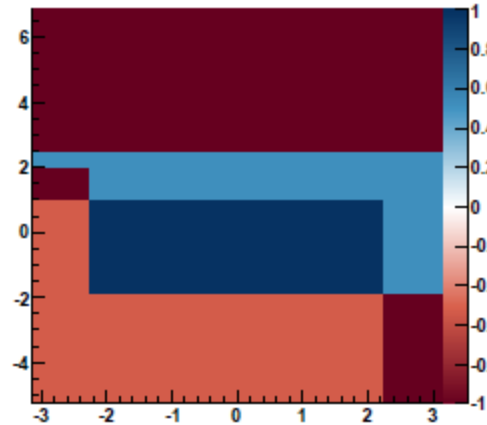


Likelihood

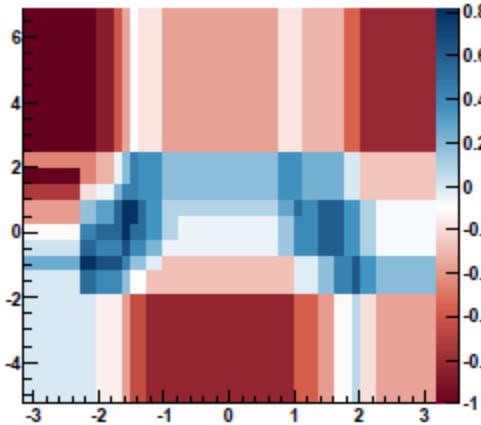


MULTIDIMENSIONS: APPROXIMATE

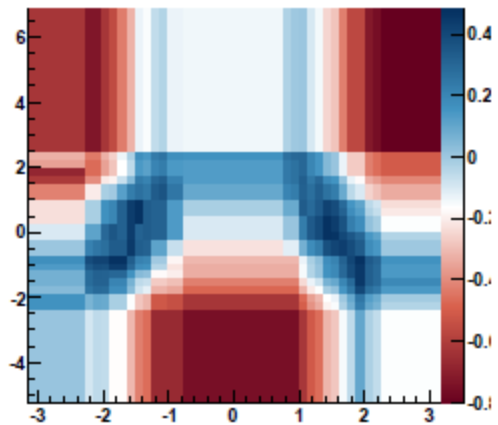
BDT 2



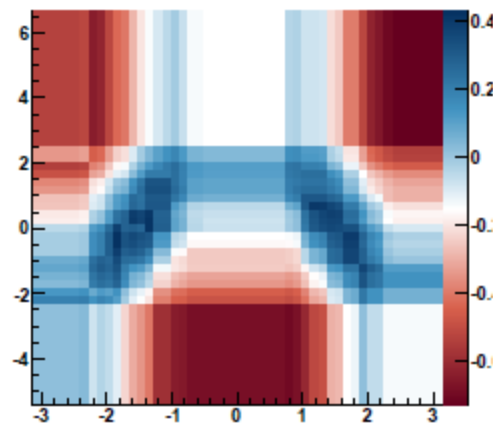
BDT 8



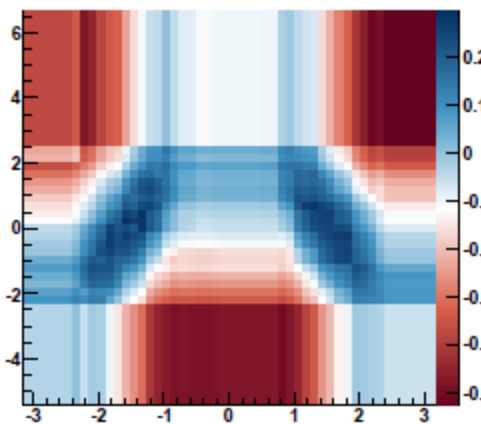
BDT 32



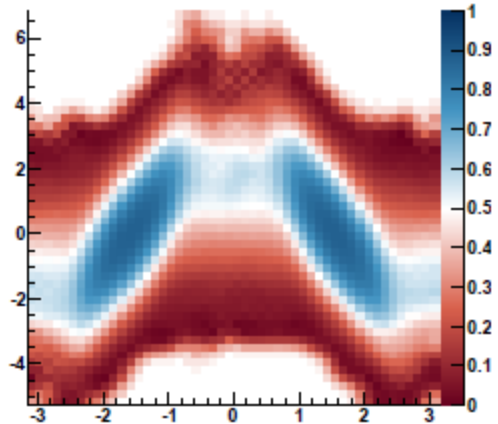
BDT 64



