Hidden Logarithms in Heavy Jet Mass

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Based on
arXiv:2506.xxxxx with A. Hoang, V. Mateu, I Stewart
arXiv:2502.12253 with M. Benitez, A Bhattacharya, A. Hoang, V. Mateu, I Stewart and X. Zhang.
arXiv:2205.05702 (PRD106.074011) with A Bhattacharya and X. Zhang
arXiv:2306.08033 with A Bhattacharya, J. Michel, I Stewart and X. Zhang
arXiv:1403.4949 with H-X Zhu
arXiv:1105.3676 with R. Kelley, R. Schabinger, and H-X Zhu
arXiv:1005.1644 with Y-T. Chien

Thrust and Heavy Jet mass

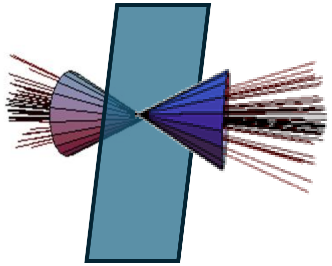
Two similar measures of energy distribution in an e+e- event

$$\tau = 1 - \max_{\vec{n}} \frac{\sum |\vec{p}_j \cdot \vec{n}|}{\sum |\vec{p}_j|}$$

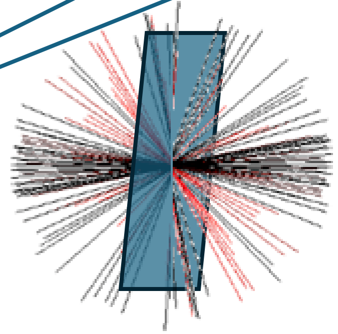
$$\rho = \max\left(\frac{(P^+_\mu)^2}{Q^2}, \frac{(P^-_\mu)^2}{Q^2}\right)$$

$$P^\pm_\mu = \sum_{\pm \vec{p}_j \cdot \vec{n} > 0} p^\mu_j$$

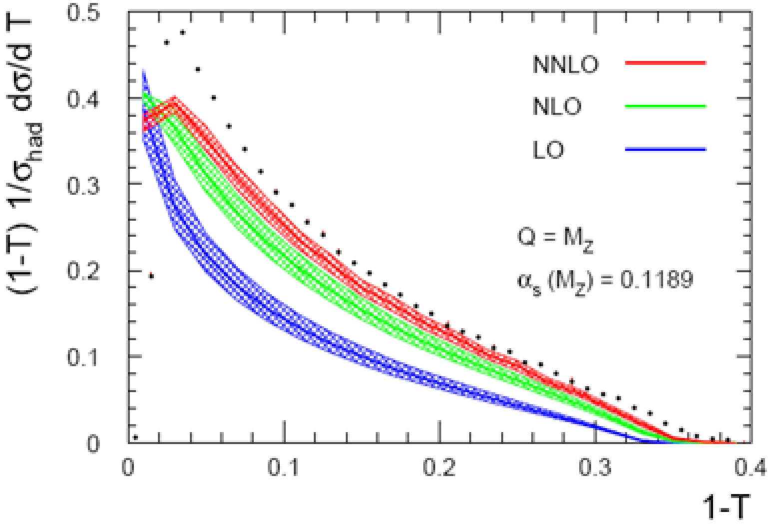
thrust axis finds most collimated plane

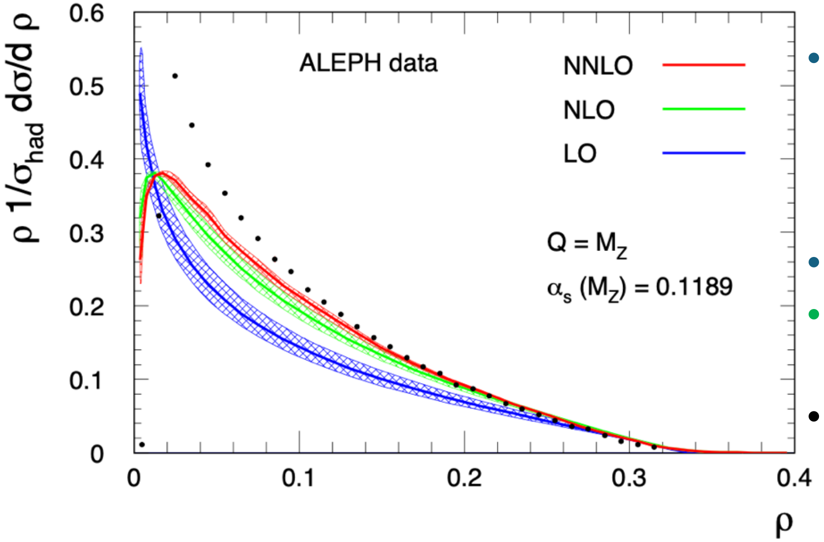


hemisphere momenta/mases determined by thrust axis



Gehrmann–De Ridder et al 2007





- Thrust and HJM identical at LO

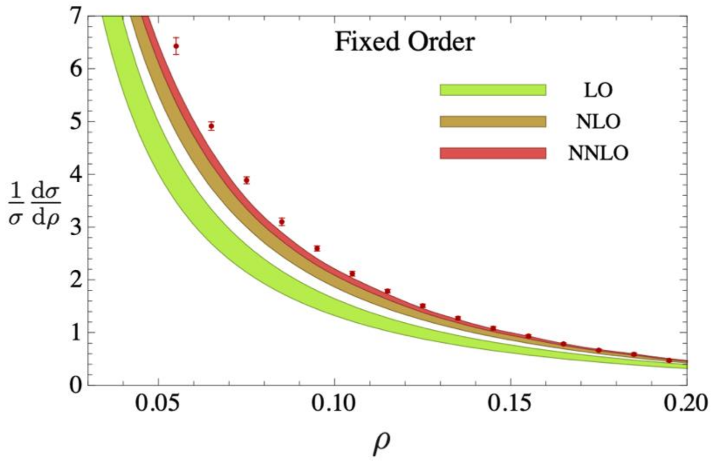
$$\frac{d\sigma}{d\rho} = \frac{d\sigma}{d\tau} = \delta(x) + \frac{\alpha}{2\pi}\left(\frac{4}{x}\ln x + \cdots\right)$$

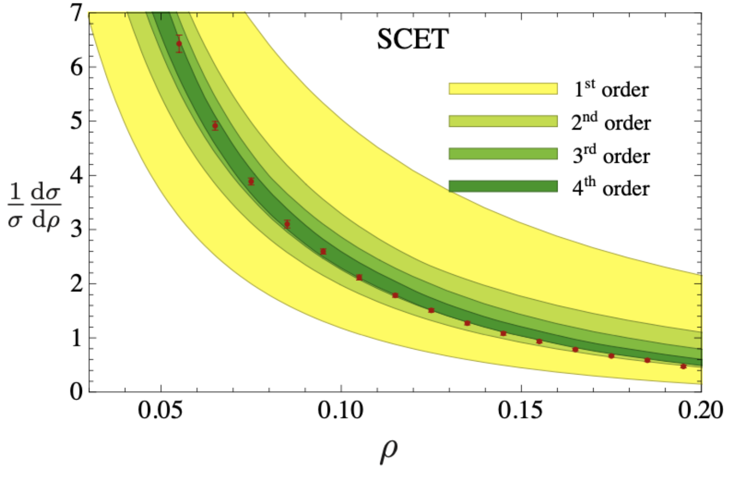
- Thrust and HJM data qualitatively similar
- NLO (1981), NNLO (2007) theory qualitatively similar
- FO convergence is horrible

Resummation

- convergence much improved (thrust and HJM similar)

- NLL resummation [Catani et al 1993]
- NNNLL resummation
  - thrust [MDS and Becher 2008]
  - HJM [MDS and Chien 2010]





Fits to data

Fixed order (combined) $\alpha_s = 0.1240 \pm 0.0040$ [Dissertori et al 2007]

with resummation

Thrust $\alpha_s = 0.1175 \pm 0.0026$ [MDS, Becher 2008]

HJM $\alpha_s = 0.1220 \pm 0.0031$ [MDS, Chien 2010]

with power corrections

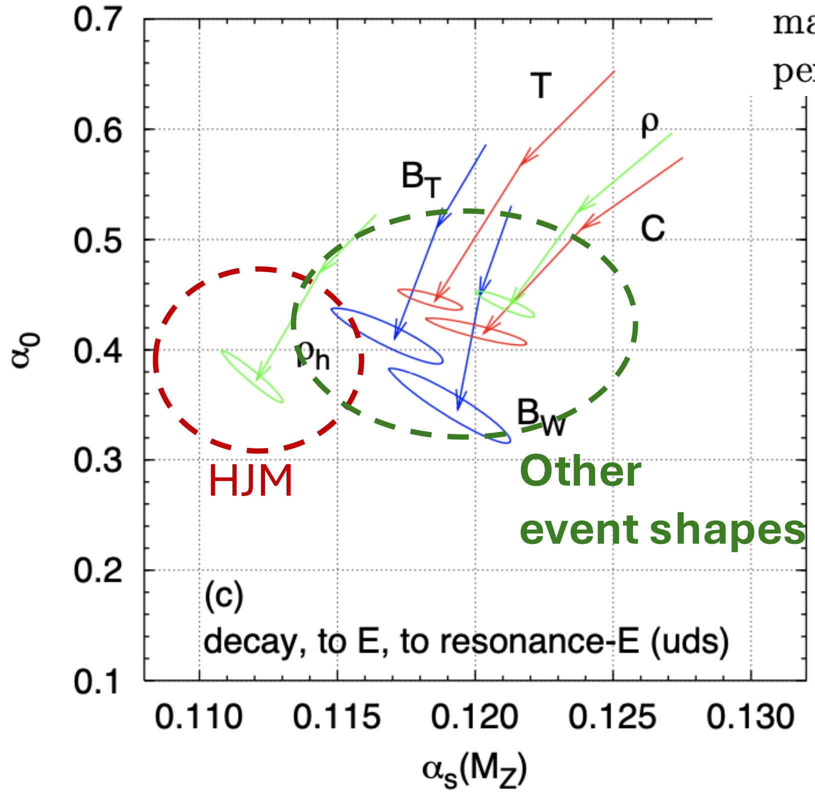
$\alpha_s(m_Z) = 0.1136 \pm 0.0012$ [Abbate et al. 2010] [Benitez et al. 2024]

HJM = ???

HJM is an outlier

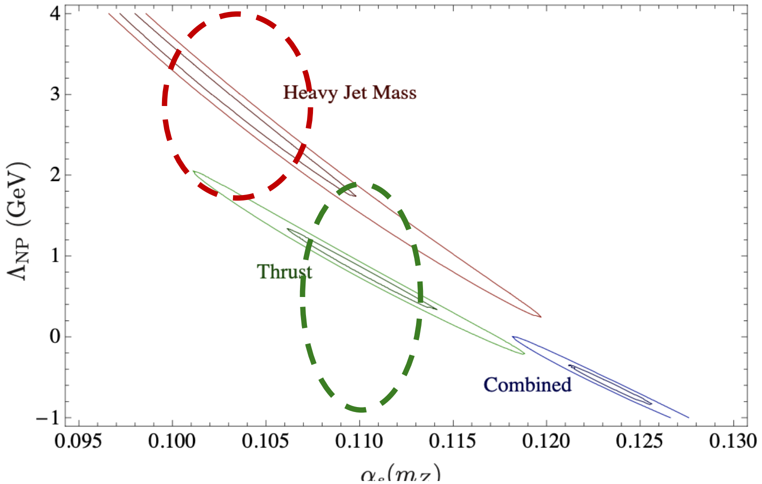
[Salam and Wicke 2001]

NLL + NLO filts



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders. But in appendix D there is evidence from Monte Carlo

- Inconsistency still there at NNNLL + NNLO [Chien and MDS 2010]



Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	χ^2/d.o.f.
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

- very low α_s
- conflict with thrust?

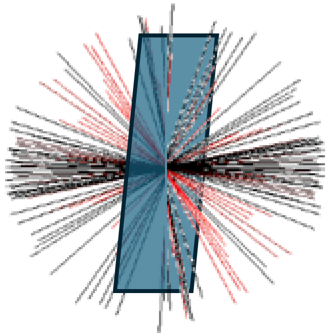
What’s the difference between τ and ρ?

1. Non-global Logarithms
2. Sudakov Shoulders
3. Negative Power Corrections

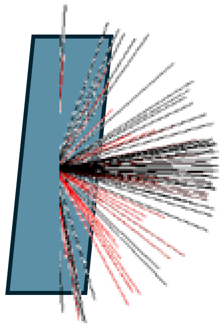
Are we fitting sensibly?

4. Theoretical uncertainty
5. Dependence on fit range

1. Non-global logarithms



Thrust is global
- All particles contribute



HJM is weakly non-global
- Only particles in heavy hemisphere contribute
- Non-global logs in heavy/light jet mass integrated over

[Dasputa and Salam 2001]
- logarithms from incomplete real/virtual cancellation

Hemisphere soft function

projection

matrix element of Wilson lines

$$S(k_L, k_R, \mu) \equiv \frac{1}{N_c} \sum_{X_s} \delta(k_R - n \cdot P_s^R) \delta(k_L - \bar{n} \cdot P_s^L) \langle 0 | \overline{Y}_{\bar{n}} Y_n | X_s \rangle \langle X_s | Y_n^\dagger \overline{Y}_{\bar{n}}^\dagger | 0 \rangle \,,$$

$$\sim \delta(k_L)\delta(k_R) + \alpha^2 C_F^2 \left[\ln^2 \frac{k_L}{k_R} \right]_+ + \cdots$$

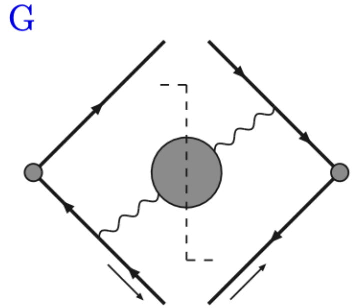
$$S_T(k, \mu) = \int \mathrm{d}k_L \mathrm{d}k_R S(k_L, k_R, \mu) \delta(k - k_L - k_R) \,.$$

HJM projection
- cannot write as a single-scale soft function
- NGLs do not completely drop out

thrust projection
- can write a single soft function
- NGLs completley drop out

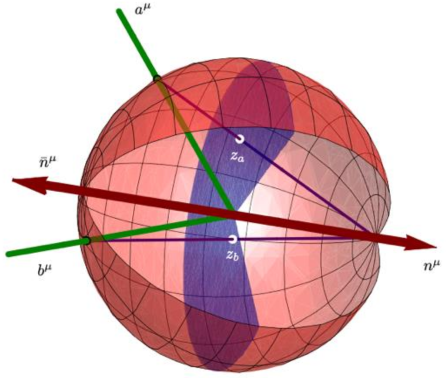
Non-global logarithms

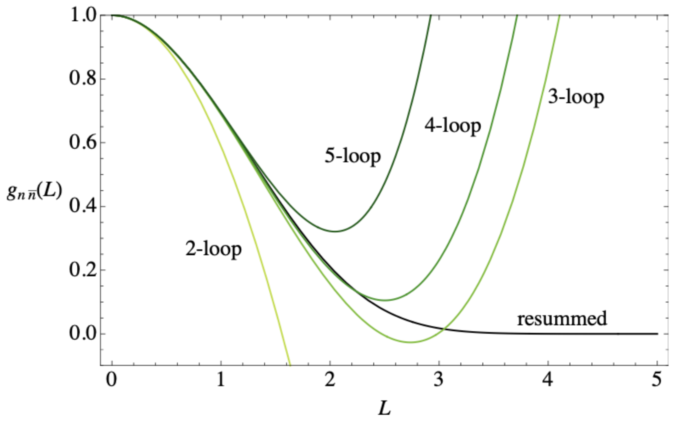
- Exact 2-loop hemisphere soft function [Kelley, MDS, et al 2011, Hornig et al 2011, Monni et al 2011]



$$\mathcal{R}_f(z) = \frac{\pi^4}{2}C_F^2 + \Big[-88\mathrm{Li}_3(-z) - 16\mathrm{Li}_4\left(\frac{1}{z+1}\right) - 16\mathrm{Li}_4\left(\frac{z}{z+1}\right) + 16\mathrm{Li}_3(-z)\ln(z+1)$$
$$+\frac{88\mathrm{Li}_2(-z)\ln(z)}{3} - 8\mathrm{Li}_3(-z)\ln(z) - 16\zeta(3)\ln(z+1) + 8\zeta(3)\ln(z) - \frac{4}{3}\ln^4(z+1)$$
$$+\frac{8}{3}\ln(z)\ln^3(z+1) + \frac{4}{3}\pi^2\ln^2(z+1) - \frac{4}{3}\pi^2\ln^2(z) - \frac{4\left(3(z-1)+11\pi^2(z+1)\right)\ln(z)}{9(z+1)}$$
$$-\frac{506\zeta(3)}{9} + \frac{16\pi^4}{9} - \frac{871\pi^2}{54} - \frac{2032}{81}\Big]C_F C_A + \Big[32\mathrm{Li}_3(-z) - \frac{32}{3}\mathrm{Li}_2(-z)\ln(z)$$
$$+\frac{8(z-1)\ln(z)}{3(z+1)} + \frac{16}{9}\pi^2\ln(z) + \frac{184\zeta(3)}{9} + \frac{154\pi^2}{27} - \frac{136}{81}\Big]C_F n_f T_F \qquad (53)$$

- Resummation of large logs at LL level [Banfi et al 2002]
- Fixed order computation of large logs to 5 loops [MDS and HX Zhu]