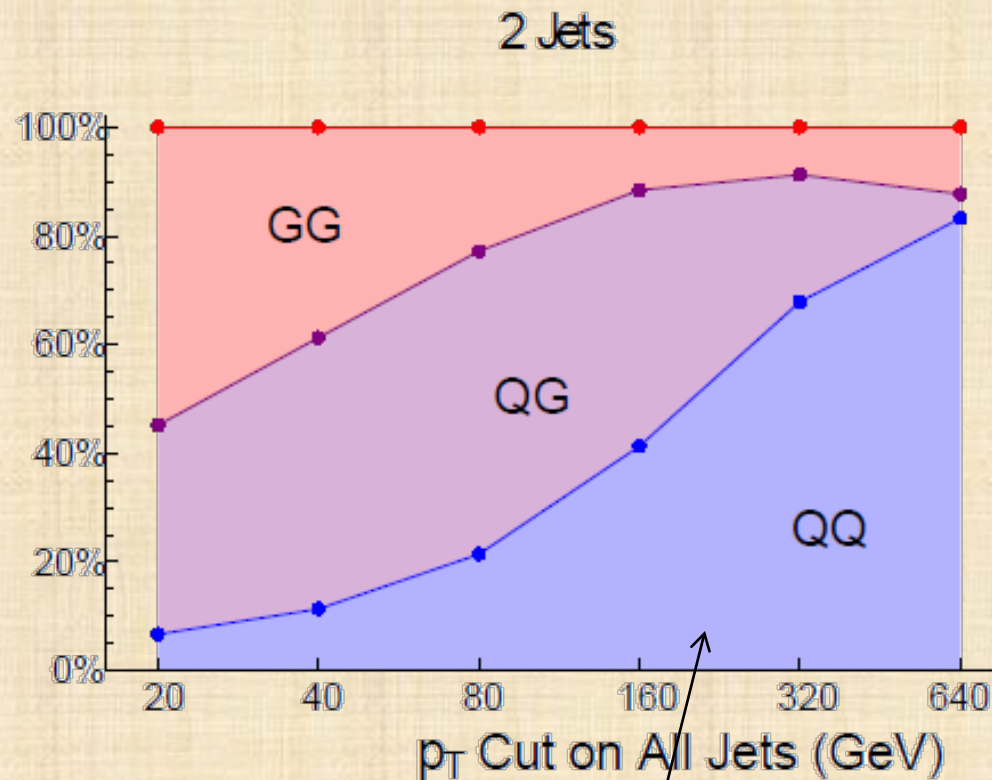
BDT 256



Likelihood



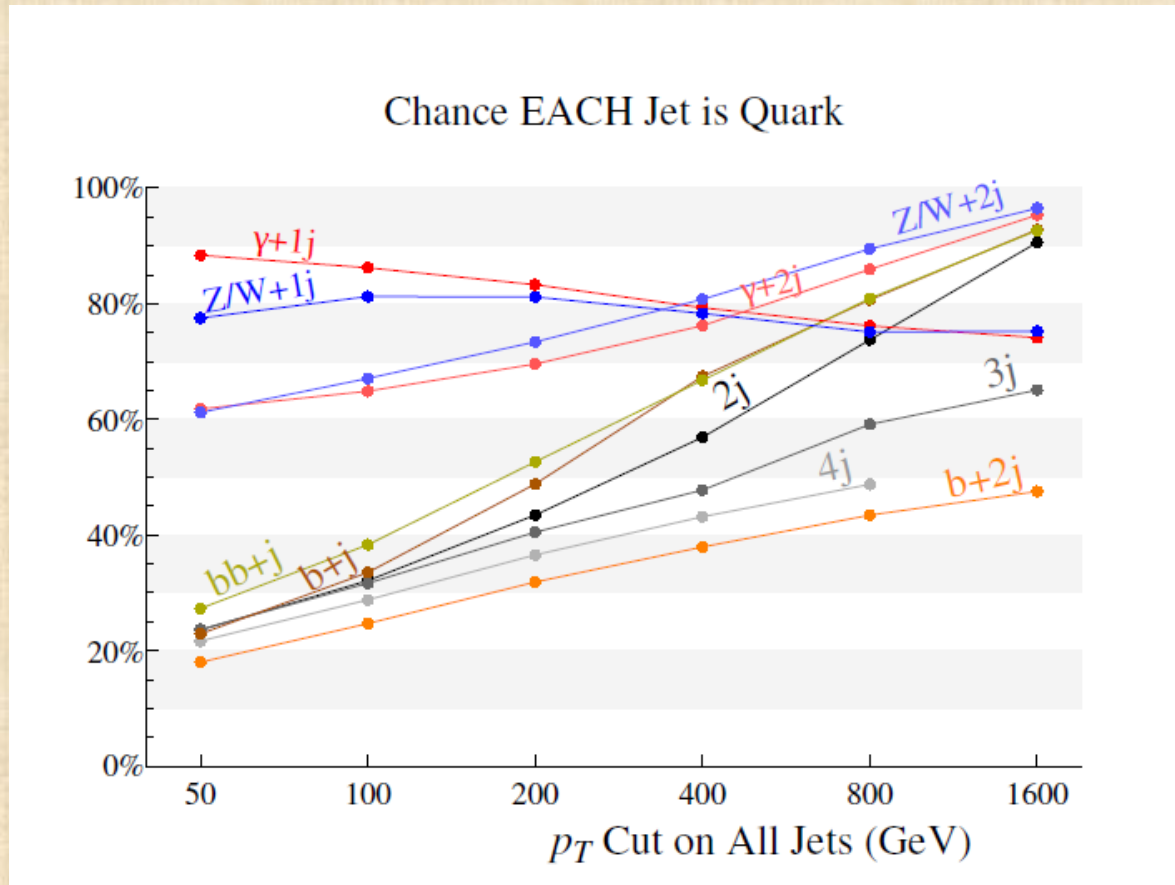
EXAMPLE: WHERE ARE THE QUARK JETS?



(Aside for Pekka:

Maybe one reason jet masses more correlated at high p_T)

LOOK AT ALL SAMPLES



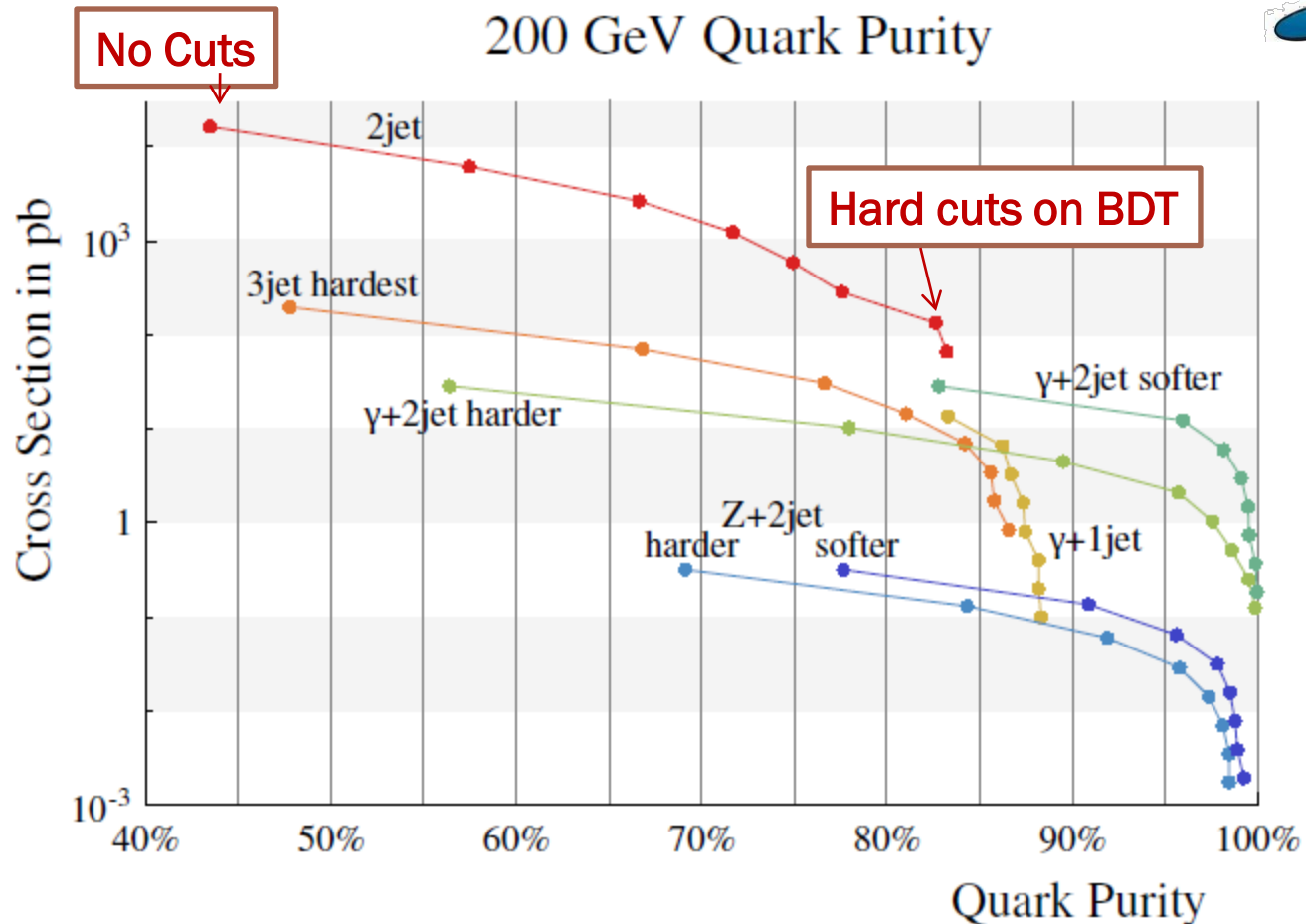
- What about cross sections?
- Can cuts purify the samples?