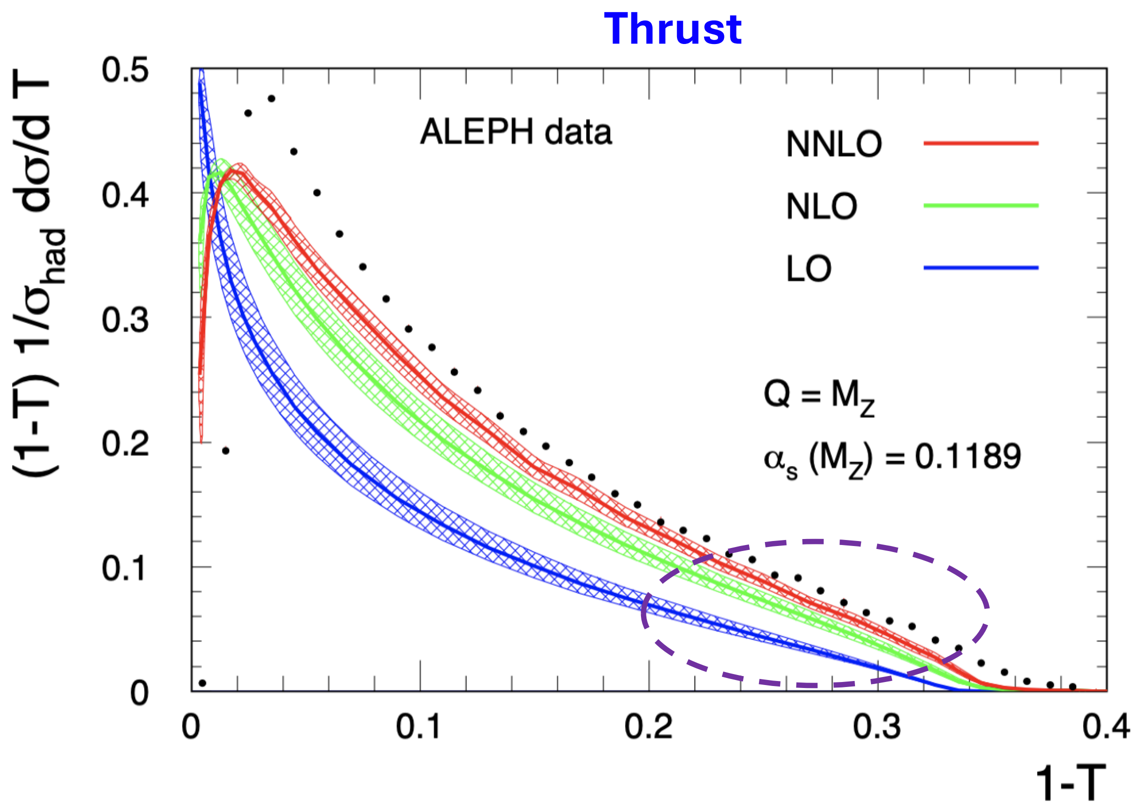


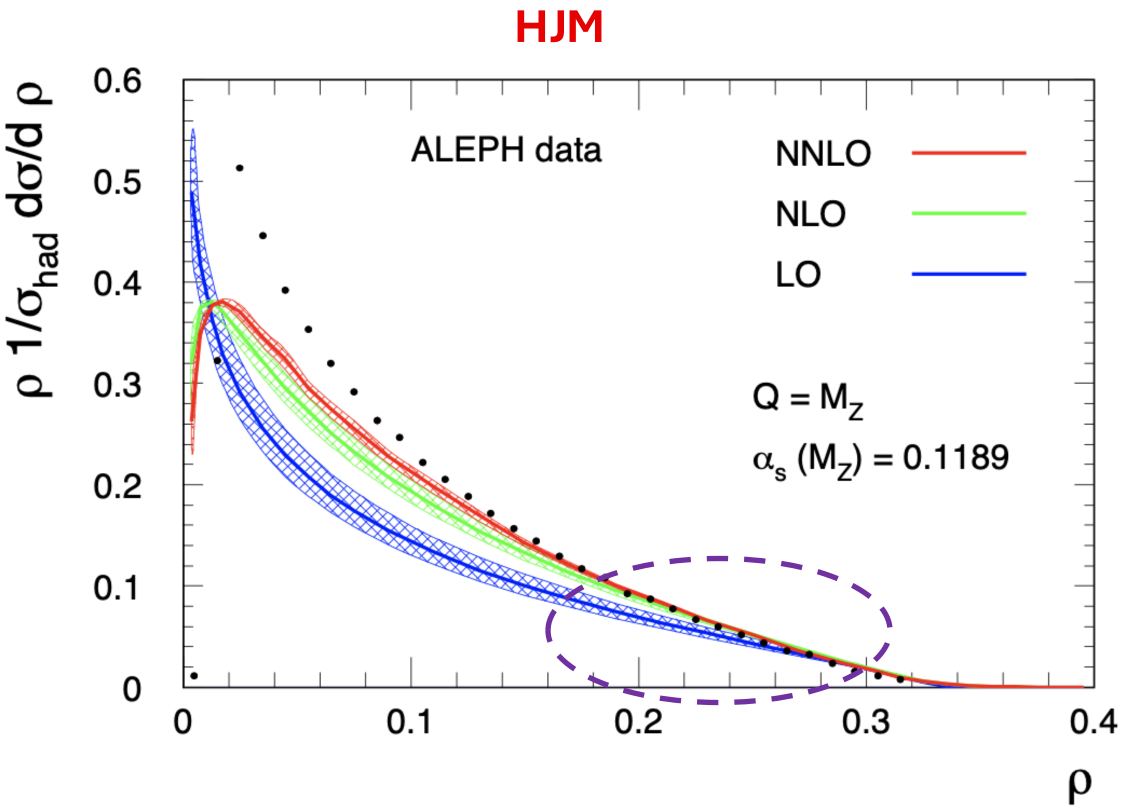
Now can include in fits
- Full 2-loop hemisphere soft function
- LL resummation

$$g_{n\bar{n}}(L) = 1 - \frac{\pi^2}{24}L^2 + \frac{\zeta(3)}{12}L^3 + \frac{\pi^4}{34560}L^4 + \left(-\frac{\pi^2\zeta(3)}{360} + \frac{17\zeta(5)}{480}\right)L^5 + \dots$$

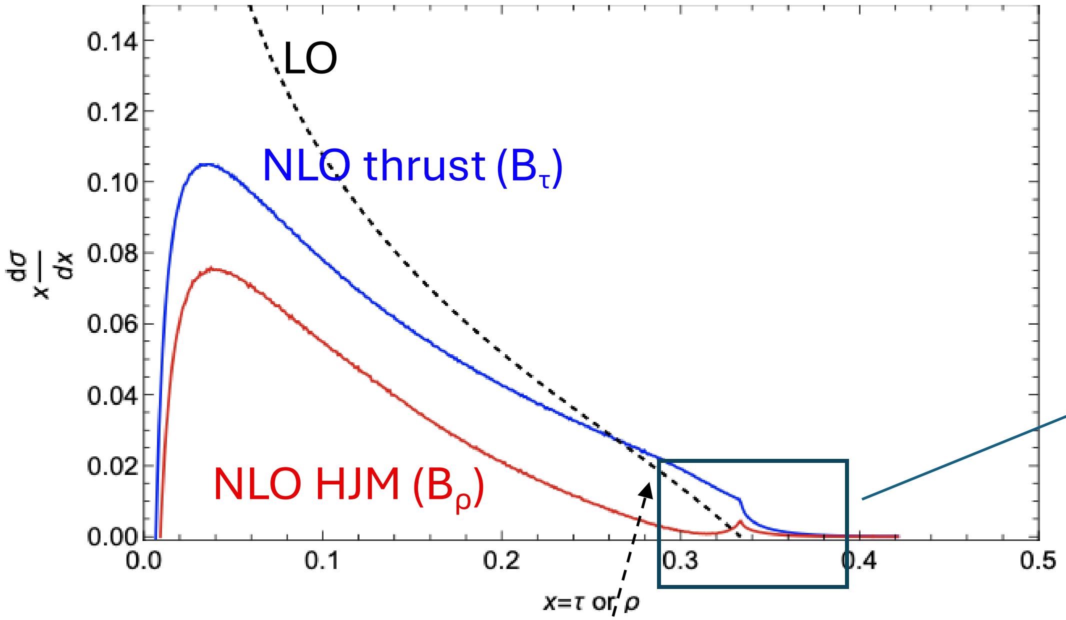
2. Sudakov shoulders

Data for thrust seems matches ***shape*** of NNLO theory better than HJM in the far tail





Zoom in on tail region at fixed order



Leading order has a linear kink

$$\left(\frac{d\sigma}{d\tau}\right)_{\rm LO} = \left(\frac{d\sigma}{d\rho}\right)_{\rm LO} \sim x\,\theta(x)$$

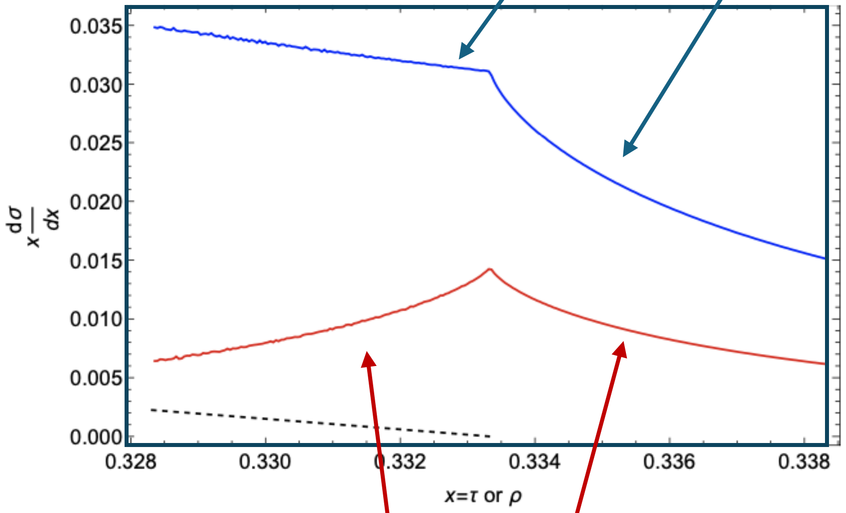
in thrust or HJM

$x = \frac{1}{3} - \tau$ $\quad x = \frac{1}{3} - \rho$

Thrust: $\left(\frac{d\sigma}{d\tau}\right)_{\rm NLO} \sim \alpha_s^2\Big[\, x\,\theta(x) + \ln^2(-x)\theta(-x)\Big]$

linear kink (no logs)

sudakov shoulder (logs)



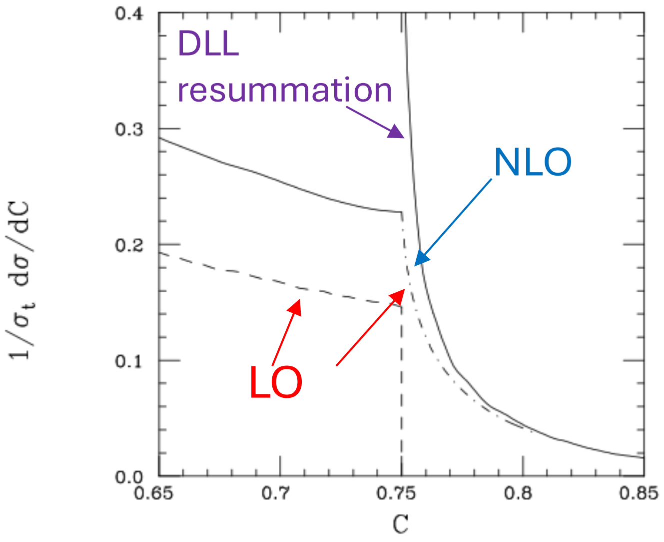
HJM has **left** and **right** Sudakov shoulders