THROW THEM INTO THE BDT

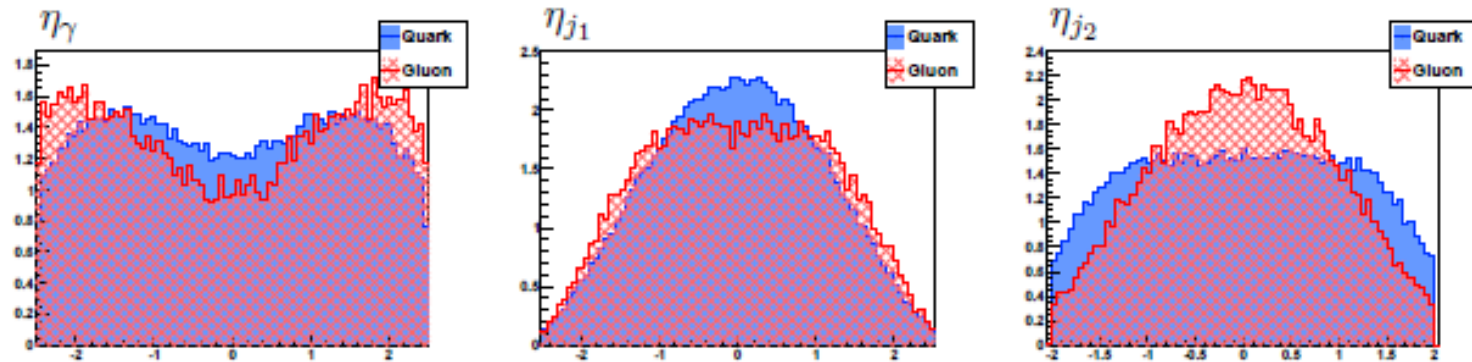
Now
you're
talking!



Optimize efficiency using BDT classifier with parton momenta as inputs (6 or 9 inputs)



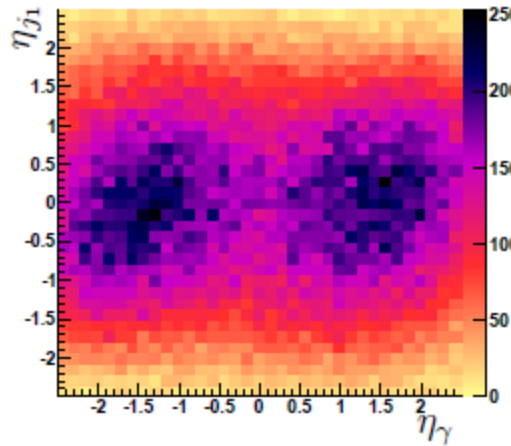
NOW LOOK AT THE $\gamma + 2$ JETS SAMPLE



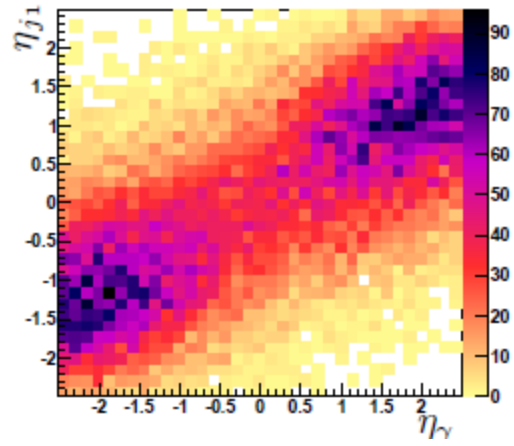
- Look at the best discriminants, ranked by cuts
- The **rapidity of the photon** and the **rapidity of the second hardest** jet look good
- But cutting on just η_γ or just η_{j2} does not help much