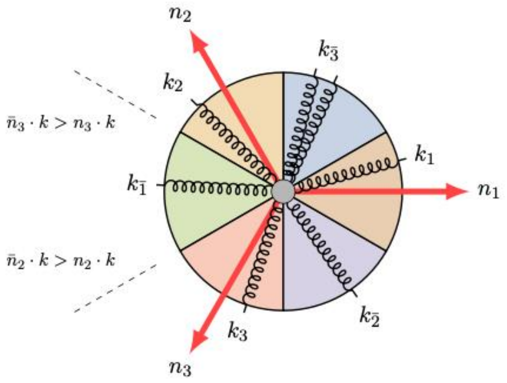
$$\left(\frac{d\sigma}{d\rho}\right)_{\rm NLO} \sim \alpha_s^2\Big[\, \ln^2 x\,\theta(x) + \ln^2(-x)\theta(-x)\Big]$$

Sudakov Shoulders

[Catani and Webber 1997]

- Sudakov Shoulders arise from finite matrix elements at phase-space boundaries
- Double-logarithmic resummation of **C-parameter** shoulder





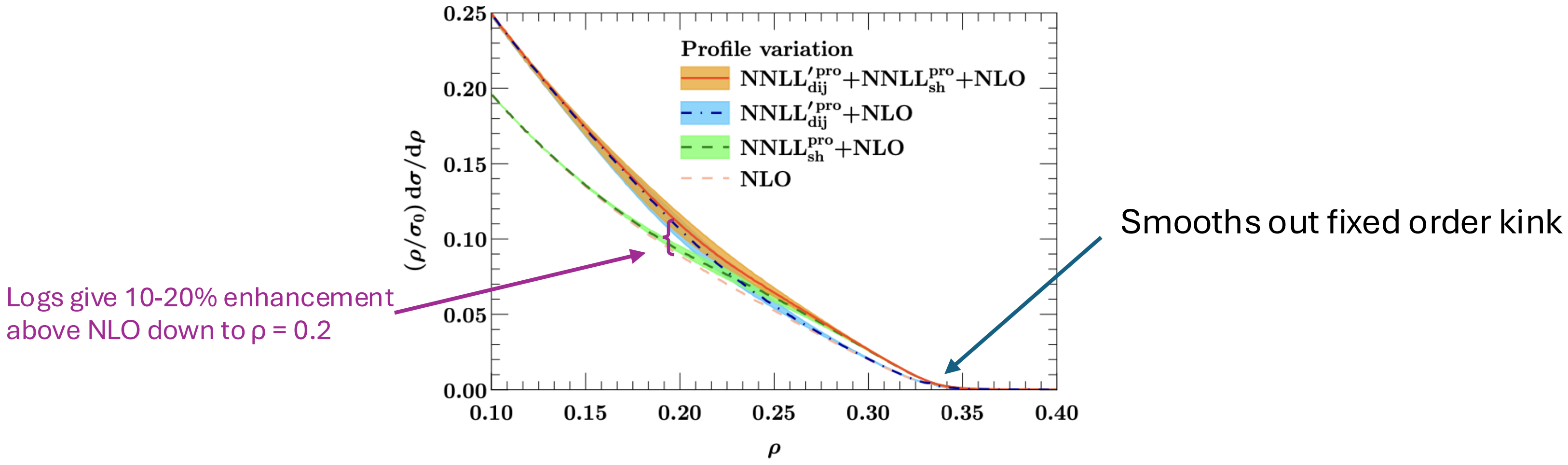
[MDS, Bhattacharya, Zhang (2022)]

- Factorization theorem in SCET
- NNLL resummation for **thrust** and HJM

$$\frac{1}{\sigma_1}\frac{d\sigma}{dr} = H(Q)\int d^3m^2 d^6q\, J(m_1^2)J(m_2^2)J(m_3^2)S_6(q_i)W(m_j,q_i,r)\theta[W(m_j,q_i,r)]$$

[MDS, Bhattacharya, Zhang, Stewart, Michel (2023)]

- Position space resummation
- Matching to dijet resummation

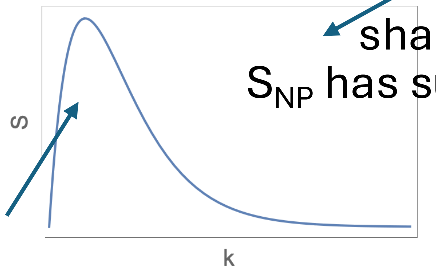


3. Power corrections

SCET approach to power corrections

soft function = matrix element of Wilson lines = pert + non-pert shape function

$$S(k) = \langle 0|Y_n Y_n^{\dagger} Y_n Y_n^{\dagger}|0\rangle = \int dk' S_P(k-k') S_{\rm NP}(k')$$



shape function

S_{NP} has support $O(\Lambda_{QCD})$

$$\Omega_1 \sim \langle 0|Y_n Y_n^{\dagger} D Y_n Y_n^{\dagger}|0\rangle$$

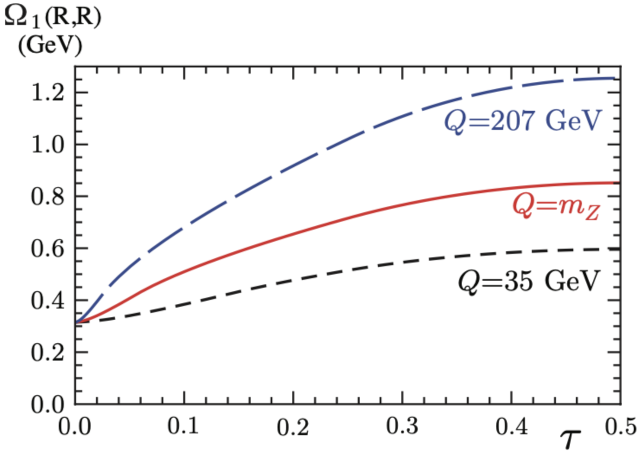
approximate S_{NP} as δ-function

$$S_{\rm NP}(k) \approx \delta(k-\Omega_1)$$

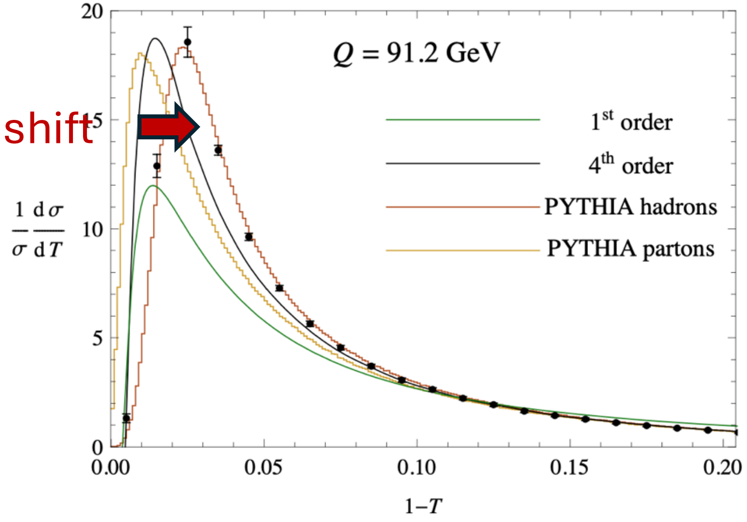
leading NP effect is a shift

$$S(k) = S_P(k-\Omega_1)$$

- Leading power correction has operator definition
- Extrapolation away form dijet limit using R-evolution [Hoang et al. 2008]



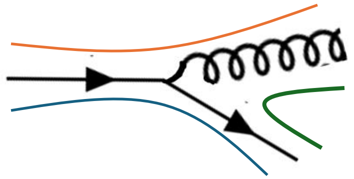
- trust up to where dijet breaks down (τ~ρ~0.15)



Gluer approach

[Lusioni et al 2020]
[Caola et al 2021, 2022]
[Nason and Zanderighi 2023, 2025]

- Approximate 3-parton configurations as dipoles



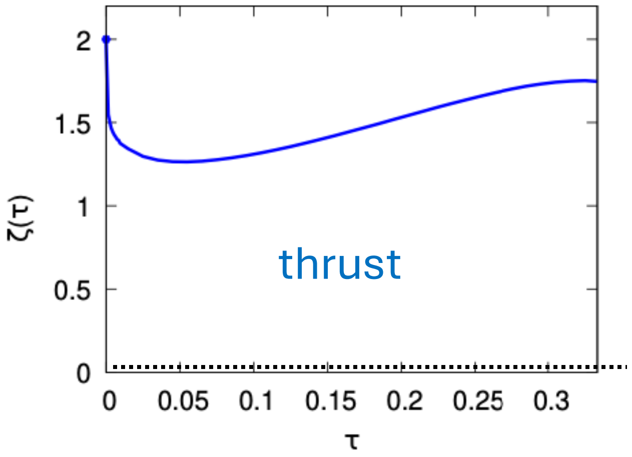
soft emission

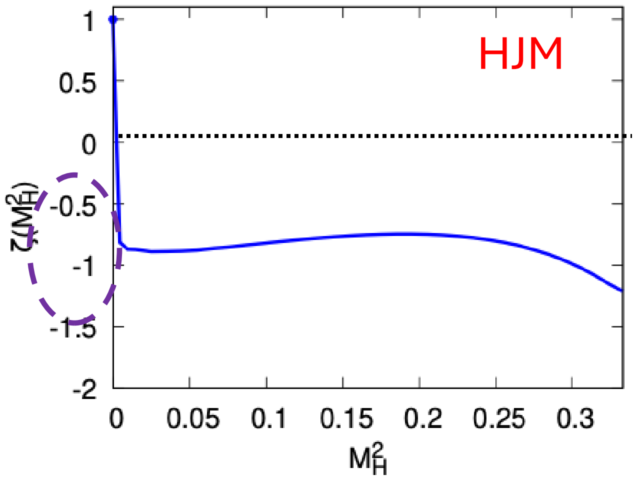
$$\delta\tau = k^0 f(s,t)$$

shift in thrust proportional to soft energy

- Leading order distribution comes from 3-parton configurations
- Compued weighted average shift

$$\Omega_1(\tau) = \Lambda\cdot\zeta(\tau) \sim \Lambda\cdot\int ds\,dt\left(\frac{d\sigma}{ds\,dt}\right)_{\mathrm{LO}}\delta(\tau-\tau(s,t))\frac{\delta\tau(s,t)}{k_0}$$