LOOK AT CORRELATIONS

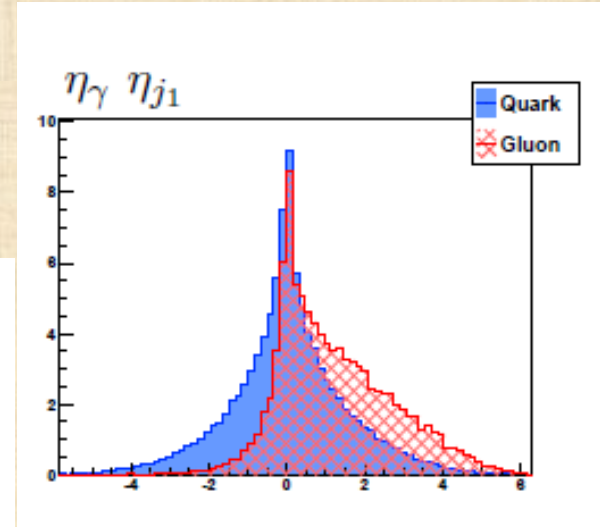
Quark



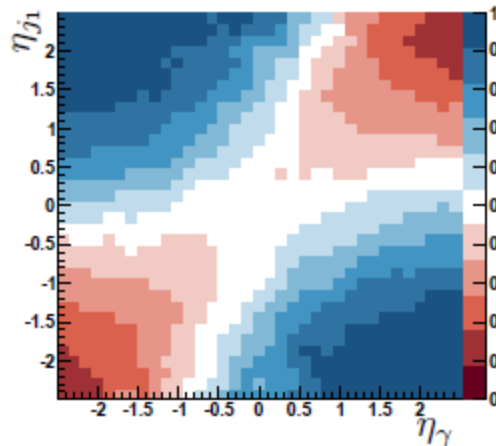
Gluon



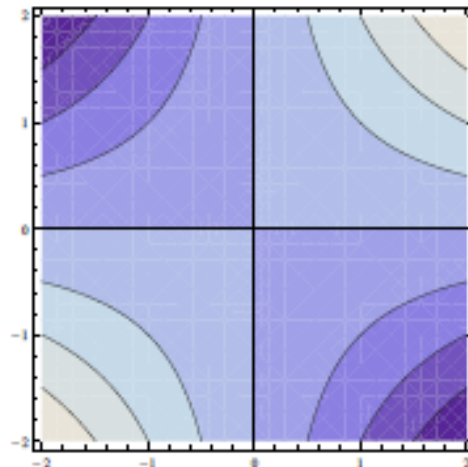
Distribution of $\eta_\gamma \eta_{j1}$



Likelihood

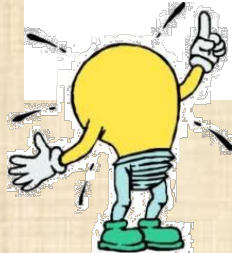
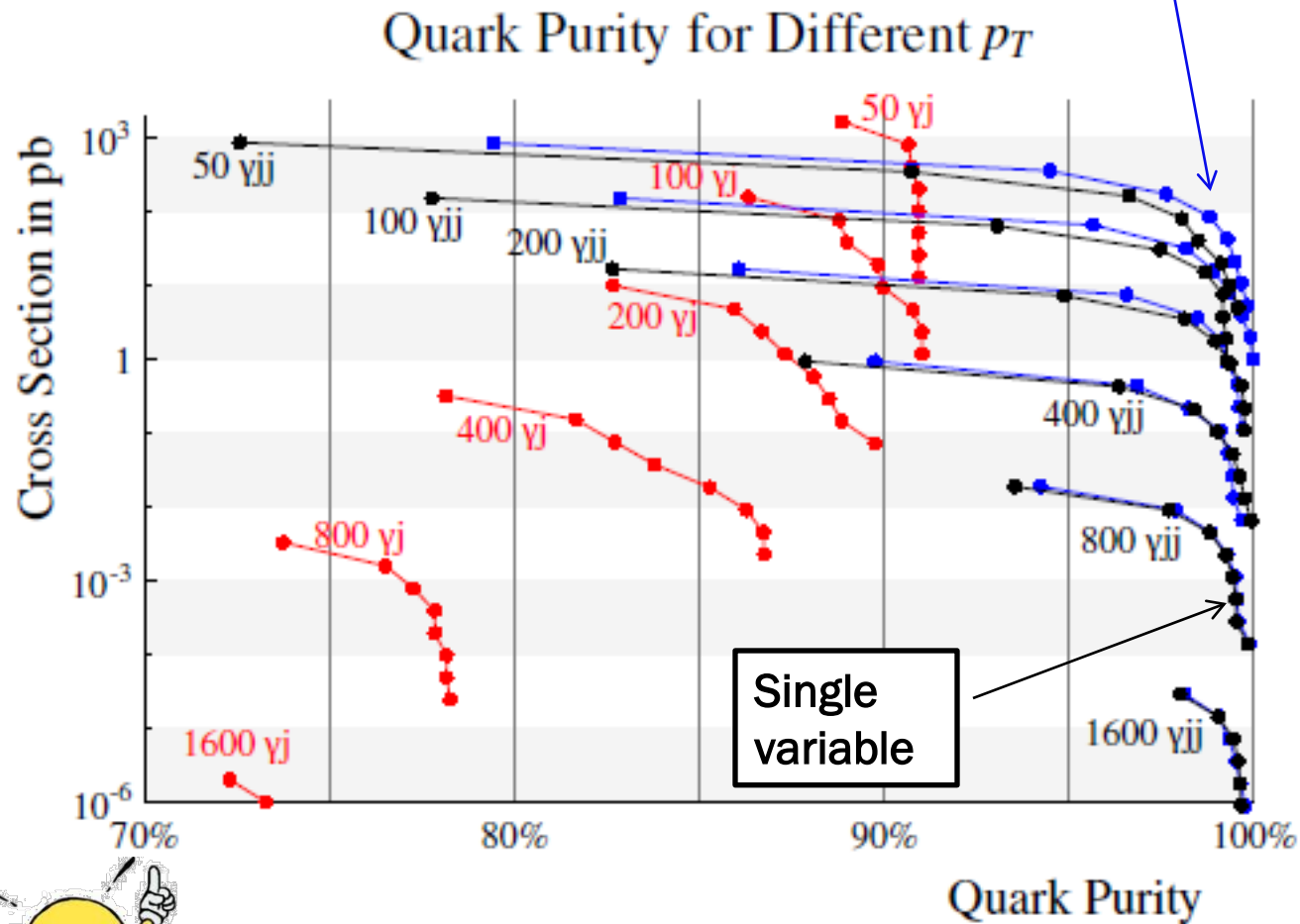