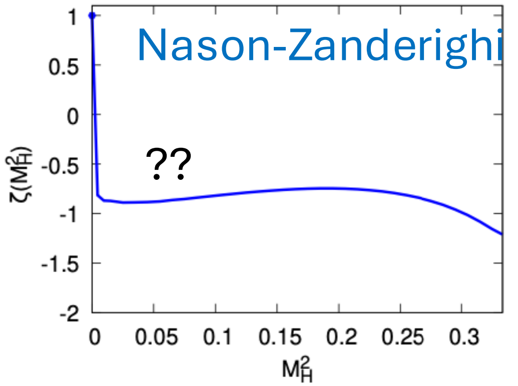


- Insipred by large N_f renormalon arguments
  - Actual calculation doesn't need renormalons
- ζ for thrust and HJM are roughly constant
- HJM shift is negative for all ρ

Concerns:

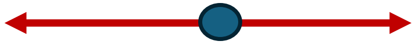
- Pertubative calculation of a NP effect
- No operator definition of Λ
  - Corrections are order 1
- Needs ρ~10^{-6} to match dijet limit

Shoulder power correction from SCET



SCET in dijet limit

2-Wilson line soft operator



$$\mathcal{S}_2 = Y_n Y_{\bar{n}}^\dagger$$

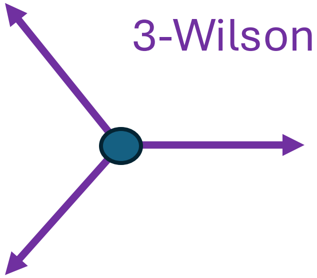
$$S(k) = \langle 0 | Y_n Y_n^\dagger Y_n Y_n^\dagger | 0 \rangle = \int dk' S_P(k-k') S_{\rm NP}(k')$$

NP shift away from ρ=0

$$S(k) = S_P(k - \Omega_1)$$

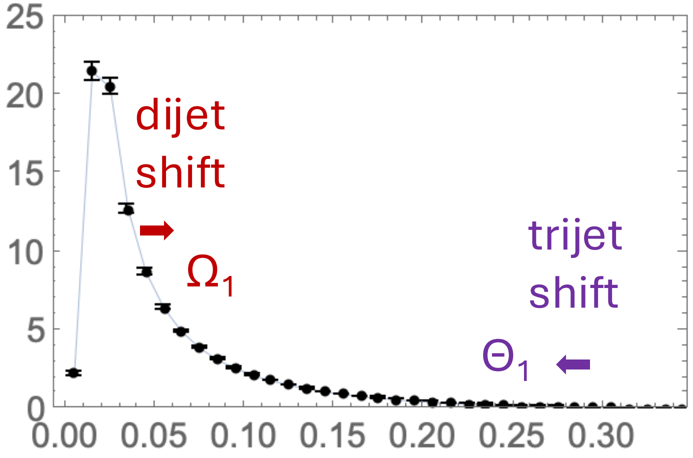
SCET in trijet (Sudakov shoulder) limit

3-Wilson line soft operator



$$\mathcal{S}_3 = Y_{n_1} \mathcal{Y}_{n_2} Y_{n_3}$$

NP shift away from ρ=1/3



Expect positive shift where dijet resummation is relevant (0 < ρ < 0.15)

NP parameter called Ω_1

- Expect negative shift

only where shoulder resummation is relevant (0.2 < ρ < 1/3)

NP parameter called Θ_1

[Mateu et al 2012]

we cannot relate $\Omega_1^\tau, \Omega_1^\rho, \Theta_1^\rho$

4. Theory uncertainty

There are as many ways to assess theory uncertainty as there are theorists

- Uncertainty Band, minimal scale variation, Brodsky-Lepage, trunctation-based, Pade-approxmant, random scan

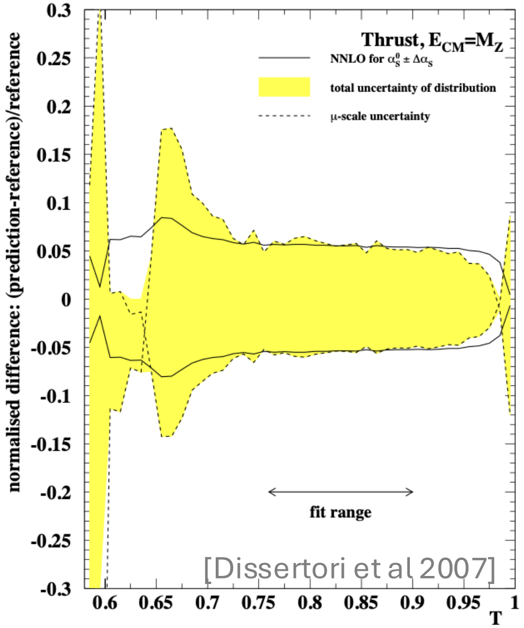
[Jones et al 2003, Dissertori et al 2007, MDS Becher 2008]

Used by ALEPH, OPAL, theorists

- Central value of α found using canonical theory parameters
- Minimize χ^2 using experimental errors only

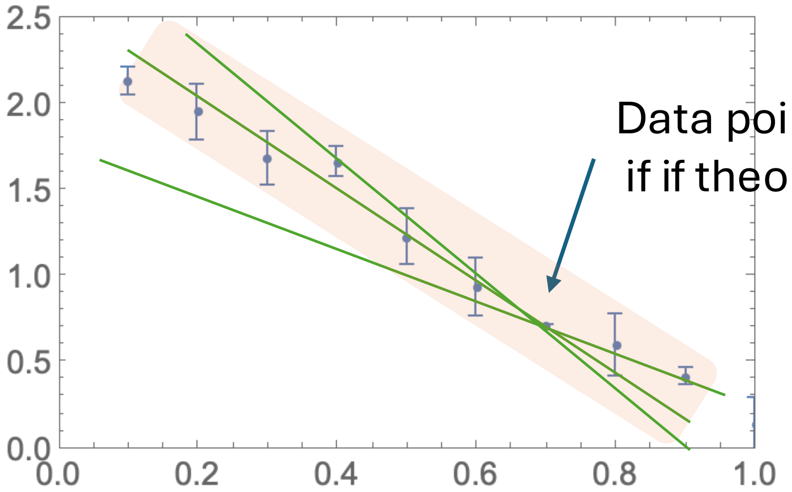
$$\chi^2(\alpha_s, \Omega, \vec{\eta}) = \sum_j \left(\frac{x_j^{\mathrm{th}} - x_j^{\mathrm{data}}}{\Delta_j} \right)^2$$

- Vary theory parameters to find band in prediction
- Vary α to stay within band



[Dissertori et al 2007]

Can lead to very bad results



Data points with tiny experimental error dominate fit if if theory error is large

- Cannot fit then find theory error afterwards
- Must include theory error during fitting

Random scan

- Decide some collection of sets of theory parameters to use (we use 5000)
- Distribute randomly as Gaussian or flat in region (doesn't matter much)

Minimize $$\chi^2 = \sum_{i,j=1}^{N_{\text{bins}}} (\bar{x}_i - x_i^{\text{exp}})\,(\bar{x}_j - x_j^{\text{exp}})\,(\sigma_{\text{tot}}^{-1})_{ij}$$

$$\sigma_{ij}^{\text{theo}} = \Delta_i^{\text{theo}}\,\Delta_j^{\text{theo}}\,r_{ij}^{\text{theo}}$$

$$\sigma_{\text{ij}}^{\text{tot}} = \sigma_{\text{ij}}^{\text{exp}} + \sigma_{\text{ij}}^{\text{theo}}$$

covariance matrix

minimum overlap model for experiment

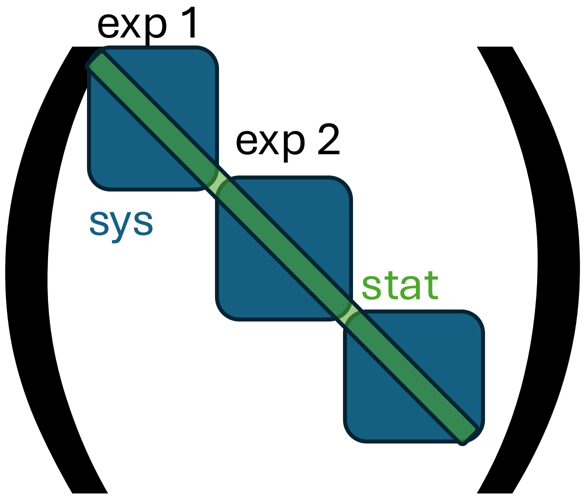
$$r_{ij}^{\text{theo}} = \frac{\langle (x_i - \bar{x}_i)(x_j - \bar{x}_j)\rangle}{\sqrt{\langle (x_i - \bar{x}_i)^2\rangle}\sqrt{\langle (x_j - \bar{x}_j)^2\rangle}}\,,$$

correlation coefficent among theoryvariations

$$\Delta_i^{\text{theo}} = (x_i^{\text{max}} - x_i^{\text{min}})/2$$

maximal variation

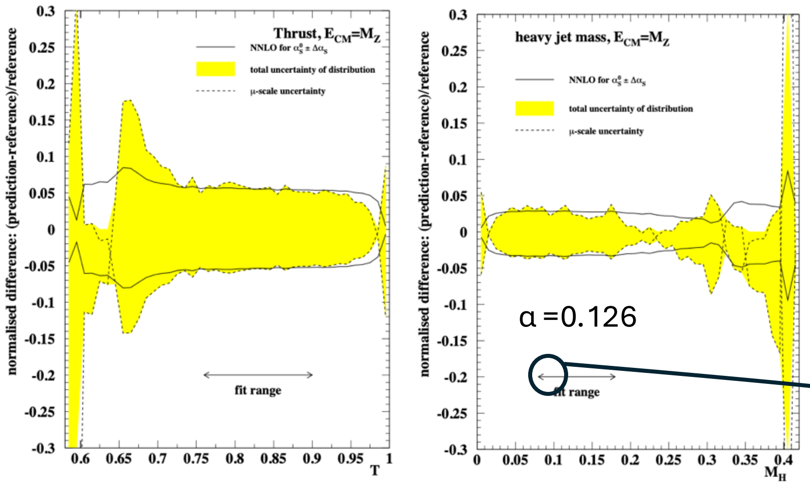
$$\sigma_{ij}^{\text{exp}} = \delta_{ij}(\Delta_i^{\text{stat}})^2 + \delta_{D_i D_j}\min(\Delta_i^{\text{sys}}, \Delta_j^{\text{sys}})^2\,.$$