Contours of $\eta_\gamma \eta_{j1}$



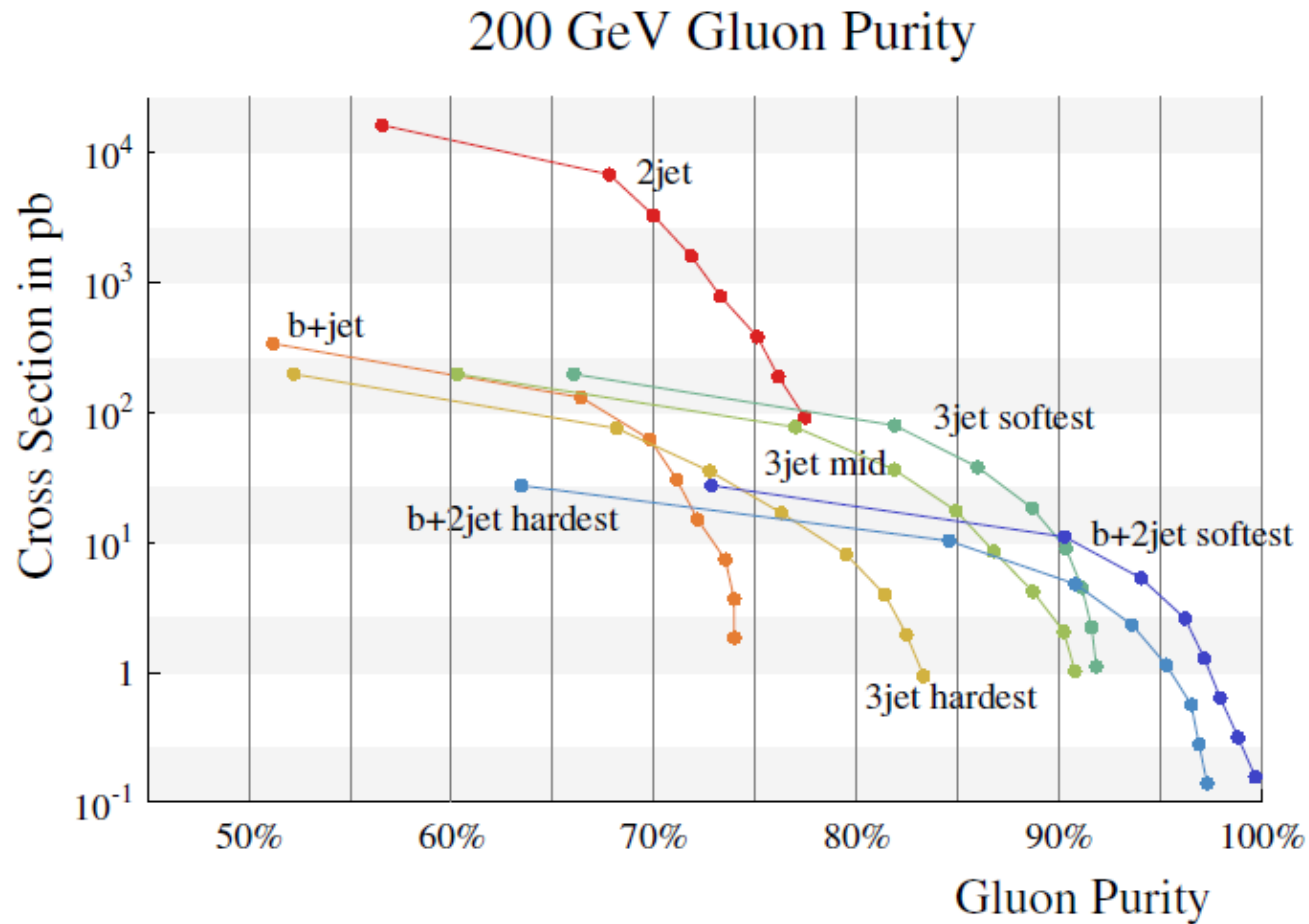
BEST SINGLE VARIABLE

BDT results



BDTs led us to the variable,
but with the variable we **don't need BDTs**

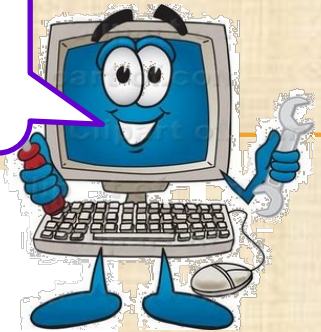
WHAT ABOUT PURE GLUONS?