Full covariance matrixed used for fitting has exp and theory uncertainties

5. Fit range

[Dissertori et al 2008]



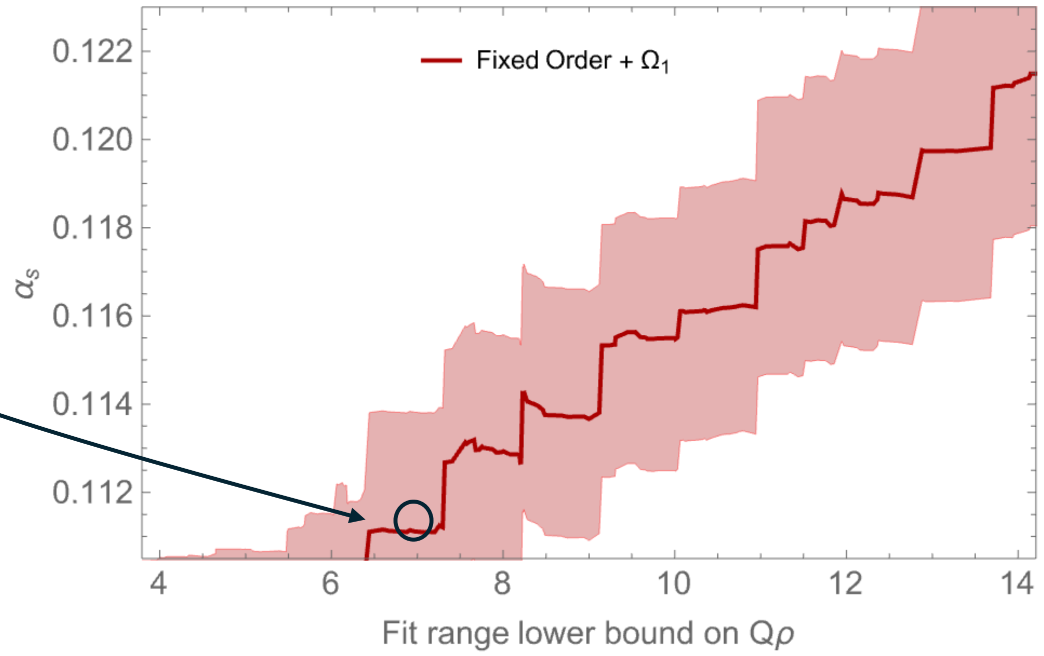
α =0.126

ρ > 0.08

ρQ =7.3

Varying the lower bound of the fit range

$$\frac{a}{Q} < \rho < 0.3$$



- Almost always a fixed fit range is used
- Chosen where theory is “accurate” (???)
- Error from fit range variation rarely included

[Nason & Zanderighi 2025]: fit range is problematic

5.2.2 Fit range

The computation of the fit range is discussed in section 2.3, and is controlled by a parameter C_{ll} that is set to two by default. We set C_{ll} to 1.5 and 3 to assess the effect of lowering/rising the lower limit. We know that our calculation must fail for very low lower limits, due to the raising importance of Sudakov logarithms. We thus expect that the χ^2 should become worse as we lower the lower limit, and be nearly constant as we raise it. This is in fact what we observe. By raising the limit the change in χ^2/d.o.f. is very small, and the fitted values of $\alpha(M_Z)$ and α_0 change roughly by 0.3% and 4% respectively. On the other hand, when lowering the limit we get a variation in $\alpha(M_Z)$ and α_0 of 1.4% and 6% respectively, accompanied by a sharp increase in χ^2/d.o.f., warning us not to venture further in that direction.

There is a very strong dependence on fit range at fixed order

Results

A Precise Determination of α_s from the Heavy Jet Mass Distribution

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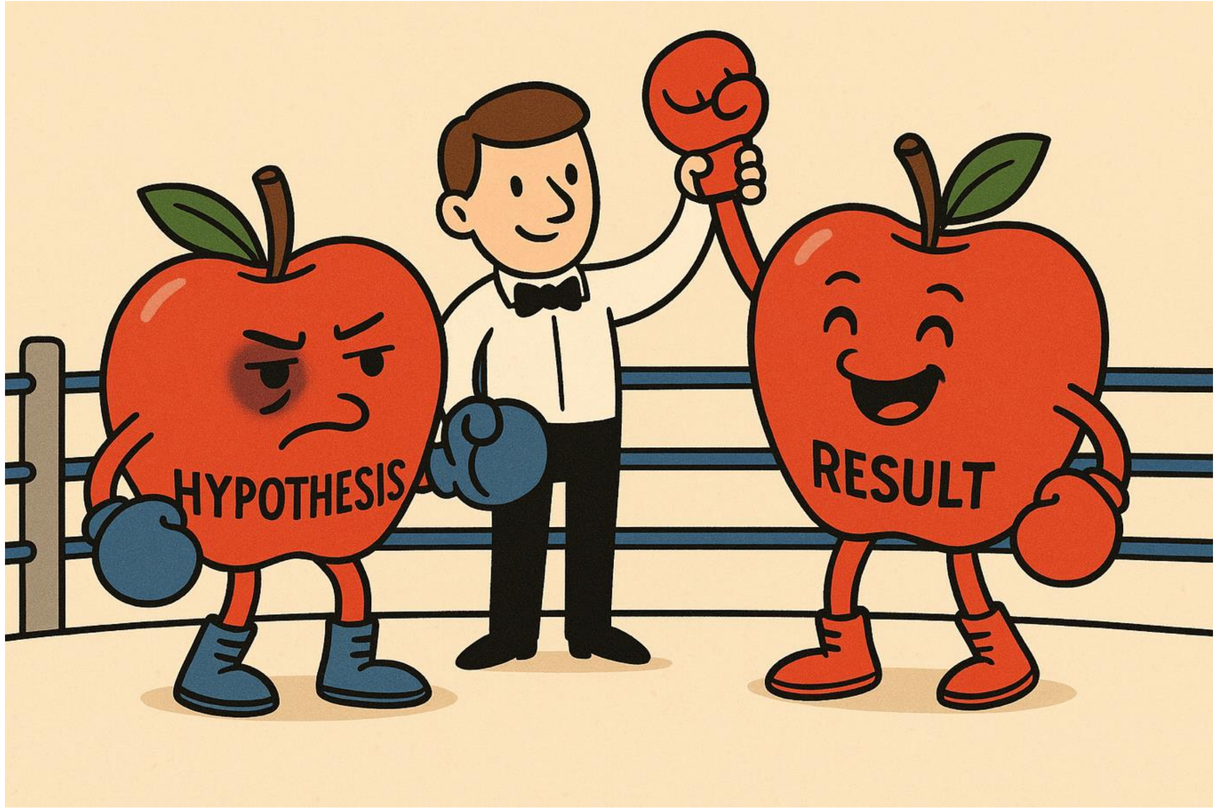
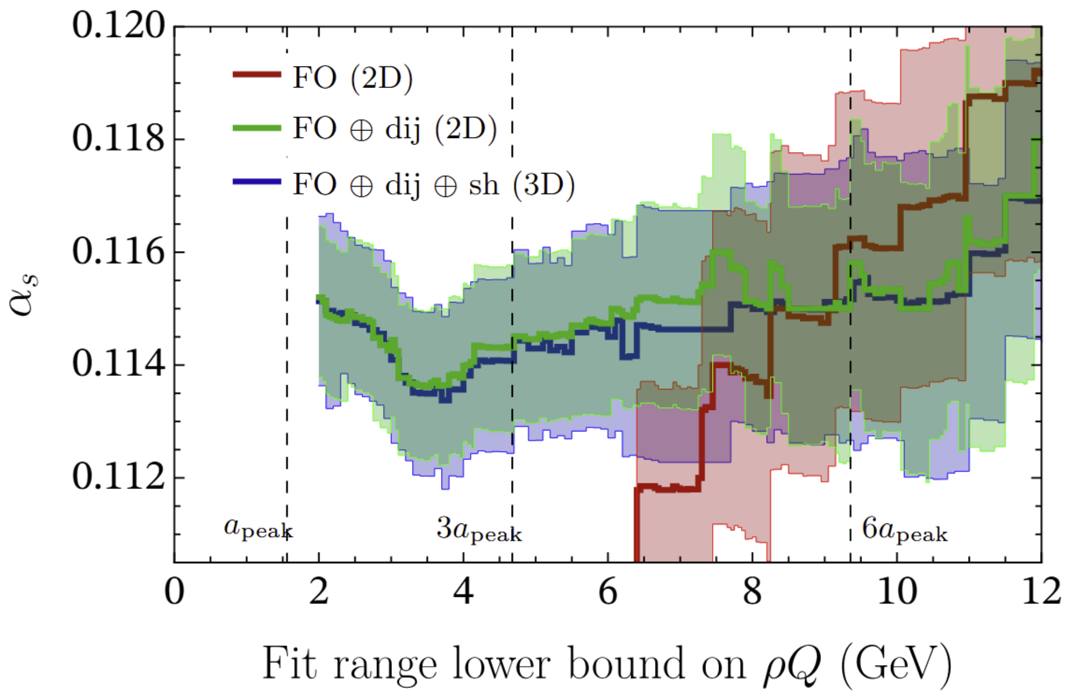
Vicent Mateu,[1] Matthew D. Schwartz,[2] Iain W. Stewart,[3,4] and Xiaoyuan Zhang[2]