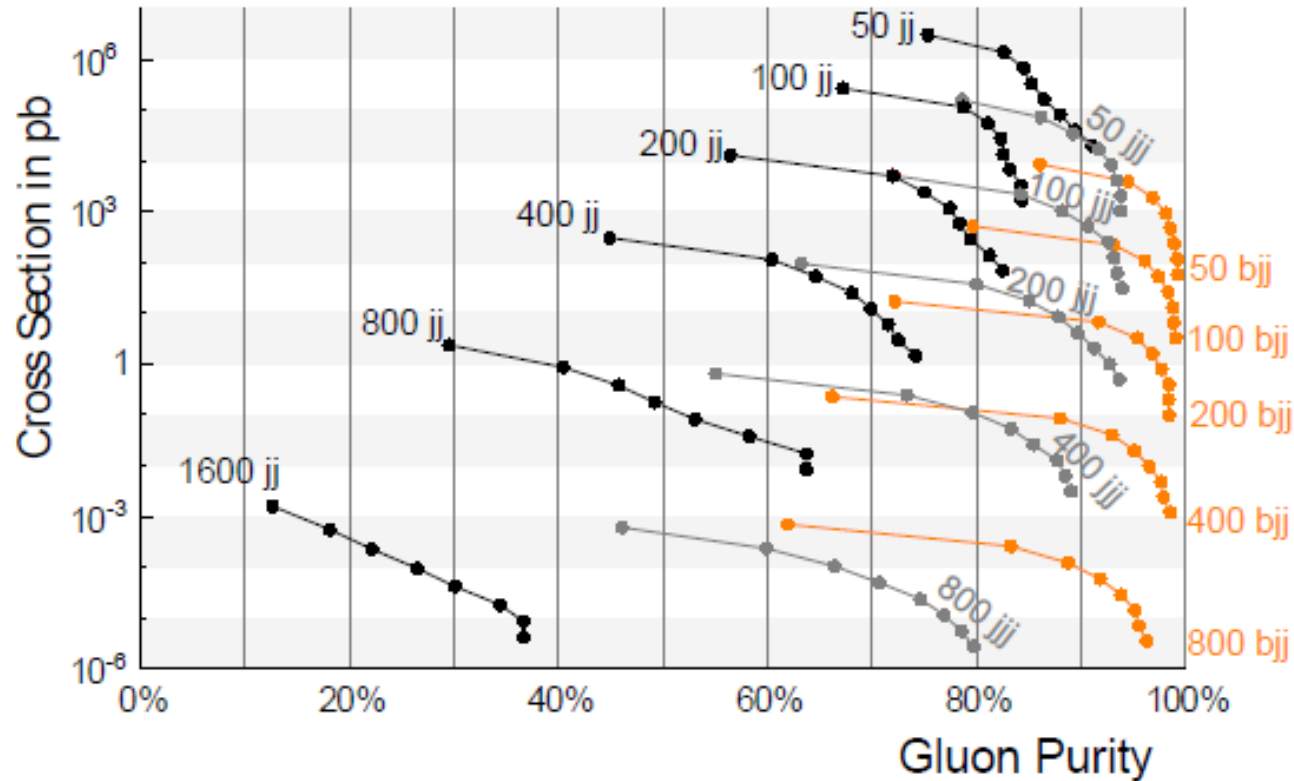
b+2 jets or trijets look promising

THROW IT AT THE BDT

This is my
favorite
part



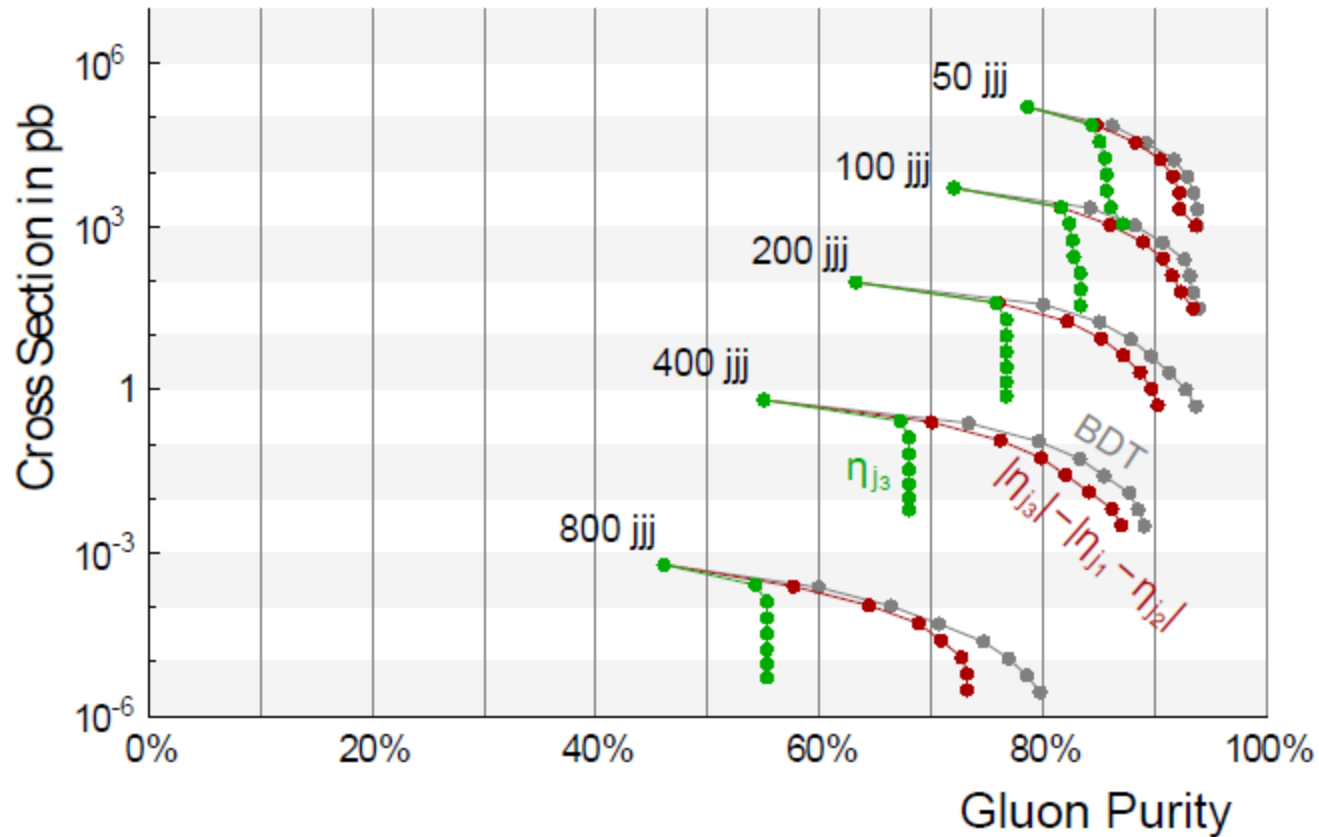
Best Samples for Gluon Purity



- Now try to find a single variable that works as well...

FINDING PURE GLUON JETS

Trijet Sample with Different Kinematic Cuts



SUMMARY OF FINDING QUARKS/GLUONS

- ✗ For quarks, look at gamma + jet

- + cut on $\eta_\gamma \eta_{j1} + \Delta R_{\gamma j2}$

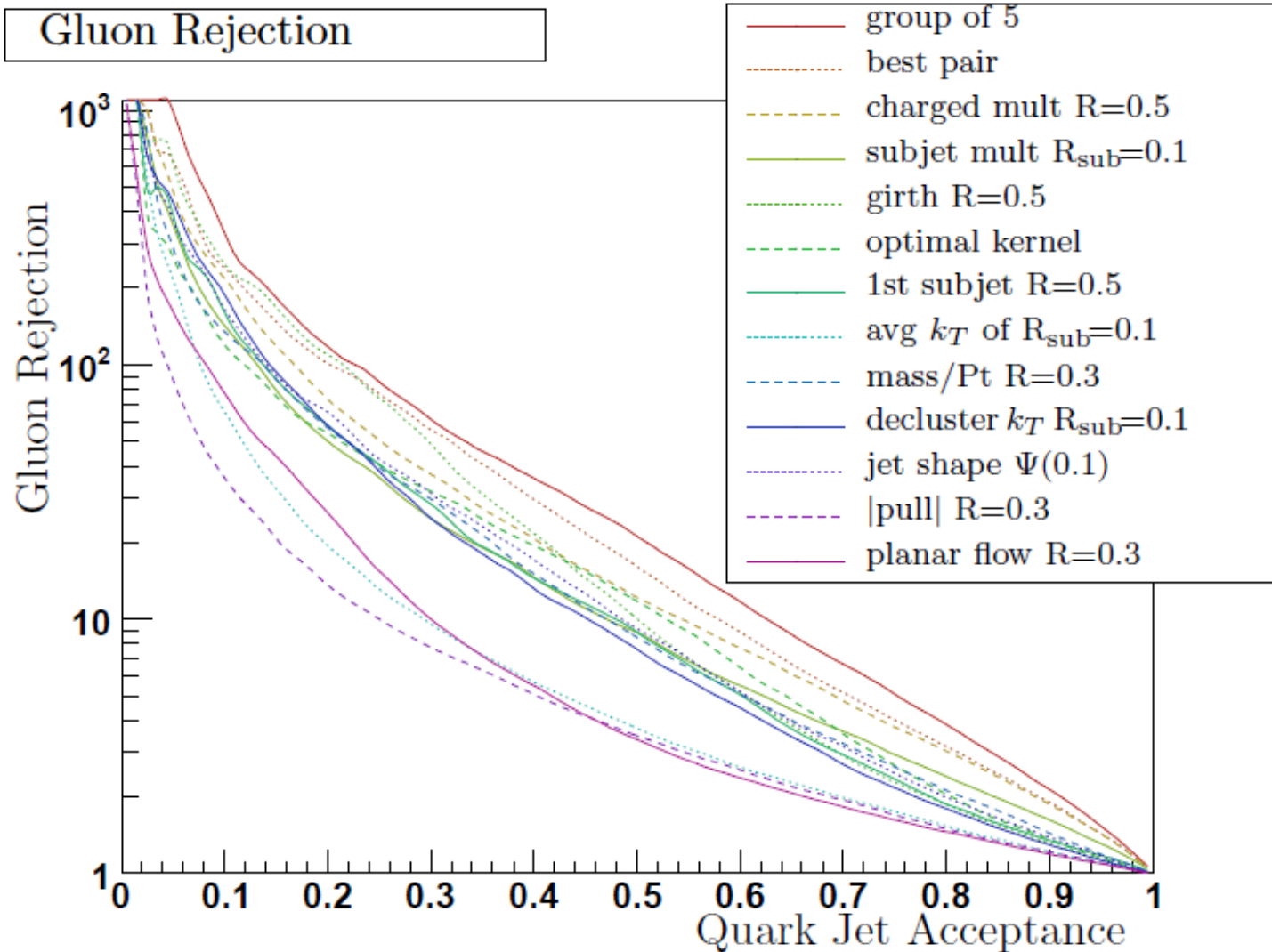
- ✗ For gluons

- + Look at b+2 jets

- + look at trijets

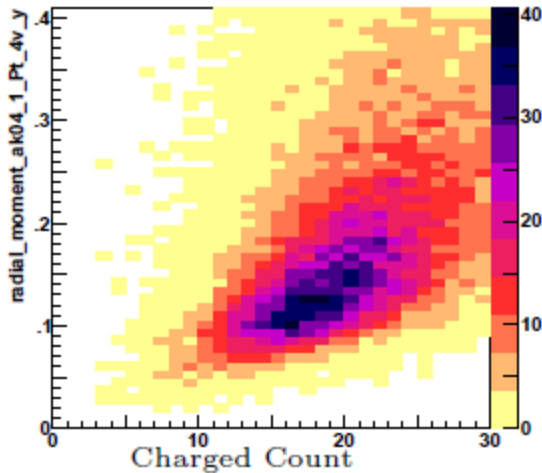
- ✗ Cut on $|\eta_{j3}| - |\eta_{j1} - \eta_{j2}|$