α_s extraction from HJM data

- NNLO fixed order
- Shape function with R-evolution in dijet region
- Dijet resummation to NNNLL
- Flat random scan used for theory errors

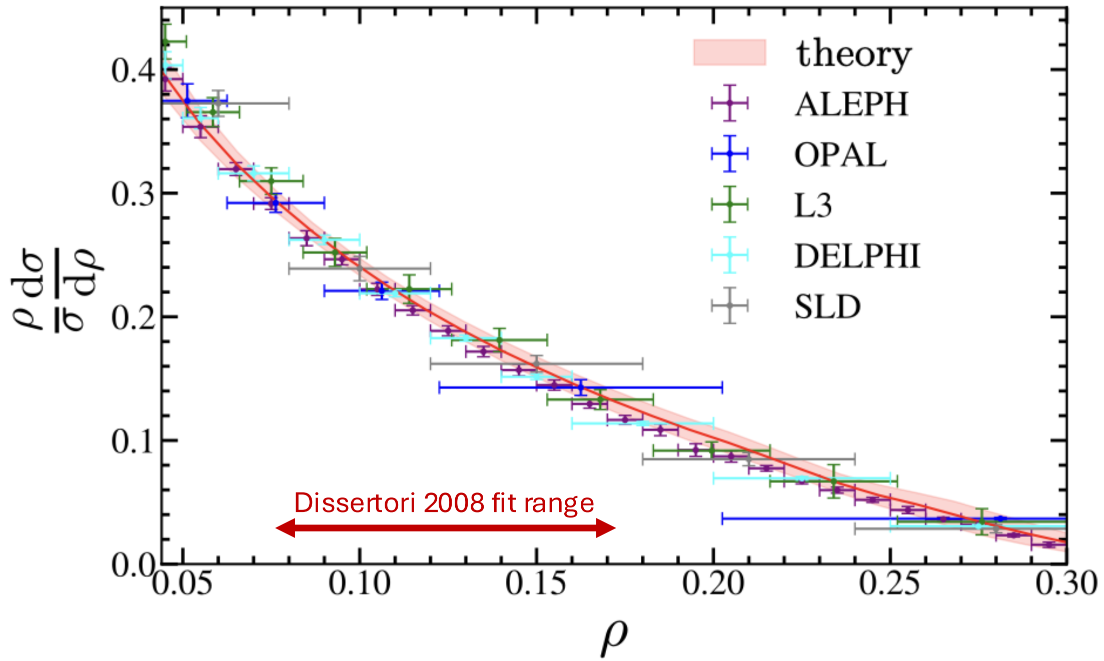


Now fit is insensitive to fit range!
Self-consistent for any lower cut

- Sudakov shoulder resummation to NNLL
- Extra NP paramter

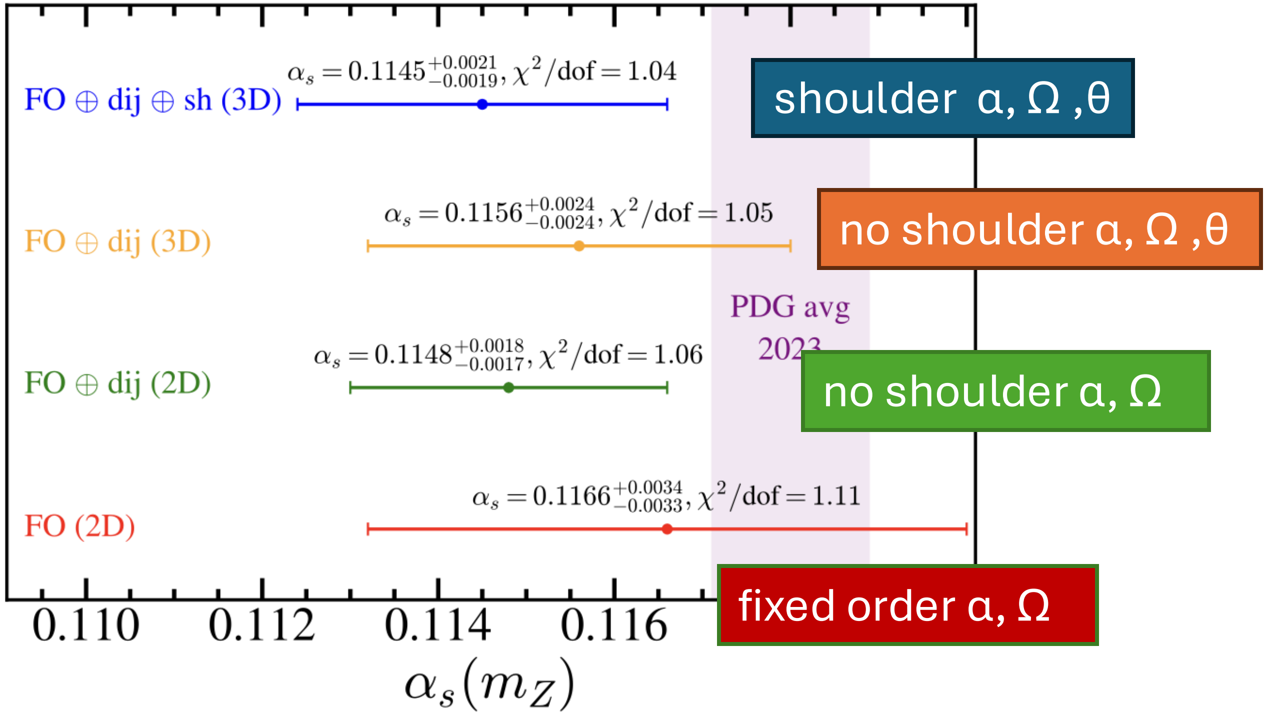
$$\alpha_s(m_Z) = 0.1145\,^{+0.0009}_{-0.0009}\,(\text{th+exp})\,^{+0.0019}_{-0.0016}\,(\Omega_1^\rho)\,^{+0.0001}_{-0.0001}\,(\Theta_1)\,^{+0.0003}_{-0.0003}\,(\text{fit range}) = 0.1145^{+0.0021}_{-0.0019}\,,$$

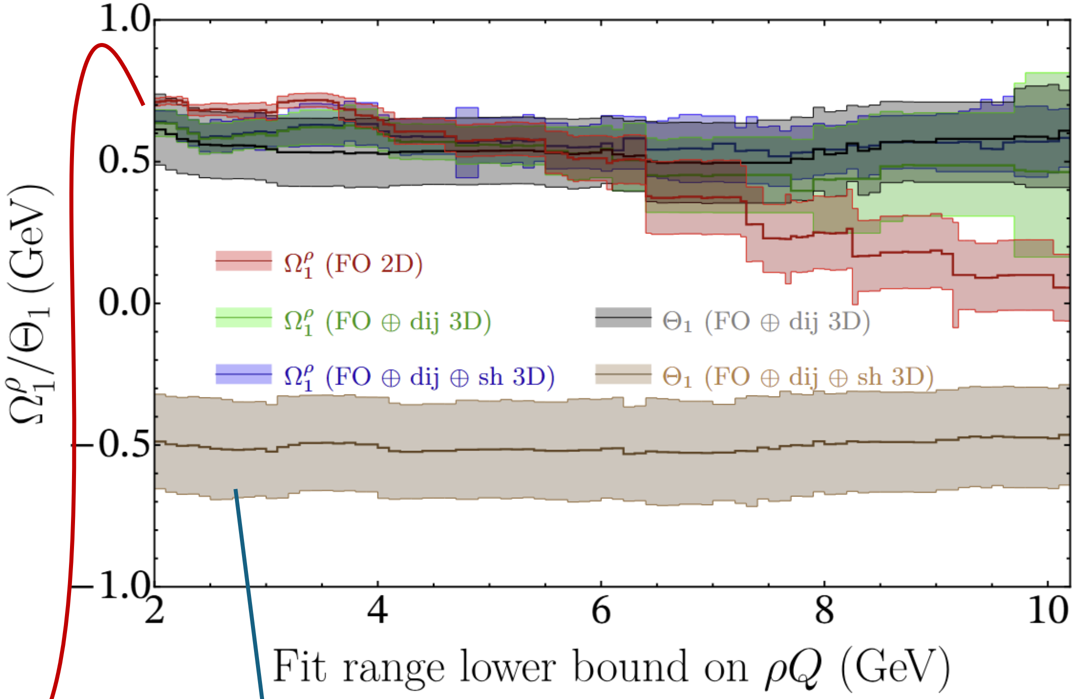
our fit range

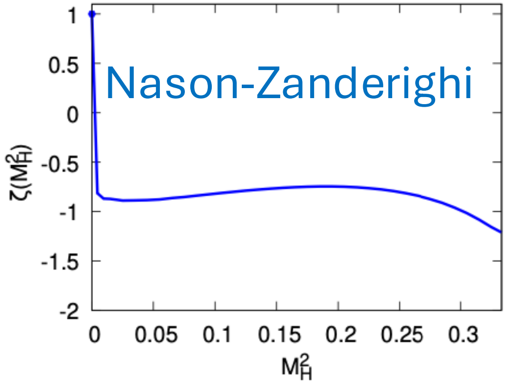


Shoulder and tail power corrections

consistent results with increasing theoretical sophistication







- Using only FO, data prefers positive shift always
  - Incosistent with Nason-Zanderighi negative shift

With Sudakov shoulder resummation data prefers right shift in peak and left shift in tail!