HOW TO TELL QUARKS FROM GLUONS

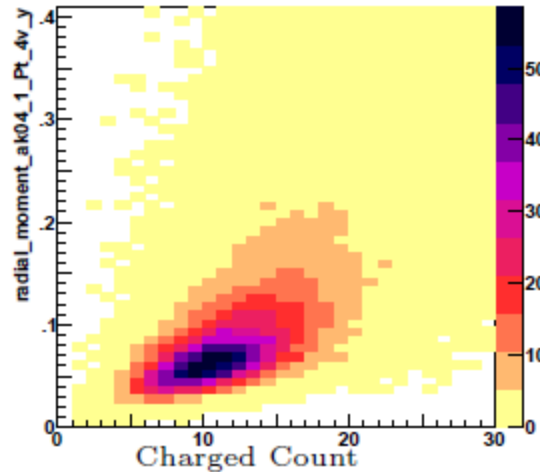


BEST PAIR WORKS PRETTY GOOD

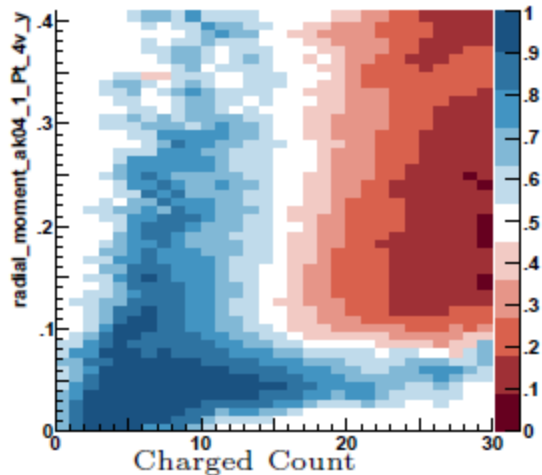
Gluon



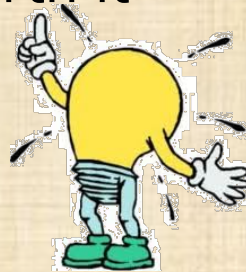
Quark



Likelihood: $q/(q + g)$



- Can get 50% quarks and 4% gluons
- Need one “count” variable and one “moment” variable
- Beyond that, probably not worth it