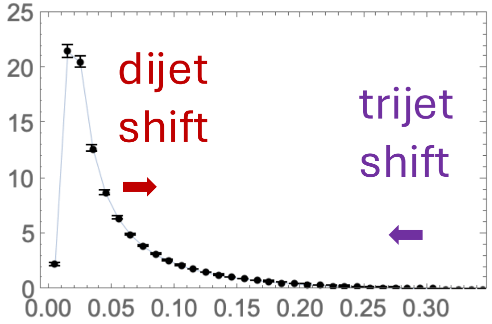
Conclusion

1. HJM **is finally consistent** with other event shapes

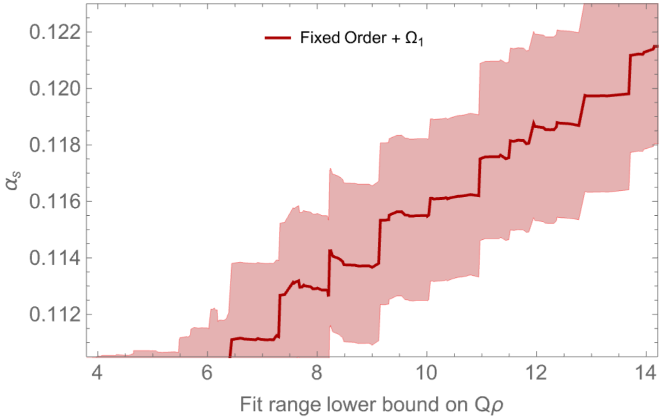
HJM $\alpha_s(m_Z) = 0.1145 \pm 0.0020$

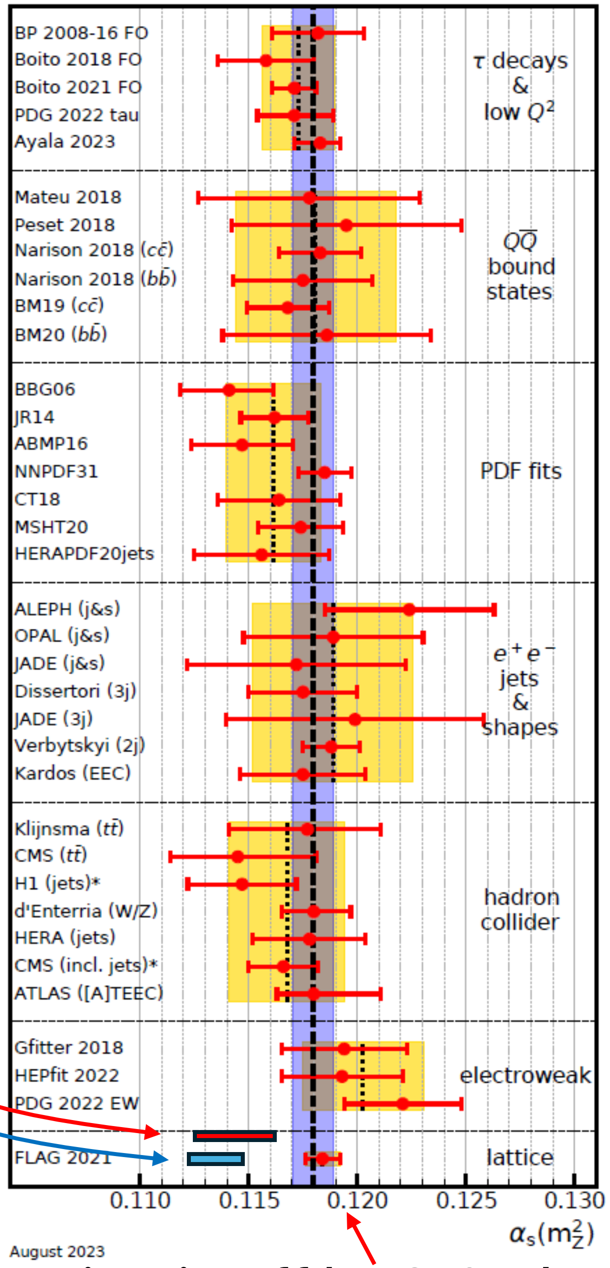
Thrust $\alpha_s(m_Z) = 0.1136 \pm 0.0012$

2. Evidence for **negative power correction in the tail**
   - Only if shoulder resummation included (can't see at fixed order)
   - Dijet power correction still positive



3. There is **no fit range** where **fixed order** can be used

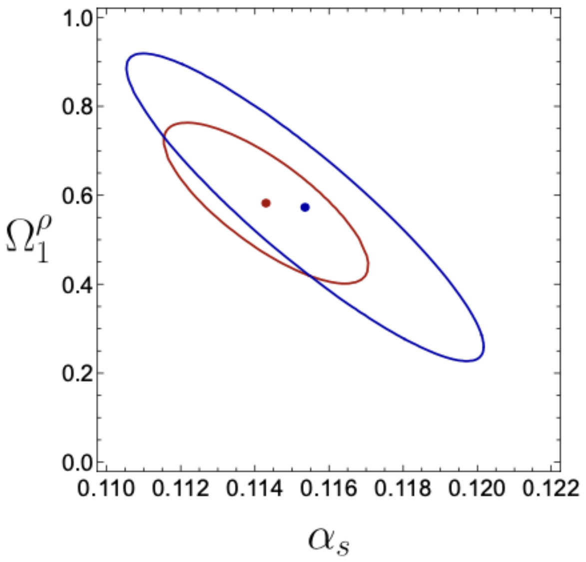




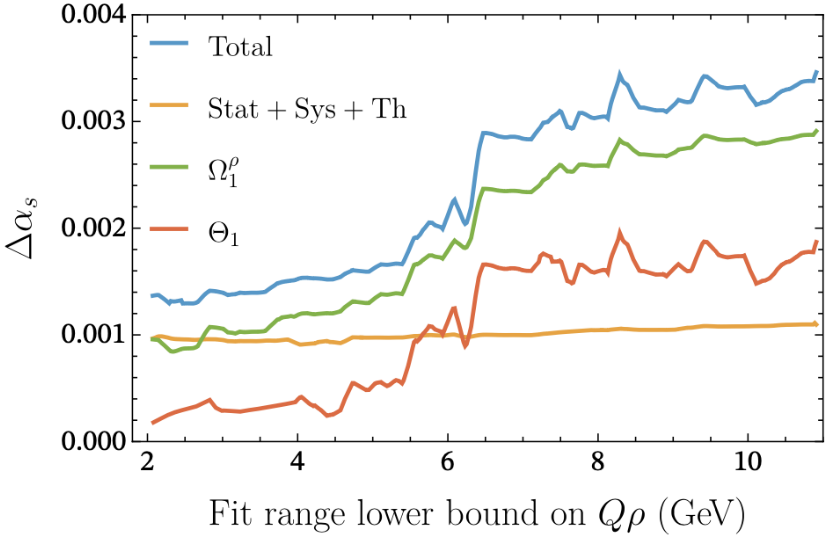
Lattice is off by 2.6 σ!

What are they doing wrong?

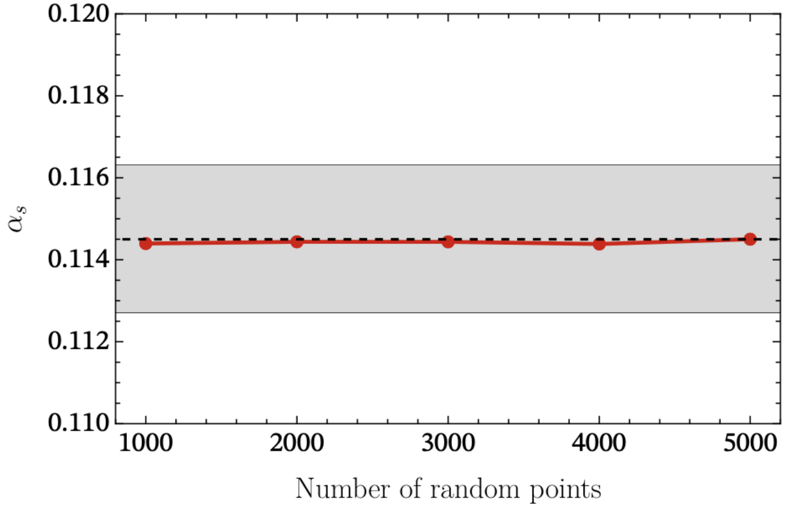
Backup



Correlation between α and Ω



Contributions to uncertainty as a function of the lower cut



Sensitivity to number of points in random scan

Hadron mass effects

$$\tau = 1 - \max_{\vec{n}} \frac{\sum |\vec{p}_j \cdot \vec{n}|}{\sum |\vec{p}_j|}$$

thrust depends only on particles' 3-momenta

$$\rho = \max\left(\frac{(P_\mu^+)^2}{Q^2}, \frac{(P_\mu^-)^2}{Q^2}\right) \qquad P_\mu^\pm = \sum_{\pm\vec{p}_j\cdot\vec{n}>0} p_j^\mu$$

HJM also depends on particles' energies

[Salam and Wicke 2001]

- Theory assumes particles are massless
- Massless -> massive depends on mass scheme

$$E(1,\vec{n}) \to \left(\sqrt{m^2+E^2\vec{n}^2}, E\vec{n}\right) \qquad E(1,\vec{n}) \to \left(E, \sqrt{E^2-m^2}\vec{n}\right)$$

"p" scheme — "E" scheme

[Mateu et al 2012]

- Scheme absorbed into NP parameter Λ
- Can no longer expect $\Lambda_\tau = 2\,\Lambda_\rho$
- Should be able to fit independently

Can use MC to correct data and redo fits

- No noticeable difference
- α_s still comes out low