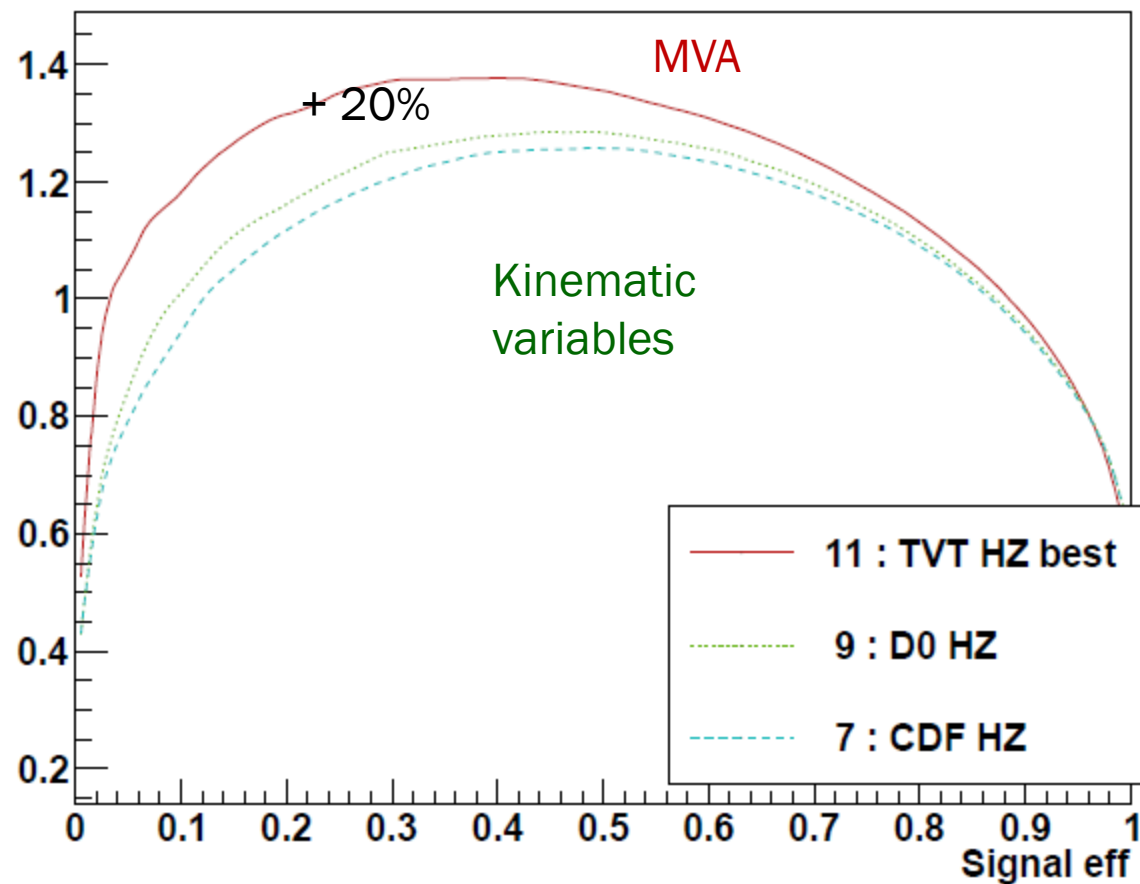
Sometimes one variable is good enough

but

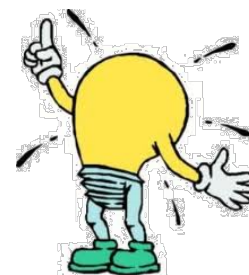
sometimes many variables are needed

MVA POWERFUL FOR HIGGS SEARCH

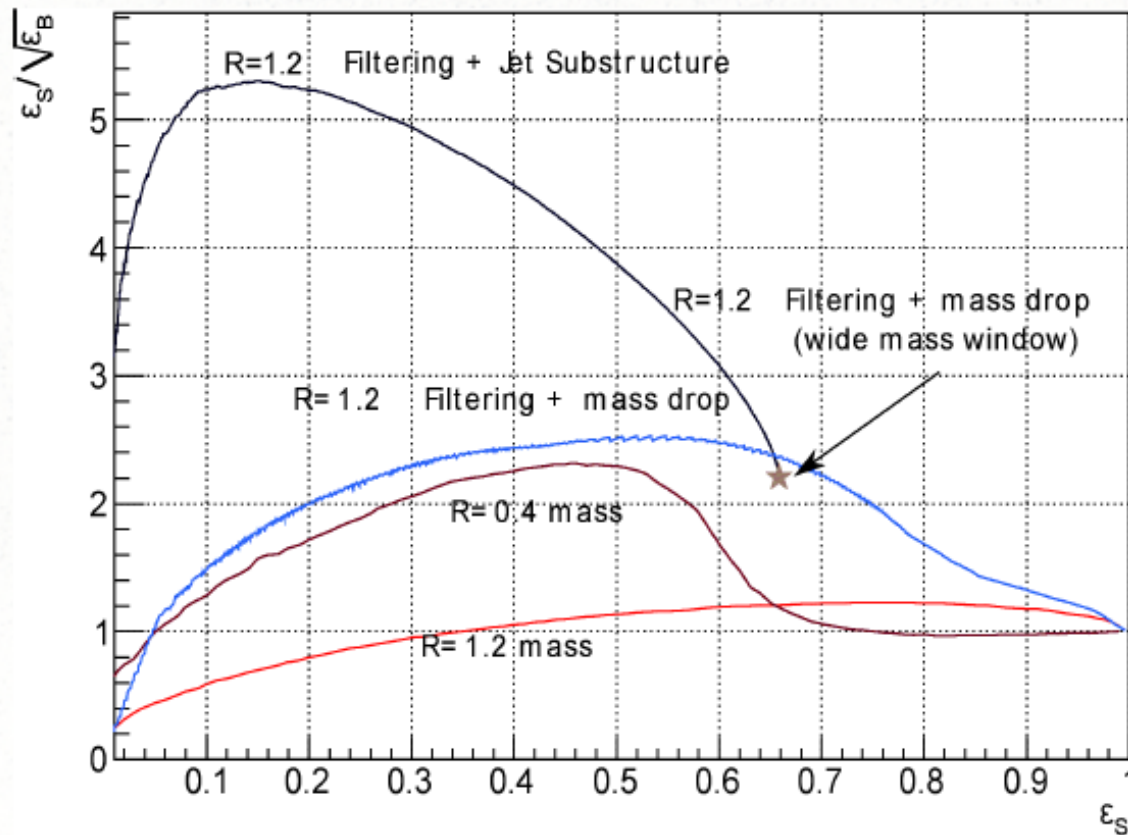
TVT HZ : Significance



Every little bit helps

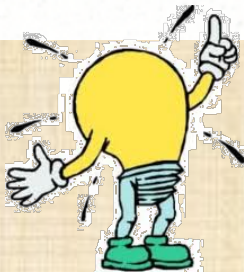


OPTIMIZE W TAGGING



7 MC insensitive variables

1. Jet mass with $R=0.5$
2. Jet mass with $R=0.4$
3. Filtered Jet mass
4. Mass of hardest subject, from filtering
5. Mass of second hardest subject, from filtering
6. Ratio of the pT's of the two hardest subjects
7. Planar Flow

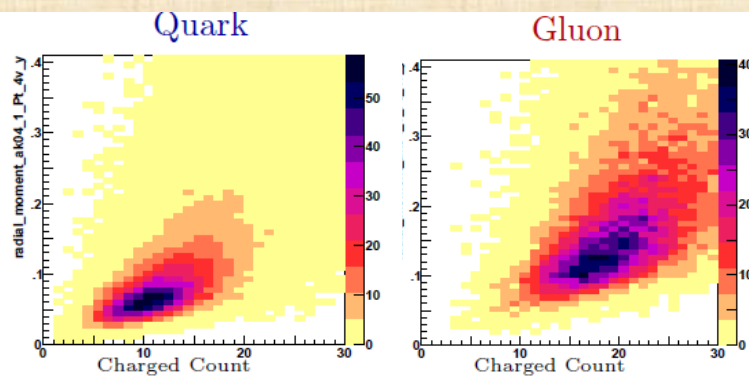
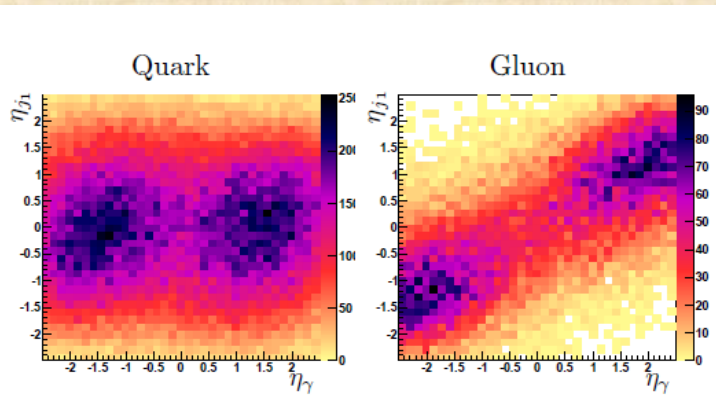


This is the **ultimate goal** of W-tagging
Challenge: can fewer variables do as well?

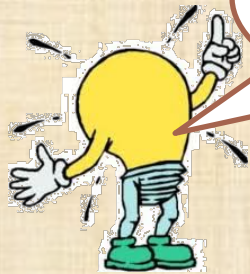
GENERAL OBSERVATIONS

- For matrix-element level (fixed # dof)
 - BDT much **easier to implement** than matrix element method
 - MVA kinematic discriminants can be **similar to cutting on one** smart variable (e.g. in finding pure samples of Quark and Gluon jets)
- For showered samples (W-tagging, HZ)
 - **Subtle correlations** make **thinking difficult**
 - Some discriminants (like pull) **don't work on their own** but **work well as the 5th or 6th variable** added to a BDT
 - Sometimes **few variables are enough** (e.g. Quark vs Glue)
 - Sometimes **many variables are needed** (e.g. W-tagging)
- We desperately need data on 2d correlations!!

Most useful
with **pure
samples**
(quark, glue,
hadronic-W,
hadronic-top)

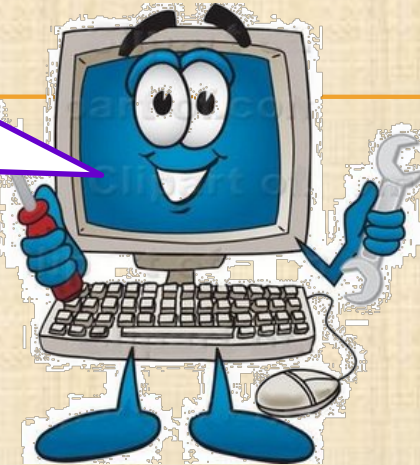


CONCLUSIONS



Boosted Decision Trees
are here to **help** you

Let me do the
work for you!



Multivariate analysis quickly tells you how well you could **possibly do**

FRAMING

See if simple variables
can do as well

POWER

Sometimes MVA is really necessary

EFFICIENCY

Saves you the trouble or looking
for good variables

- Need to have **correlations in real data**
 - If 2D distributions agree with the Monte Carlo, the BDTs can be trusted
 - If they don't agree, we have a great opportunity to improve the MCs